SOLUTIONS

PROGRESS TEST-5

CD-1801(α), CD-1801 (β)

CDK-1801 & CDS-1801

JEE MAIN PATTERN

Test Date: 09-09-2017

Mentors® Eduserv™

Corporate Office: Paruslok, Boring Road Crossing, Patna-01
Kankarbagh Office: A-10, 1st Floor, Patrakar Nagar, Patna-20
Bazar Samiti Office : Rainbow Tower, Sai Complex, Rampur Rd.,
Bazar Samiti Patna-06
Call : 9569668800 | 7544015993/4/6/7

PHYSICS

1. (B)

Direction of electric field or magnetic field may be parallel or antiparallel to the direction of velocity.

2. (A)

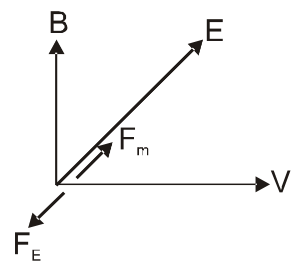
3. (B)

$F_m = F_E$

eVB = eE

$V = \frac{E}{B} = \frac{3.2\times10^4}{2\times10^{-3}}$

$= 16 \times 10^6$ m/s



4. (C)

The horizontal force on curve current carrying were will be cancelled. The only vertical upward (to wards Y-axis) will act.

Hence,

$F = i\ell B = i(2L)B$

5. (A)

$\sqrt{2}B_0 i\ell$

$\vec{F} = i\left(\overrightarrow{d\ell}\times\vec{B}\right)$

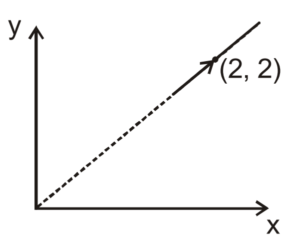
$= i\left\{1j\times B_0\ell\left(\hat{i}+\hat{j}+\hat{k}\right)\right\}$

$= i\left(-B_0\ell\hat{k}+B\ell\hat{i}\right)$

$|\vec{F}| = \sqrt{2}\, B_0\ell i$

6. (B)

Effective length will 2m along 45^o with x-axis.



$F = i(d\ell \times B)$

7. (D)

Due to symmetry of the circuit, field will be zero at centre.

8. (C)

$$F = \frac{mv}{dt}$$

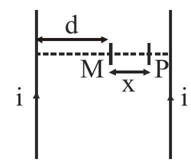
$$i\ell B = \frac{mv}{dt}$$

$$\frac{q}{dt}\ell B = \frac{mv}{dt}$$

$$v = \frac{q\ell B}{m}$$

9. (B)

The magnetic field due to two wires at P



$$B_1 = \frac{\mu_0 i}{2\pi(d+x)}$$

$$B_2 = \frac{\mu_0 i}{2\pi(d-x)}$$

Both the magnetic fields act in opposite direction.

$$\therefore \quad B = B_2 - B_1 = \frac{\mu_0 i}{2\pi}\left[\frac{1}{d-x} - \frac{1}{d+x}\right] = \frac{\mu_0 i}{2\pi}\left[\frac{d+x-d+x}{d^2-x^2}\right] = \frac{\mu_0 ix}{\pi(d^2-x^2)}.$$

10. (A)

magnetic field due to straight wire will be zero.

B_{net} = magnetic field due to curved were

$$= B_1 + B_2 + B_3$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{I}{r}\cdot\theta\otimes + \frac{\mu_0}{4\pi}\cdot\frac{I}{2r}\theta\odot + \frac{\mu_0}{4\pi}\cdot\frac{I}{3r}\otimes$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{I}{r}\cdot\theta\left(1 - \frac{1}{2} + \frac{1}{3}\right)$$

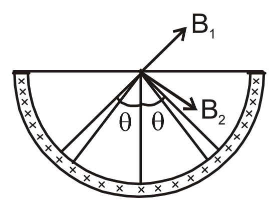
$$=\frac{\mu_0}{4\pi}\cdot\frac{I}{r}\cdot\theta\left(\frac{6-3+2}{6}\right)=\frac{5}{6}\times\frac{\mu_0}{4\pi}\frac{I}{r}\theta$$

$$B_{net}=\frac{5}{24}\frac{\mu_0}{\pi}\frac{I}{r}\theta$$

11. (C)

Due to symmetry

The vertical component



will cancelled out

$$B_{net}=B\,(\text{Horizontal})$$

$$dB_{net}=\frac{\mu_0}{4\pi}\cdot\frac{2(dI)\cos\theta}{R}$$

$$=\frac{\mu_0}{4\pi}\cdot\frac{2I}{\pi R}\cos\theta\,d\theta$$

$$B_{net}=\int dB_{net}=\frac{\mu_0}{4\pi^2}\cdot\frac{2I}{R}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta$$

$$=\frac{\mu_0}{4\pi^2}\cdot\frac{2I}{R}.2=\frac{\mu_0 I}{\pi^2 R}$$

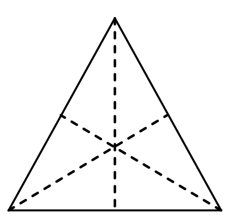
12. (C)

There will be no current in the circuit. Hence

$B_\rho = 0$

13. (A)

$$B=3\left[\frac{\mu_0 i}{4\pi r}(\sin 60^\circ+\sin 60^\circ)\right]$$



$$= \frac{9\mu_0 i}{2\pi a}$$

$\therefore$ **(A)**

14. (C)

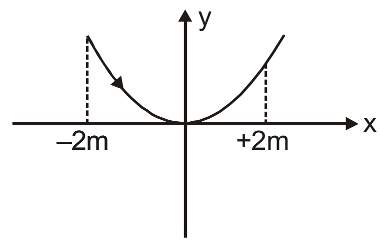
The effective $d\ell$ will be along +ve x-axis

$F = i\ell B$

$= 2 \times 4 \times 0.02$

0.16 upward

0.16j



$$a = \frac{F}{m} = \frac{0.16}{100} \times 1000 = 1.6J$$

15. (B)

$$R = \frac{mv}{qB} = \frac{\sqrt{2m.K.E.}}{qB}$$

$$R_p = \frac{\sqrt{2m_p \times K.E}}{qB} \quad R_\alpha = \frac{\sqrt{2.4mp.K.E}}{2qB}$$

$$R_D = \frac{\sqrt{2.2m_p.KE}}{qB}$$

$$R_p : R_D : R_\alpha = 1 : \sqrt{2} : 1$$

16. (C)

Sections *AB* and *DE* produce no field at *O*. Sections *BC* and *EF* produce equal fields at *O*,

$$B = \frac{\mu_0 I}{2\pi a}.$$

$\therefore$ **(C)**

17. (D)

The Magnitude of magnetic field due to circular loop at the center *C* is

$$B = \frac{\mu_0 I}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\therefore$ **(D)**

18. (A)

$\vec{F} = \vec{F}_e + \vec{F}_m$, $\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B}) = 0 \Rightarrow$ $B = \frac{E}{v}$ **=** 10^3 Wb/m^2

$\therefore$ **(A)**

19. (A)

Small circles ¾ th . Big circles ¼ th part produces effective magnetic fields

$$|\vec{B}_{net}| = \frac{3}{4} \times \frac{\mu_0 I}{2R} + \frac{1}{4}\frac{\mu_0 I}{2(2R)} = \frac{7\mu_0 I}{16R}$$

20. (B)

$$T = \frac{2\pi m}{qB}, \frac{T_\alpha}{T_p} = \frac{m_\alpha}{m_p}.\frac{q_p}{q_\alpha} = 2$$

21. (C)

$Q_1 + Q_2 = 80$

$$\frac{Q_1}{2} = \frac{Q_2}{3}$$

$3Q_1 = 2Q_2$

$$\frac{2Q_2}{3} + Q_2 = 80 \qquad Q_2 = \frac{80 \times 3}{5} = 48\mu C$$

22. (D)

Inital Energy stored $= \frac{1}{2}CV^2 = \frac{1}{2}2.V^2 = V^2$

Charge on capactor = 2V

When switch is turn to point (2).

$$\frac{Q_0 - Q}{2} = \frac{Q}{8}$$

$$\frac{2V - Q}{2} = \frac{Q}{8} \Rightarrow V = \frac{Q}{8} + \frac{Q}{2}$$

$$\frac{Q+4Q}{8}=V$$

$$Q=\frac{8V}{5}$$

now final Energy

$$\frac{1}{2}\frac{\left(Q_0-Q\right)^2}{2}+\frac{1}{2}\frac{Q^2}{8}$$

$$=\frac{1}{2}\frac{\left(2V-8V/5\right)^2}{2}+\frac{1}{2}\left(\frac{8V}{5}\right)^2\times\frac{1}{8}$$

$$\frac{1}{4}\times\left(\frac{2V}{5}\right)^2+\frac{1}{16}\times\left(\frac{8V}{5}\right)^2=\frac{1}{4}\times\frac{36}{25}V^2+\frac{1}{16}\times\frac{16}{25}V^2$$

$$=\frac{2}{5}V^2$$

$$=\frac{1}{4}\times\frac{4}{25}V^2+\frac{1}{16}\times\frac{64}{25}V^2=\left(\frac{1+4}{25}\right)V^2$$

$$=\frac{1}{5}V^2=0.2V^2$$

% of Energy less = $\frac{V^2-0.2V^2}{V^2}\times 100=80\%$

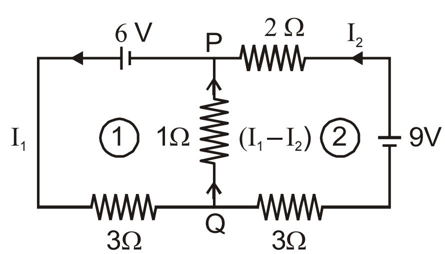
23. (C)

24. (C)

Here fig. R_1 and R_2 are same

Henec $P_2 \geq P_1 > P_3$

25. (B)



In loop 1

$3I_1 + 1(I_1 - I_2) = 6$

$4I_1 - I_2 = 6$ ....(i)

In loop 2

$3I_2 - 9 + 2I_2 - (I_1 - I_2) = 0$

$6I_2 - I_1 = 9$ ....(ii)

Solving eq^n (i) and (ii)

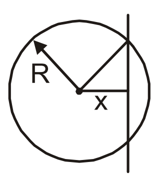
we get $(I_1 - I_2)$ = 0.13A from Q to P.

26. (B)

Potential difference between two pionts due to a infinitely long charge wire of charge per unit length λ is $\frac{\lambda}{2\pi\varepsilon_0}\ell n\left(\frac{r_2}{r_1}\right)$

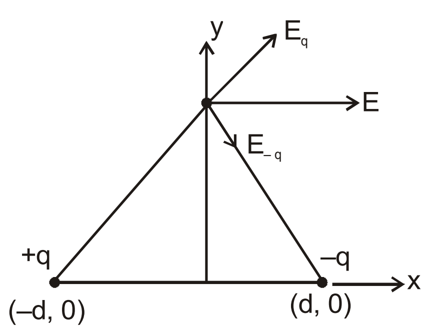
27. (D)

$$\phi = \frac{q_{inclosed}}{\varepsilon_0}$$



$$= \frac{\pi\left(R^2 - x^2\right)\sigma}{\varepsilon_0}$$

28. (C)



29. (A)

Charge distribution $\propto$ radius

q_{total} = 100 + 50 = 150 μC

$q_1' : q_2' = R_1 : R_2 = 1 : 2$

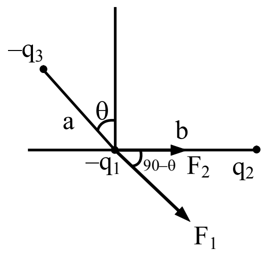
$$q_1' = \frac{1}{3} \times 150 = 50\,\mu C$$

$$q_2' = \frac{2}{3} \times 150 = 100\,\mu C$$

(50 μC shifted from small to big sphere)

30. (C)

x component of force on – q $\Rightarrow$

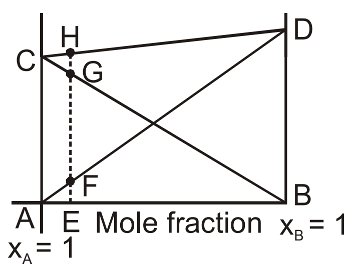


$F_1 \cos(90 - \theta) + F_2$

$= \dfrac{Kq_1q_3}{a^2} \sin\theta + \dfrac{Kq_1q_2}{b^2}$

CHEMISTRY

31. (D)



Plots AD shows vapour pressure of B containing A volatile component. Plots BC show vapour pressure of A containing B volatile component. Plot CD show vapour pressure of liquid solution containing A and B volatile components

$P = P_A + P_B$

$\therefore E_H = EF + EG$

32. (A)

Elevation in boiling point $\propto$ concentration of a solution. Thus, the order of concentration of solutions is I < II < III

33. (C)

$T_B^\circ > T_A^\circ$ [From diagram]

B $\rightarrow$ Residue

A $\rightarrow$ Distillate

34. (B)

The mixture will show positive deviation from Raoult's law hence boiling point of an azeotropic mixture of water-ethanol is less than that of both water and ethanol.

35. (D)

On changing external pressure, composition of azeotrope alters.

36. (B)

$\Delta G_1^\circ = -2\Delta G_2^\circ$

$E^\circ_{Cl^-|Cl_2} = -1.36\,V$

37. (C)

$\Delta G_1^\circ = -1 \times F(x) = -RT\ln K_1$

$\Delta G_2^\circ = -1 \times F(y) = -RT\ln K_2$

$\Delta G_2^\circ = 2\Delta G_1^\circ = -2RT\ln k_1$

$\therefore K_2 = K_1^2, x = y$

38. (B)

$$E = E^\circ - \frac{0.0591}{2}\log\frac{[Zn^{2+}]}{[Ag^+]^2}$$

y = C – mx

39. (A)

(A) True $[B]_t = k_I t$; $0.25 = \frac{1}{\sqrt{3}}t$; $t = 0.25\sqrt{3}$

$[D]_t = k_{II}t = \sqrt{3} \times 0.25 \times \sqrt{3} = 0.75M$

(B) False if [C] = [A] then at that time [B] < [D]

(C) False $t_{100\%} = \frac{a}{k}$ (for zero order)

$$\frac{(t_{100\%})_I}{(t_{100\%})} = \frac{a_I}{a_{II}} \cdot \frac{k_I}{k_{II}} = \frac{0.5}{1} \times \frac{\sqrt{3}}{1/\sqrt{3}} = \frac{3}{2}$$

(D) $[A]_t = [A]_0 - k_I t$ or $[A]_t = 0.5 - \frac{1}{\sqrt{3}}t$

$[C]_t = [C]_0 - k_{II} t$ or $[C]_t = 1 - \sqrt{3}t$

if $[A] = [C]_t$

i.e., $0.5 - \frac{1}{\sqrt{3}}t = 1 - \sqrt{3}t$ or $\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)t = 0.5$

$t = \frac{\sqrt{3}}{4}$ min.

40. (B)

$A + B \underset{\text{fast}}{\overset{\text{slow}}{\rightleftharpoons}} IAB$; So $E_{a(f)}$ is high and $E_{a(b)}$ is low. $k_1 << k_2$; So, E_a for this step is very high and for next step is low and overall reaction is exothermic.

41. (D)

$$CH_3-\overset{\overset{O}{||}}{C}-CH_2-\overset{\overset{O}{||}}{C}-CH_3 \rightleftharpoons CH_3-\overset{\overset{OH}{|}}{C}-CH-\overset{\overset{O}{||}}{C}-CH_3$$

(enolform)

(Due to presence of 2nd > C = O size of conjugation becomes larger. There is intramolecular H–bonding also in enolic form.

42. (B)

Due to +I effect of two CH_3 group attached with nitrogen atom its K_b will be high

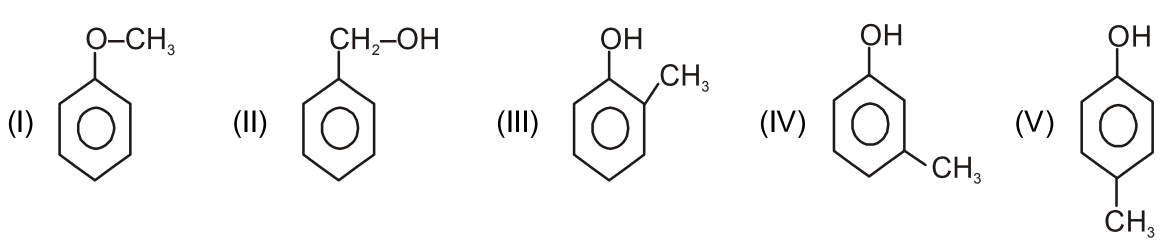
43. (B)

Where –Ve charge can be created by base attack and –Ve charge goes after resonace from these sites H atom can be replaced by D.

44. (C)

Due to strong –I effect of $\overset{\oplus}{N}H_3$ at C_2 position.

45. (A)



46. (D)

I and (III) satisfy conditions for showing geometrical isomerism.

47. (D)

Acid–Base reaction always shifts towards weak acid and weak base

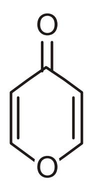
48. (D)

$$CH_3-\overset{\displaystyle O}{\overset{||}{C}}-O-\overset{2}{C}H_2-\overset{1}{C}OOH$$

(2–Ethynoyle ethanoic acid)

49. (C)

Compound (c) is Aromatic



(i) Cyclic (ii) Planar (iii) Cyclic delocalisation

(iv) $(4n+2)\pi = 6\pi$ n = 1

50. (B)

is most stable due to electron withdrawing effect of NO_2 group.

51. (C)

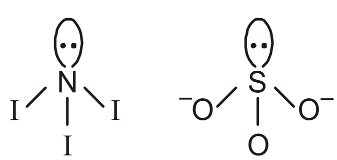
H_2O has maximum dielectric constant.

52. (D)

$H-C\equiv N: \;\rightarrow\; L.P. = 1, \sigma\text{-bonds} = 2, \pi\,\text{bonds} = 2$

$H-\overset{+}{N}\equiv\overset{-}{C}: \;\rightarrow\; L.P. = 2, \sigma\text{-bonds} = 2, \pi\,\text{bonds} = 2$

53. (B)



(sp^3) (sp^3)

54. (D)

$N_b = 4, N_a = 0$

$\therefore$ Bond order $= \frac{1}{2}[N_b - N_a] = 2$

55. (C)

Electronegativity of S < O < F.

56. (A)

$$\underset{\underset{sp^2}{\uparrow}}{AlH_3} \longrightarrow \underset{\underset{sp^3}{\uparrow}}{AlH_4^-}$$

57. (C)

F.C. = $V - \frac{s}{2} - u$

V - Valence electrons, s - shared electrons, u - unshared electron.

58. (D)

Cr has maximum oxidation state (+6) in K_2CrO_4 and thus has minimum radius.

59. (A)

Mn^{2+} ($3d^5$) is more stable than Mn^{3+} ($3d^4$).

60. (C)

Cs is highly electropositive element hence it forms Cs^+ ion.

$$CsI_3 \rightleftharpoons Cs^+ + I_3^-$$

MATHEMATICS

61. (C)

f(0) = 0

62. (A)

$$I = \int_{\sin^{-1}a}^{\cos^{-1}a} \left(\frac{[2x]}{[2x] + [\pi - 2x]} \right) dx$$

$$I = \int_{\sin^{-1}a}^{\cos^{-1}a} \left(\frac{[\pi - 2x]}{[\pi - 2x] + [2x]} \right) dx \quad \left(\int_a^b f(x)dx = \int_a^b f(a+b-x)dx \right)$$

$$2I = \int_{\sin^{-1}a}^{\cos^{-1}a} 1.dx$$

$$I = \frac{1}{2}(\cos^{-1} a - \sin^{-1} a) = \frac{\pi}{4} - \sin^{-1} a$$

63. **(D)**

Given equation is satisfied if cosx dx = d (f(x)) $\Rightarrow$ f(x) = sinx

64. **(D)**

$$\int (x^7 + x^5)\sqrt{2x^2+3}\,dx = \frac{1}{12}\int (12x^5 + 12x^3)\sqrt{2x^6+3x^4}\,dx$$

Put $2x^6 + 3x^4 = t$ $\quad \therefore (12x^5 + 12x^3)\,dx = dt$

$$= \frac{1}{12}\int \sqrt{t}\,dt$$

$$= \frac{x^6}{18}(2x^2+3)^{3/2} + C$$

65. **(A)**

$$\lim_{x\to 1^-} f(x) = 3 = \lim_{x\to 1^+} f(x) = f(1)$$

LHD at (x = 1) = $\lim_{x\to 1} \frac{f(x)-f(1)}{x-1} = 3\ell n3$

RHD at (x = 1) = –1.

66. **(A)**

Required sum = $\int_0^1 \frac{1+x}{1+x^2}dx$

$$= \left[\tan^{-1} x\right]_0^1 + \left[\frac{1}{2}\ell n(1+x^2)\right]_0^1 = \frac{\pi}{4} + \frac{1}{2}\ell n2$$

67. **(C)**

Put ln x = t

$$I = \int e^t\left(\frac{t-1}{t^2+1}\right)^2 dt = \int e^t\left(\frac{1}{t^2+1} - \frac{2t}{(t^2+1)^2}\right)dt \quad = \frac{e^t}{t^2+1} + c = \frac{x}{(\ln x)^2+1} + c$$

Hence (c) is the correct answer.

68. **(D)**

$$\lim_{x\to 0} \frac{x - e^x + 1 - (1-\cos 2x)}{x^2} = -\frac{1}{2} - 2 = -\frac{5}{2}.$$

$\therefore$ For continuity, f(0) = $-\frac{5}{2}$.

$\therefore \; [f(0)]\{f(0)\} = \frac{-3}{2}$

69. (A)

Substituting x = p^6, dx = $6p^5$ dp, we have

$$I = \int \frac{6p^5(p^6+p^4+p)}{p^6(1+p^2)}dp = \int \frac{6(p^5+p^3+1)}{(p^2+1)}dp = \int 6p^3 dp + \int \left(\frac{6}{p^2+1}\right)dp$$

$$= \frac{6p^4}{4} + 6\tan^{-1} p = \frac{3}{2}x^{2/3} + 6\tan^{-1}\left(x^{1/6}\right) + c$$

Hence (a) is the correct answer.

70. (A)

It is possibe only when f(x) is differentiable at x = 1 and $f'(1) \neq 0$.

$\Rightarrow \quad 1 + \frac{b^3 - b^2 + b - 1}{b^2 + 3b + 2} = -1 \quad \Rightarrow b^3 + b^2 + 7b + 3 = 0$

f(b) = $b^3 + b^2 + 7b + 3$

$\therefore \quad f'(b) = 3b^2 + 2b + 7 > 0 \;\; \forall b \in R$

$\Rightarrow$ f(b) = 0 will have exactly one real root.

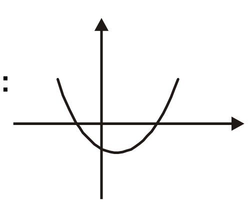
71. (D)

$f'(x) = 6x^2 - 6(a-3)x + 6a$

$f'(x) = 0$ must have two distinct real roots of which at least one is negative root.

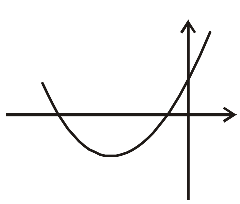
$y = f'(x)$:

case 1 :



$6a < 0 \Rightarrow a < 0$

Case 2 :



$$\left.\begin{array}{l} D>0 \Rightarrow a \in(-\infty, 1) \cup(9, \infty) \\ \text{and } -\dfrac{B}{2A}<0 \Rightarrow a \in(-\infty, 3) \\ \text{and } C \geq 0 \quad \Rightarrow a \in[0, \infty) \end{array}\right\} \Rightarrow a \in[0,1)$$

$a \in (-\infty, 1)$

72. **(D)**

$$\tan^{-1}\left(\frac{1}{2r^2}\right)=\tan^{-1}\left(\frac{2}{4r^2}\right)=\tan^{-1}\left(\frac{(2r+1)-(2r-1)}{1+(2r+1)(2r-1)}\right)=\tan^{-1}(2r+1)-\tan^{-1}(2r-1)$$

Thus,

$$\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{2r^2}\right)=\sum_{r=1}^{n}\left[\tan^{-1}(2r+1)-\tan^{-1}(2r-1)\right]=\tan^{-1}(2n+1)-\tan^{-1}(1)$$

$$=\tan^{-1}(2n+1)-\frac{\pi}{4}$$

$$\therefore \lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{2r^2}\right)=\lim_{n\to\infty}\left[\tan^{-1}(2n+1)-\frac{\pi}{4}\right]$$

$$= \tan^{-1}(\infty)-\frac{\pi}{4} = \frac{\pi}{2}-\frac{\pi}{4} = \frac{\pi}{4}.$$

73. **(D)**

74. **(A)**

$t = \sin^2 x$.

$$I=\frac{1}{2}\int e^t(2-t)\,dt=\frac{3}{2}e^t-t\frac{e^t}{2}+C=\frac{1}{2}e^{\sin^2 x}(3-\sin^2 x)+C$$

75. **(D)**

$f'(x)=3x^2-12x+9=3(x-1)(x-3)$

$f(-1) = -21$

$f(1) = -1$

$f(3) = -5$

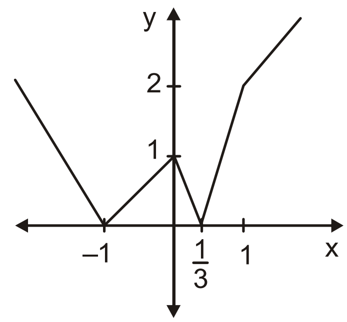
$f(5) = 15$

$\because \quad x \notin (-1, 5)$

$\Rightarrow$ Absolute maximum value of f(x) does not exist.

76. **(D)**

From graph the function has local minimum at x = –1, 1/3 and maximum at x = 0.



77. **(C)**

$$\int_0^1 \cos^{-1}\cos[x]dx + \int_1^2 \cos^{-1}(\cos[x])dx + \int_2^3 \cos^{-1}\cos[x]dx + \int_3^4 \cos^{-1}\cos[x]dx + \int_4^5 \cos^{-1}\cos[x]dx$$

$$= \int_0^1 \cos^{-1}(1)dx + \int_1^2 1.dx + \int_2^3 2dx + 3\int_3^4 dx + \int(2\pi - 4)dx = 0 + 1 + 2 + 3 + 2\pi - 4 = 2\pi + 2$$

78. **(B)**

The equation of the curve is $y - e^{xy} + x = 0$

$$\Rightarrow \frac{dy}{dx} - e^{xy}\left(y + x\frac{dy}{dx}\right) + 1 = 0 \Rightarrow \frac{dy}{dx}(1 - xe^{xy}) = ye^{xy} - 1 \Rightarrow \frac{dx}{dy} = \frac{1 - xe^{xy}}{ye^{xy} - 1}$$

Clearly, $\frac{dx}{dy} = 0$ at (1, 0). So, required point is (1, 0).

79. **(C)**

$$I = \int_0^{\pi/2} \frac{\cos\theta - \sin\theta}{(1+\cos\theta)(1+\sin\theta)}d\theta$$

$$= \int_0^{\pi/2} \frac{\sin\theta - \cos\theta}{(1+\sin\theta)(1+\cos\theta)}d\theta \qquad \text{[using property]} \quad \int_0^a f(x)dx = \int_0^a f(a-x)dx$$

$2I = 0 \Rightarrow I = 0$

80. **(D)**

$$g'(0^+) = \lim_{h\to 0} \frac{\cosh - 1}{h} = 0$$

$$g'(0^-) = \lim_{h\to 0} \frac{-h + b - 1}{-h}$$ and for existence of limit, b = 1

$\Rightarrow \quad g'(0^-) = 1.$

81. (A)

$$I = \int_0^{2\pi} \cos^{-1}(\sin x)\,dx = \int_0^{2\pi} \cos^{-1}(-\sin x)\,dx$$

$$= \int_0^{2\pi} (\pi - \cos^{-1}(\sin x))\,dx$$

$$2I = \int_0^{2\pi} \pi\,dx \Rightarrow I = \pi^2$$

82. (B)

Put $1 + x^2 = t^2$

83. (B)

$$f(x) = \begin{cases} x - 2k\pi; & 2k\pi - \dfrac{\pi}{2} \le x \le 2k\pi + \dfrac{\pi}{2} \\ (2k+1)\pi - x; & 2k\pi + \dfrac{\pi}{2} < x \le 2k\pi + \dfrac{3\pi}{2} \end{cases}$$

84. (C)

$$I = \int_{-\pi/2}^{\pi/2} [\tan x]\,dx = \int_{-\pi/2}^{\pi/2} [-\tan x]\,dx = \int_{-\pi/2}^{\pi/2} (-1 - [\tan x])\,dx \quad (\because [-x] = -1 - [x] \ \forall\, x \notin I)$$

$$= -\pi - I \quad \Rightarrow \quad I = -\frac{\pi}{2}$$

85. (B)

$$f'(x) = 8x^3 - 9(a-3)x^2 + 12ax + a$$

$$f''(x) = 24x^2 - 18(a-3)x + 12a$$

$$= 6 \cdot \{4x^2 - 3(a-3)x + 2a\}$$

$f''(x) = 0$ has roots of opposite signs $\Rightarrow \quad a \in (-\infty, 0)$

86. (A)

Let $I = \int \dfrac{3 + 2\cos x}{(2 + 3\cos x)^2}\,dx$

Multiplying N^r. & D^r. by cosec^{2}x, we get

$$\Rightarrow \quad I = \int \frac{(3\,\text{cosec}^2 x + 2\cot x\,\text{cosec}\,x)}{(2\,\text{cosec}\,x + 3\cot x)^2}\,dx$$

$$=-\int \frac{-3\operatorname{cosec}^2 x - 2\cot x \operatorname{cosec} x}{(2\operatorname{cosec} x + 3\cot x)^2} dx = \frac{1}{2\operatorname{cosec} x + 3\cot x} = \left(\frac{\sin x}{2+3\cos x}\right) + c$$

Hence (a) is the correct answer.

87. (B)

It is given that the tangent at each point of the curve y = $\frac{2}{3}x^3 - 2ax^2 + 2x + 5$ makes an acute angle with the positive direction of x-axis.

$\therefore \quad \frac{dy}{dx} \geq 0$ for all x

$\Rightarrow \quad 2x^2 - 4ax + 2 \geq 0$ for all x

$\Rightarrow \quad x^2 - 2ax + 1 \geq 0$ for all x

$\Rightarrow \quad 4a^2 - 4 \leq 0 \quad \Rightarrow \quad a^2 - 1 \leq 0 \quad \Rightarrow \quad -1 \leq a \leq 1$

88. (A)

$I = \int \left(x + \frac{1}{x}\right)^{n+5} \left(\frac{x^2 - 1}{x^2}\right) dx$, put $x + \frac{1}{x} = p$ then, $\left(1 - \frac{1}{x^2}\right) dx = dp$

$$\Rightarrow \int p^{n+5}\, dp = \frac{p^{n+6}}{n+6} + c = \frac{\left(x + \frac{1}{x}\right)^{n+6}}{n+6} + c.$$

Hence (a) is the correct answer.

89. (A)

We have, $2x^2 + y^2 = 12$ ... (i)

Differentiating w.r.t. x, we get

$$4x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2x}{y} \Rightarrow \left(\frac{dy}{dx}\right)_{(2,2)} = -2$$

The equation of the normal at (2, 2) is

$$y - 2 = \frac{1}{2}(x - 2) \Rightarrow x - 2y + 2 = 0 \qquad \text{... (ii)}$$

Solving (i) and (ii), we obtain that the coordinates of their points of intersection are (2, 2) and (–22/9, –2/9).

Hence, the normal to the curve at (2, 2) cuts it again at (–22/9, –2/9).

90. **(D)**

$$I = \int_0^{\pi/2} \sin x \sin 2x \sin 3x \, dx \qquad \text{......(i)}$$

Using $\int_0^a f(x) = \int_0^a (a-x)\, dx$

$$I = -\int_0^{\pi/2} \cos x \sin 2x \cos 3x \, dx \qquad \text{......(ii)}$$

Adding (i) and (ii) we get,

$$2I = \int_0^{\pi/2} \sin 2x(\sin x \sin 3x - \cos x \cos 3x)dx$$

$$= -\int_0^{\pi/2} \sin 2x \cos 4x \, dx = -\frac{1}{2}\int_0^{\pi/2} (\sin 6x - \sin 2x)dx$$

$$= \left(\frac{\cos 6x}{12} - \frac{\cos 2x}{4}\right)_0^{\pi/2} = \left(-\frac{1}{12}+\frac{1}{4}\right)-\left(\frac{1}{12}-\frac{1}{4}\right) = -\frac{1}{6}+\frac{1}{2} = \frac{1}{3} \Rightarrow I = \frac{1}{6}$$