SOLUTIONS

PROGRESS TEST-2

CD-1802

JEE MAIN PATTERN

Test Date: 29-07-2017

Mentors® Eduserv™

Corporate Office: Paruslok, Boring Road Crossing, Patna-01
Kankarbagh Office: A-10, 1st Floor, Patrakar Nagar, Patna-20
Bazar Samiti Office : Rainbow Tower, Sai Complex, Rampur Rd., Bazar Samiti Patna-06
Call : 9569668800 | 7544015993/4/6/7

PHYSICS

1. (C)
2. (A)
3. (B)
4. (C)
5. (A)
6. (B)
7. (A)

We know that $\frac{g_h}{g}=\left(\frac{R}{R+h}\right)^2$; But $g_h=\frac{g}{2}$

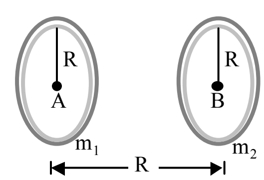
$\therefore \quad \frac{1}{2}=\left(\frac{R}{R+h}\right)^2$

or $\frac{R}{R+h}=\frac{1}{\sqrt{2}}$ or $\frac{R+h}{R}=\sqrt{2}$ or $\frac{h}{R}=\sqrt{2}-1=0.414$

$h=0.414\times R=0.414\times 6400\,\text{km}$ or h = 2649.6 km

$\therefore$ At a height of 2649.6 km from the Earth's surface, the acceleration due to gravity will be half its value on the surface.

8. **(B)**



$$V_A=\begin{pmatrix}\text{Potential at}\\ \text{A due to A}\end{pmatrix}+\begin{pmatrix}\text{Potential at}\\ \text{A due to B}\end{pmatrix}$$

$$\Rightarrow \quad V_A=-\frac{Gm_1}{R}-\frac{Gm_2}{\sqrt{2}R}\text{ and}$$

$$V_B=\begin{pmatrix}\text{Potential at}\\ \text{B due to A}\end{pmatrix}+\begin{pmatrix}\text{Potential at}\\ \text{B due to B}\end{pmatrix}$$

$$\Rightarrow \quad V_B=-\frac{Gm_2}{R}-\frac{Gm_1}{\sqrt{2}R}$$

Since $W_{A\to B}=m(V_B-V_A)$

$$\Rightarrow \quad W_{A\to B}=\frac{Gm(m_1-m_2)(\sqrt{2}+1)}{\sqrt{2}R}.$$

9. (C)

We know that

$$T^2 \propto R^3 \qquad \text{or} \qquad (T_2/T_1) = (R_2/R_1)^{3/2}$$

$$\text{or} \qquad \frac{T_2}{T_1} = \left(\frac{6400}{36000}\right)^{3/2}$$

$$\text{or} \qquad T_2 = \left(\frac{6400}{36000}\right)^{3/2} \times 24 \approx 2\,\text{hr.}$$

10. (C)

Total energy = kinetic energy + Potential energy

$$E_0 = \frac{1}{2}mv^2 - \frac{GMm}{r} \qquad \text{...(i)}$$

Further, $\frac{mv^2}{r} = \frac{GMm}{r^2}$

or $\frac{1}{2}mv^2 = \frac{GMm}{2r}$ ...(ii)

Substiuting the value of $\frac{1}{2}mv^2$ in equation (i) from equation (ii), we get

$$E_0 = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$$

Therefore, $P.E. = -\frac{GMm}{r} = 2E_0$.

11. (C)

12. (C)

$$E = \frac{V}{d} = \frac{5\times10^3}{10\times10^{-3}} = 5\times10^5\ \text{V/m}$$

13. (A)

When one plate is fixed, the other is attracted towards the first with a force

$$F = \frac{q^2}{2A\varepsilon_0} = \text{constant}$$

Hence, an external force of same magnitude will have to be applied in opposite direction to increase the separation between the plates.

$$\therefore \quad W = F(2d - d) = \frac{q^2 d}{2A\varepsilon_0}$$

14. (B)

$$\frac{q_1}{C_1} = \frac{q_2}{C_2} \;;\; q_1 + q_2 = 2Q_0$$

$$C_1 = \frac{\varepsilon_0 A}{d_0 + vt};\; C_2 = \frac{\varepsilon_0 A}{d_0 - vt}$$

$$\frac{q_1}{q_2} = \frac{d_0 - vt}{d_0 + vt}$$

$$q_2\left(\frac{d_0 - vt}{d_0 + vt}\right) + q_2 = 2Q_0$$

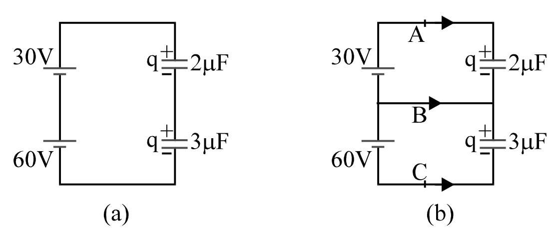
$$q_2\left[\frac{2d_0}{d_0 + vt}\right] = 2Q_0$$

$$q_2 = \frac{2Q_0}{2d_0}(d_0 + vt)$$

$$I = \frac{dq_2}{dt} = \frac{Q_0 v}{d_0} = 20 \text{ amp}$$

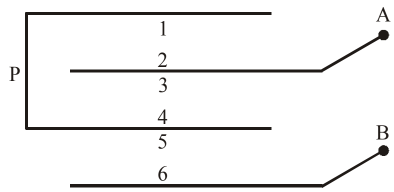
15. (A)

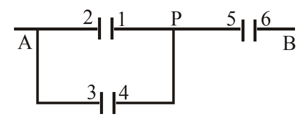
Let us draw two figures and find the charge on both the capacitors before closing the switch and after closing the switch.



16. (D)

Let us call the isolated plate as P. A capacitor is formed by a pair of parallel plates facing each other. Hence we have three capacitor formed by the pairs (1, 2), (3, 4) and (5, 6). The surface 2 and 3 are at same potential as that of A. The arrangement can be redrawn as a network of three capacitors.





$$C_{AB} = \frac{2C.C}{2C + C} = \frac{2C}{3}$$

$$= \frac{2}{3}\frac{\varepsilon_0 A}{d}$$

17. **(D)** **18.** **(B)** **19.** **(A)** **20.** **(C)**

21. **(B)** **22.** **(D)** **23.** **(B)** **24.** **(C)**

25. **(A)** **26.** **(B)** **27.** **(C)** **28.** **(D)**

29. **(B)** **30.** **(A)**

CHEMISTRY

31. **(A)**

$$P_{H_2} = K_H \cdot \frac{n_{H_2}(\text{mili mole})}{n \cdot H_2O(\text{mili mole})}$$

$$0.987 = 76480 \times \frac{n_{n_2}(\text{mili mole})}{55.55 \times 10^3}$$

$$n_{n_2} = \frac{55.55 \times 987}{76480}$$

= 0.716

32. **(A)**

$$P = Ae^{\frac{-\Delta H}{RT}}$$

33. **(A)**

$$\text{rate} = \frac{-d[N_2O_5]}{dt} = \frac{1}{2}\frac{d[NO_2]}{dt} = \frac{2d[O_2]}{dt} = k[N_2O_5]$$

So, $k_1 = k$; $k_2 = 2k$; $k_3 = \frac{k}{2}$

So, $2k_1 = k_2 = 4k_3 = 2k$

34. (A)

35. (A)

$$P_{CO_2} = K_H \times \frac{n_{CO_2}}{n_{H_2O}}$$

$$3.2\times10^5 = 1.6\times10^8 \times \frac{w/44}{500/18}$$

$$w = 3.2\times10^5 \times \frac{500}{18}\times\frac{44}{1.6\times10^8}$$ = 2.44 g

36. (D)

37. (C)

For 1st reaction

Ea_f = 800 Cal/mol $\quad k_f = A_1 e^{\frac{-800}{RT}}$ ....(i)

Ea_r = 200 Cal/mol

A_1

for 2nd reaction

$Ea_f' = 200$ Cal / mol $\quad k_f' = A_2 e^{\frac{-200}{RT}}$ ....(ii)

Ea_r = 200 Cal/mol

A_2

$$\frac{k_f}{k_f'} = \frac{A_1}{A_2} e^{\left(\frac{200-800}{RT}\right)}$$ (depends on T)

for 1st reaction,

$$k_{eq} = \frac{k_f}{k_r} = \frac{A\,e^{-\frac{800}{RT}}}{A_1'\,e^{-\frac{200}{RT}}} = A\,e^{\frac{-600}{RT}}$$

for 2nd reaction,

$$k_{eq}' = \frac{k_f'}{k_r'} = A' e^{\frac{600}{RT}}$$

So, $k_{eq} \times k_{eq}'$ = AA' e^o = const. (independent of T)

at 300 k

for 1st reaction,

$k_{eq} = Ae^{\frac{-600}{2\times300}} = Ae^{-1}$

for 2nd reaction,

$k_{eq} = A'e^{\frac{-600}{2\times300}} = A'e$

[A is not same for forward reverse reaction]

38. (B)

39. (B)

$\frac{1}{2}A \longrightarrow 2B$

$\text{rate} = \frac{-1}{½}\frac{d[A]}{dt} = \frac{1}{2}\frac{d[B]}{dt}$

or, $\frac{-d[A]}{dt} = \frac{1}{4}\frac{d[B]}{dt}$

40. (A)

rate = k [R]

$= 4\times10^{-3} \times 0.02$

$= 8 \times 10^{-5}\ Ms^{-1}$

41. (A) **42. (C)** **43. (C)** **44. (C)**

45. (A) **46. (B)** **47. (B)** **48. (C)**

49. (B) **50. (C)**

51. (B)

$O^{2-} > Mg^{2+} > Al^{3+}$; (Z/e) ratio.

52. (B)

I.E. of Mg > I.E. of Al (Due to electronic configuration)

53. (C)

$CsBr_3$ exist as $Cs^+\ Br_3^-$, due to lattice energy effect (large cations stabilises by large anion)

54. (C)

55. (D)

56. (C)

57. (A)

58. (C)

59. (D)

60. (D)

MATHEMATICS

61. (B)

$$\sin^3 x \cdot \sin 3x = \frac{1}{2}\sin^2 x(\cos 2x - \cos 4x)$$

$$= \frac{1}{4}(1 - \cos 2x)(\cos 2x - \cos 4x)$$

$$= \frac{1}{4}(\cos 2x - \cos 4x - \cos^2 2x + \cos 2x.\cos 4x)$$

$$= \frac{1}{4}\left(\cos 2x - \cos 4x - \frac{1+\cos 4x}{2} + \frac{1}{2}(\cos 6x + \cos 2x)\right)$$

$$= \frac{1}{4}\left(\frac{3}{2}\cos 2x - \frac{3}{2}\cos 4x + \frac{1}{2}\cos 6x - \frac{1}{2}\right) = -\frac{1}{8} + \frac{3}{8}\cos 2x - \frac{3}{8}\cos 4x + \frac{1}{8}\cos 6x$$

$\Rightarrow \quad n = 6$

62. (B)

use $x^2 - 5x + 7 < 1$ and $x^2 - 5x + 7 > 0$

63. (A)

For y to be defined, we must have

(a) $\log_{10}\left(\frac{5x - x^2}{4}\right) \ge 0 \;\Rightarrow\; \frac{5x - x^2}{4} \ge 10^0$

$\Rightarrow\; 5x - x^2 \ge 4 \;\Rightarrow\; x^2 - 5x + 4 \le 0 \;\Rightarrow\; (x - 1)(x - 4) \le 0 \;\Rightarrow\; 1 \le x \le 4$

(b) $\frac{5x - x^2}{4} > 0 \;\Rightarrow\; 5x - x^2 > 0 \Rightarrow\; x(x - 5) < 0 \Rightarrow\; 0 < x < 5$

From (a) and (b), we get the domain of f = $[1, 4] \cap (0, 5) = [1, 4]$

64. (B)

$\sqrt{9 - x^2}$ is defined for

$9 - x^2 \ge 0 \Rightarrow (3 - x)(3 + x) \ge 0$

$\Rightarrow (x - 3)(x + 3) \le 0$

$\Rightarrow -3 \le x \le 3$ ... (1)

$\sin^{-1}(3 - x)$ is defined for

$-1 \le 3 - x \le 1 \Rightarrow -4 \le -x \le -2 \Rightarrow 2 \le x \le 4$ ... (2)

Also, $\sin^{-1}(3 - x) = 0 \Rightarrow 3 - x = 0$ or $x = 3$ ... (3)

From (1), (2) and (3), we get the domain of f :

$([-3, 3] \cap [2, 4]) \setminus \{3\} = [2, 3).$

65. (D)

$$|4-3x| \le \frac{1}{2} \Rightarrow -\frac{1}{2} \le 4-3x \le \frac{1}{2}$$

$$4-3x \le \frac{1}{2} \text{ and } 4-3x \ge -\frac{1}{2}$$

$$\frac{7}{2} - 3x \le 0;\ \frac{9}{2} - 3x \ge 0$$

$$x \ge \frac{7}{6};\ x \le \frac{3}{2}$$

$$x \in \left[\frac{7}{6}, \frac{3}{2}\right]$$

$\therefore$ Option (D) is correct.

66. (C)

67. (C)

68. (A)

$$\lim_{x \to 2^-} \frac{\cos(2x-4) - 33}{2} = -16$$

$$\lim_{x \to 2^-} \frac{x^2 |4x-8|}{x-2} = -16$$

$\therefore$ By sandwich theorem ; $\lim_{x \to 2^-} f(x) = -16$

69. (C)

70. (C)

$$f(x) = \begin{cases} -1 & ; \quad -1 < x < 1 \\ 0 & ; \quad x = 1, -1 \\ 1 & ; \quad |x| > 1 \end{cases}$$

71. (B)

$3 = \lim_{x\to 0}(1+a\sin x)^{\operatorname{cosec} x}$ $[1^{\infty}\text{ form}] \Rightarrow e^{\lim_{x\to 0} a\sin x.\operatorname{cosec} x} = e^a$

$\therefore$ $e^a = 3 \Rightarrow a = \log_e 3 = \ln 3$.

Hence (B) is the correct answer.

72. (D)

Since f(x) is an odd function,

$\left[\frac{x^2}{a}\right] = 0$ for all $x \in [-10, 10] \Rightarrow 0 \le \frac{x^2}{a} < 1$ for all $x \in [-10, 10]$

$\Rightarrow$ a > 100 Hence, (D) is the correct answer

73. (A)

$$\tan^{-1}\frac{1}{\sqrt{2}} - \tan^{-1}\frac{\sqrt{\left(\sqrt{3}-\sqrt{2}\right)^2}}{1+\sqrt{3}.\sqrt{2}} = \tan^{-1}\frac{1}{\sqrt{2}} - \tan^{-1}\sqrt{3} + \tan^{-1}\sqrt{2}$$

$$= \cot^{-1}\sqrt{2} + \tan^{-1}\sqrt{2} - \tan^{-1}\sqrt{3} \qquad = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$$

74. (D)

f is not one-one as f(0) = 0 and f(–1) = 0. f is also not onto as for y = 1 there is no $x \in R$ such that f(x) = 1. If there is such an $x \in R$, then $e^{|x|} - e^{-x} = e^x + e^{-x}$ .Clearly $x \neq 0$.For x > 0, this equation gives $e^{-x} = 0$ which is not possible and for x < 0, $\frac{e^{2x}+1}{e^x} = 0$, which is also not possible.

Hence (D) is the correct answer.

75. (D)

$$\cos\left(\tan^{-1}x\right) = x$$

$$\Rightarrow \frac{1}{\sqrt{1+x^2}} = x$$

$$\Rightarrow x^2 = \frac{\sqrt{5}-1}{2} \quad \Rightarrow \frac{x^2}{2} = \frac{\sqrt{5}-1}{4} = \sin\frac{\pi}{10}.$$

76. (C)

$2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Range of right hand side is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow -\frac{\pi}{2} \le 2\sin^{-1}x \le \frac{\pi}{2} \Rightarrow -\frac{\pi}{4} \le \sin^{-1}x \le \frac{\pi}{4} \qquad \Rightarrow x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$.

77. (C)

We have, $\sin^{-1}x > \cos^{-1}x$

$\Rightarrow \sin^{-1}x > \frac{\pi}{2} - \sin^{-1}x$

$2\sin^{-1}x > \frac{\pi}{2} \Rightarrow \sin^{-1}x > \frac{\pi}{4}$.

$\Rightarrow \sin(\sin^{-1}x) > \sin\frac{\pi}{4} \quad \Rightarrow x > \frac{1}{\sqrt{2}} \Rightarrow x \in \left(\frac{1}{\sqrt{2}}, 1\right] \quad$ since $-1 \le x \le 1$

78. (C)

Sine of integral multiple of $\pi = 0$

79. (B)

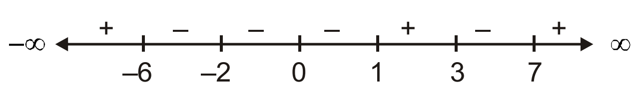
$\frac{|1+\{x\}|}{1+\{x\}} = 1$

80. (C)

$x^{\log_x a \times \log_a y \times \log_y z} = x^{\frac{\log a}{\log x}\cdot\frac{\log y}{\log a}\cdot\frac{\log z}{\log y}} = x^{\log_x z} = z$

81. (A)

$(x-1)^3(x+2)^4(x-3)^5(x+6) \ge 0$



$\therefore x \in (-\infty, -6] \cup [1, 3] \cup (7, \infty) \cup \{-2\}$

82. (A)

$$f(-x) = \frac{\cos(-x)}{\left[-\frac{x}{\pi}\right]+\frac{1}{2}} = \frac{\cos x}{-\left[\frac{x}{\pi}\right]-1+\frac{1}{2}} \qquad \left(\text{as } x \neq n\pi \Rightarrow \frac{x}{\pi} \notin I, \text{ so as} \left[-\frac{x}{\pi}\right] = -\left[\frac{x}{\pi}\right]-1\right)$$

$$= -\frac{\cos x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} = -f(x) \Rightarrow f(x) \text{ is an odd function.}$$

83. (B)

$$f(x) = \sqrt{3}\sin x - \cos x + 2 = 2\sin\left(x - \frac{\pi}{6}\right) + 2$$

Since f(x) is one–one and onto, f is invertible.

Now $fof^{-1}(x) = x \Rightarrow 2\sin\left(f^{-1}(x) - \frac{\pi}{6}\right) + 2 = x$

$$\Rightarrow \sin\left(f^{-1}(x) - \frac{\pi}{6}\right) = \frac{x}{2} - 1 \Rightarrow f^{-1}(x) = \sin^{-1}\left(\frac{x}{2} - 1\right) + \frac{\pi}{6}$$

Because $\left|\frac{x}{2} - 1\right| \leq 1$ for all $x \in [0, 4]$

84. (A)

$3x^2 - 10x + 3 = 0 \Rightarrow x = 3, 1/3$

or, $|x - 5| = 1 \Rightarrow x = 6, 4.$

85. (D)

$$\tan\left(\frac{2\pi}{5} - \frac{\pi}{15}\right) = \frac{\tan\frac{2\pi}{5} - \tan\frac{\pi}{15}}{1 + \tan\frac{2\pi}{5}\cdot\tan\frac{\pi}{15}}$$

$$\sqrt{3}\left(1 + \tan\frac{2\pi}{5}\cdot\tan\frac{\pi}{15}\right) = \tan\frac{2\pi}{5} - \tan\frac{\pi}{15}$$

86. (B)

We have,

$$\cos\theta \cos 2\theta \cos 2^2\theta \ldots \cos 2^{n-1}\theta = \frac{\sin 2^n\theta}{2^n \sin\theta} = \frac{\sin(\pi - \theta)}{2^n \sin\theta} \qquad [\because 2^n\theta = \pi - \theta]$$

$$= \frac{1}{2^n}$$

87. (B)

Clearly, $f(x) \geq 0$ for any $x \in R$. Moreover, $(x^2 - 1)^2 = x^4 - 2x^2 + 1 \geq 0$ for any $x \in R$,

so $\frac{x^2}{x^4+1} \leq 1/2$. Hence the range of f is [0, 1/2].

88. (A)

$\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$

squaring and adding, we get

$\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta + \cos^2\alpha + \cos^2\beta + 2\cos\alpha . \cos\beta$

$= \left(-\frac{21}{65}\right)^2 + \left(-\frac{27}{65}\right)^2$

$\Rightarrow 2 + 2\cos(\alpha - \beta) = \frac{1170}{4225}$

$\Rightarrow \cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{1170}{4\times 4225} = \frac{9}{130}$

$\Rightarrow \cos\left(\frac{\alpha-\beta}{2}\right) = \frac{-3}{\sqrt{130}}$ $\left(\because \pi < \alpha - \beta < 3\pi \Rightarrow \frac{\pi}{2} < \left(\frac{\alpha-\beta}{2}\right) < \frac{3\pi}{2}\right)$

89. (A)

tan 50 – tan 40 = k tan 10

$\frac{\sin 50}{\cos 50} - \frac{\sin 40}{\cos 40} = k\tan 10$

$\Rightarrow \frac{\sin 50\cos 40 - \sin 40\cos 50}{\cos 50\cos 40} = k\tan 10 \Rightarrow \frac{\sin 10}{\cos 50\cos 40} = \frac{k\sin 10}{\cos 10}$

$k = \frac{\cos 10}{\cos 50\sin 50} = \frac{2\cos 10}{\sin 100} = 2$

90. (C)

$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$

$\therefore \cos\frac{5\pi}{8} = \cos\left(\pi - \frac{3\pi}{8}\right) = -\cos\frac{3\pi}{8}$

$$\cos\frac{7\pi}{8}=\cos\left(\pi-\frac{\pi}{8}\right)=-\cos\frac{\pi}{8}$$

$$\therefore\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right)$$

$$=\left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)=\left(\sin^2\frac{\pi}{8}\right)\left(\sin^2\frac{3\pi}{8}\right)$$

$$=\frac{\left(1-\cos\frac{\pi}{4}\right)\left(1-\cos\frac{3\pi}{4}\right)}{4}=\frac{\left(1-\frac{1}{\sqrt{2}}\right)\left(1+\frac{1}{\sqrt{2}}\right)}{4}=\frac{1-\frac{1}{2}}{4}=\frac{1}{8}$$