SOLUTIONS

WEEKLY TEST-7

RBA

(JEE ADVANCED PATTERN)

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Corporate Office: Paruslok, Boring Road Crossing, Patna-01
Kankarbagh Office: A-10, 1st Floor, Patrakar Nagar, Patna-20
Bazar Samiti Office : Rainbow Tower, Sai Complex, Rampur Rd., Bazar Samiti Patna-06
Call : 9569668800 | 7544015993/4/6/7

1. (3)

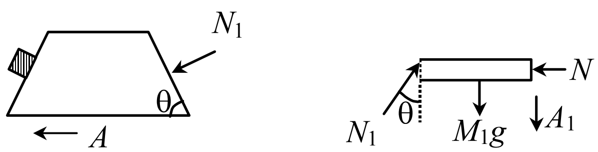
Let M_1 be the mass of the rod.

A … (i)

$N_1 \sin\theta = (M + M)A$ … (ii)

$A = g\tan\theta$ … (iii)

relation between A_1 and A



$A_1 = A\tan\theta$

So by solving these equations $\boldsymbol{M_1 = 3M}$

N = 3

2. (5)

Power of equivalent mirror $P'_M = 2P_2 + P_M$

$$\frac{1}{f_2} = \left(\frac{3/2}{4/3} - 1\right)\left(\frac{1}{R} + \frac{1}{R}\right) = \frac{1}{8} \times \frac{2}{R} = \frac{1}{4R},\ f_2 = 4R \text{ and } P_2 = \frac{1}{4R}$$

$$-\frac{1}{f'_m} = 2 \times \frac{1}{4R} + \frac{2}{R} = \frac{5}{2R}$$

$$f'_m = -\frac{2}{5}R$$

$$|h| = 2|f'_m|$$

$$h = 2 \times \frac{2}{5}R = 4\text{m}$$

R = 5m

3. (4)

$a = 4t$

$a = 4\ \text{m/s}^2 = \mu_s g$ at (t = 1s)

$\mu_s = 0.4$

$\frac{dv}{dt} = 4t - \mu_k g$ (v is relative velocity)

$\int_0^v dv = [2t^2 - \mu_k g t]_1^2$

$v_2 = 2(2^2 - 1^2) - \mu_k g\,(1)$

$v_2 = 6 - \mu_k g$

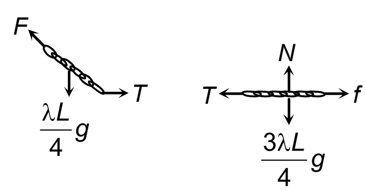
$v = 0 = v_2 - \mu_k g$

$\mu_k = 0.3$

$\therefore \quad \frac{3\mu_s}{\mu_k} = 4$

4. (1)

$F\cos 37^\circ = \frac{\lambda L}{4} g$



(where λ is the mass/length of the chain).

$F\sin 37^\circ = T = f \le \mu N$

$\Rightarrow \quad \mu \ge \frac{1}{4} \quad \Rightarrow \quad \mu_{min} = \frac{1}{4}$

$\therefore \quad n = 1$

5. (5)

$\frac{T}{2} - mg = ma_1$

$\frac{T}{4} - mg = ma_2$

$\frac{T}{2^n} - mg = ma_n$

$2^{n-1} \times a_1 + 2^{n-2} \times a_2 + ... + a_n + a_n = 0$

$$2^{n-1}\left(\frac{T}{2}-mg\right)+2^{n-2}\left(\frac{T}{2^2}-mg\right)+......+\left(\frac{T}{2^n}-mg\right)+\left(\frac{T}{2^n}-mg\right)=0$$

$$T\left(2^{n-2}+2^{n-4}+2^{n-6}+...........+2^{-n}+2^{-n}\right)=mg\left(2^{n-1}+2^{n-2}+.......1+1\right)$$

$T = 3\,mg$

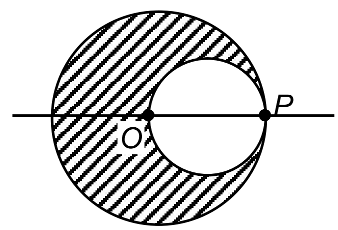
$$\frac{3mg}{2}-mg=ma_1$$

$$\frac{mg}{2}=ma_1$$

$$a_1=\frac{g}{2} = 5\text{ m/s}^2$$

6. **(6)**

7. **(2)**



Let refractive index of glass be μ

Let after first refraction, image distance be v then

$$\frac{\mu}{v}-\frac{1}{\infty}=\frac{\mu-1}{R} \quad\Rightarrow\quad v=\frac{\mu R}{\mu-1}$$

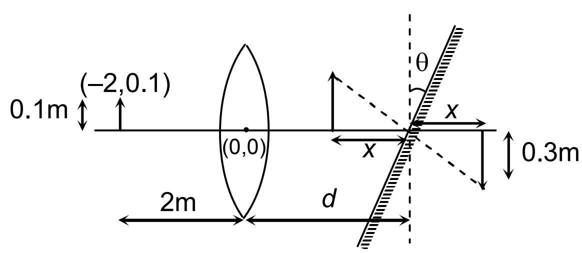
Now second refraction will take place.

So distance of first image from O is $u_1=\frac{\mu R}{\mu-1}-R=\frac{R}{\mu-1}$ and image is formed at R

$$\therefore \qquad \frac{1}{R}-\frac{\mu(\mu-1)}{R}=\frac{2(1-\mu)}{R} \quad\Rightarrow\quad \mu^2-3\mu+1=0\,.$$

So $\quad \mu=\frac{3+\sqrt{5}}{2}=\frac{2}{3-\sqrt{5}}$, there for n = 2

8. **(5)**



$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}-\frac{1}{-2}=\frac{1}{1.5};\quad v = 6\text{m}$$

$$m=\frac{v}{u}=-3$$

$$x=6-d\,;\qquad \tan\theta=\frac{0.3}{x}$$

$\Rightarrow \qquad d = 5$m.

9. (A,C)

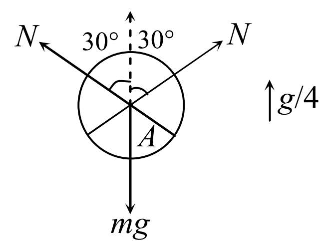
10. (A,C,D)

$F - T - \mu_2 m_2 g = m_2 a$, $T - \mu_2 m_2 g = m_1 a$

for just equilibrium $a = 0$, $F = 2\mu_2 m_2 g = 4\,\text{N}$

If F = 6N, a = 1 m/s^2 $\Rightarrow$ T = 3N

11. (B, D)



Net upward force on three spheres applied by bottom =

$3mg + \frac{3}{4}mg = \frac{15mg}{4}$

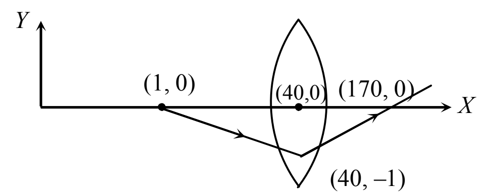
For sphere A, $N\sqrt{3} = mg + \frac{mg}{4}$, $N = \frac{5mg}{4\sqrt{3}}$

12. (B)

Acceleration of block m with respect to inclined plane = 6

Acceleration of inclined plane = $\frac{2}{\sqrt{3}}$

13. (B, C)



The ray is intersecting y = 0 line at x = 1 and x = 40 line at y = –1.

$\therefore$ u = 39 cm

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ $\Rightarrow$ v = 130 cm

$\therefore$ Equation of refracted ray is

$$130\,y = x - 170$$

If space on the right of the lens is filled with liquid of μ = 4/3, then

$$\frac{1.5}{v_1} + \frac{1}{39} = \frac{0.5}{30}$$

$$\frac{4}{3v} - \frac{1.5}{v_1} = \frac{\left(\frac{4}{3} - 1.5\right)}{-30}$$

v = –390 cm

$\therefore$ Equation of refracted ray is $390y + x + 330 = 0$

14. (A, B, C, D)

The two parts of the lens will have different focal lengths. So, there are two images.

$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{100}$; $m_1 = \frac{v_1}{u}$, $\frac{1}{v_2} - \frac{1}{u} = \frac{1}{200}$; $m_2 = \frac{v_2}{u}$

For same height of images $m_1 = -m_2$ $\Rightarrow$ $v_1 = -v_2$

$\Rightarrow$ $u = \frac{-400}{3}$ cm, $m_1 = -3$, $m_2 = 3$, $m_1 m_2 = -9$

15. (A, C)

If the image is real and magnified means object is between *f* and 2*f*.

When lens immersed in water focal length, $f_1 = \frac{(\mu - 1)}{\left(\frac{\mu}{\mu_r} - 1\right)} f = 4f$

Now object is between pole and focus so image is virtual and magnified.

16. (A, C)

Path difference = 0

$$\frac{d^2}{D} = \frac{yd}{2D} - \left(\frac{\mu_2}{\mu_3} - 1\right)t$$

y = 2 mm

When slab is removed then path difference = $\frac{d^2}{D} - \frac{y_1 d}{2D}$ = 0, $y_1 = 2d$ = 4mm

17. (C)

18. (B)

19. Smaller image will be brighter as intensity $\propto \frac{1}{\text{Area}}$

$\therefore$ **(A)**

20. $\frac{v}{u} = 2$, $v + u = 180\,\text{cm}$ $\Rightarrow$ $f = 40\,\text{cm}$

Using the formula $f = \frac{D^2 - L^2}{4D}$ $\Rightarrow L$ = 60 cm

$\therefore$ **(C)**

CHEMISTRY

21. (7)

He, Be, N, Ne, Mg, Ar, Ca, all are positive electron gain enthalpy.

22. (0)

Electronic configuration of Pd : $[Ar]5s^0 4d^{10}$

23. (4)

24. (3)

$$Z = \frac{PV_m}{RT} = \frac{3}{8}\left(\frac{P_r V_r^m}{T_r}\right) = \frac{3}{8} \times 2 = \frac{3}{4}$$

$$\Rightarrow \quad V_m = \left(\frac{3}{4}\right)\left(\frac{0.0821 \times 200}{8.21}\right) = \frac{3}{2} L$$

$\Rightarrow$ Volume occupied by 2 mole = $\frac{3}{2} \times 2 = 3L$

25. (2)

$$Z = 1 - \frac{a}{RTV_m} \qquad \text{Slope} = \frac{a}{R}$$

$$\therefore \; a = \frac{0.4}{1.64} \times 0.082 = 2 \times 10^{-2}$$

26. (5)

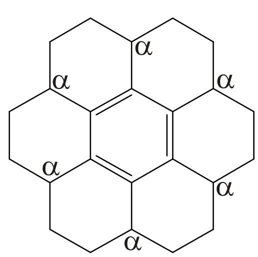
$$\underset{\substack{80 \\ \text{milli mole}}}{4X} + 7NaOH + NaNO_3 \longrightarrow 4Na_2XO_2 + \underset{20}{NH_3} + 2H_2O$$

$$\underset{20}{2NH_3} + \underset{10}{H_2SO_4} \longrightarrow (NH_4)_2SO_4$$

$$\underset{24}{2NaOH} + \underset{12}{H_2SO_4} \longrightarrow Na_2SO_4 + 2H_2O$$

Gram = $80 \times 10^{-3} \times 62.5 = 5$

27. (6)



28. **(7)**

HF, —OH, HCl, CH_3–COOH, H_2CO_3, HBr, CH_3–SH

29. **(A), (C), (D)**

30. **(A), (B), (C), (D)**

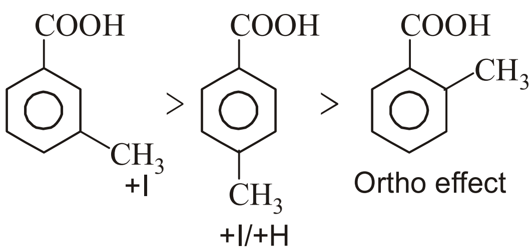
31. **(B), (C)**

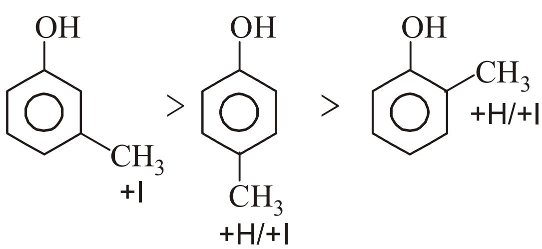
PF_5 : Exists as PF_5 molecular form in its solid phase.

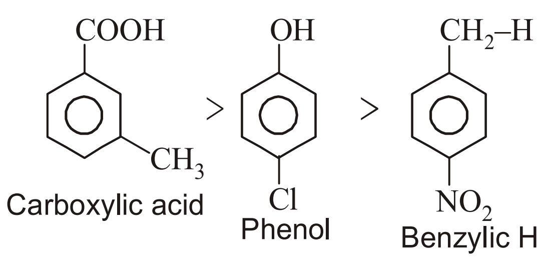
32. **(A), (C), (D)**

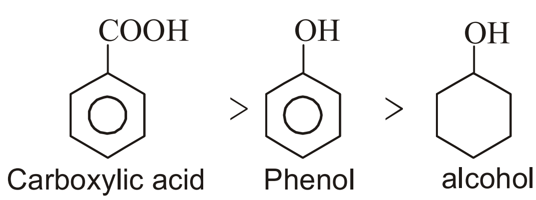
Marshall's acid contain peroxy linkage

33. **(B), (C), (D)**

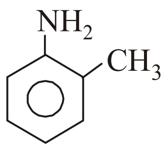
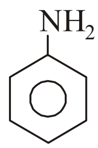
(A) 

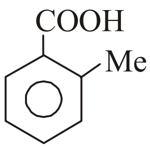
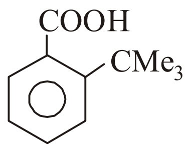
(B) 

(C) 

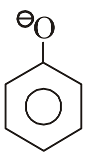
(D) 

34. (A), (B), (D)

(A) (SIP)  <  (Basic order)

(B)  <  (SIR) (Acidic order)

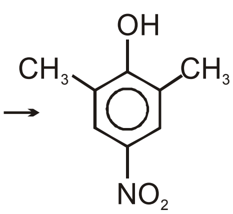
(C) HC ≡ CH > NH_3 (Acidic order)
sp hybrid carbon
more elctronegative

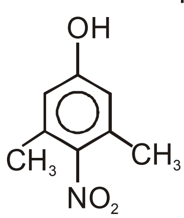
(D)  < $Me - \overset{O}{\overset{\|}{C}} - O^{\ominus}$ (Equivalent R.S.)

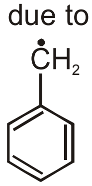
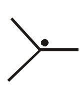
Non equivalent
R.S.

35. (A), (B), (D)

(A) $+M \rightarrow -NH_2 > -OH$

(B) A.S ⟶ 
–M

(C) 
–I due to SIR

(D)   
Resonance
stabilised

36. (A,B,D)

Third virial coefficient

(C) = b^2 = 1200 $cm^6\ mol^{-2}$

b=34.64 $cm^3\ mol^{-1}$

Second virial coeefficnet.

$$(B) = b - \frac{a}{RT}$$

$\therefore$ a = RT (b – B)

a = (8.314×10^7) 300 (34.64–(–21.7)

a = 1.40×10^{12} dyne $cm^4\ mol^{-2}$

$$T_b = \frac{a}{Rb}\ V_c = 3b$$

37. (C)

Balanced reaction : $6Fe^{+2} + 14H^+ \longrightarrow 2Cr^{+3} + 6Fe^{+3} + 7H_2O$

Millimole of dichromate ion is 2.35 hence millimole of Fe^{+2} ion = 6×2.35=14.1

$$6Fe^{+2} + Cr_2O_7^{2-} + 14H^+ \longrightarrow 2Cr^{+2} + 6Fe^{+3} + 7H_2O$$

Fe^{+2} = 6×2.35=14.1

38. (C)

Milimole of $FeSO_4.7H_2O$ = 14.1

So, weight of $FeSO_4.7H_2O$ = 1.41×10^{-2} × 278 = 3.91g

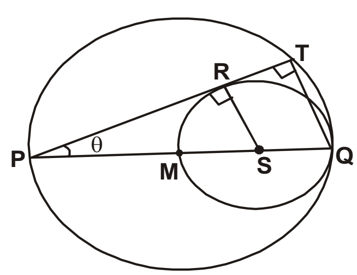
Hence % purity = (3.91/4.2) × 100 = 93.0

39. (D)

40. (D)

MATHEMATICS

41. (6)



Take $\angle QPT = \theta$

than $\sin\theta = \frac{RS}{PS} = \frac{r}{3r} = \frac{1}{3}$ $\Rightarrow \cos\theta = \frac{2\sqrt{2}}{3}$

$\Rightarrow \frac{PT}{PQ} = \frac{2\sqrt{2}}{3}$ $\Rightarrow \frac{PT}{12} = \frac{2\sqrt{2}}{3}$

$\Rightarrow PT = 8\sqrt{2}$ $\Rightarrow$ m = 8 , n = 2

42. **(3)**

Let P be point 't' i.e $(at^2, 2at)$ then point $Q \equiv -\frac{1}{t}$

point $P' \equiv -t - \frac{2}{t}$ and point $Q' \equiv \frac{1}{t} + 2t$

$$P'Q' = a\left|3\left(t+\frac{1}{t}\right)\right|\sqrt{\left(t-\frac{1}{t}\right)^2+4} = 3a\left(t+\frac{1}{t}\right)^2$$

$$PQ = a\left(t+\frac{1}{t}\right)^2$$

43. **(8)**

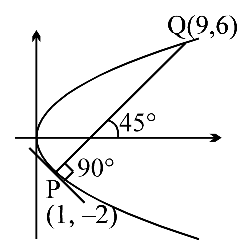
Normal : $y + tx = 2t + t^3$; slope of the normal is 1

hence $-t = 1$ Þ $t = -1$ Þ coordinates of P are (1, –2)

Hence parameter at Q = $t_2 = -t_1 - 2/t_1 = 1 + 2 = 3$

$\therefore$ Coordinates at Q are (9, 6)

$\therefore$ ℓ(PQ) = $\sqrt{64+64} = 8\sqrt{2}$



44. **(4)**

Let the equation of line be $\frac{x}{\cos\theta} = \frac{y}{\sin\theta} = r$

$(OA\cos\theta, OA\sin\theta)$

$(OB\cos\theta, OB\sin\theta)$ will be satisfying

y – x – 10 = 0 and y – x – 20 = 0 respectively

if $P(r\cos\theta, r\sin\theta)$ then

$$\frac{1}{r^2} = \left(\frac{\sin\theta-\cos\theta}{10}\right)^2 + \left(\frac{\sin\theta-\cos\theta}{20}\right)^2$$

$\Rightarrow (r\cos\theta - r\sin\theta)^2 = 80$

Locus of P is $(y - x)^2 = 80$

45. (1)

Given $f\left(\frac{5x-3y}{2}\right) = \frac{5f(x)-3f(y)}{2}\ \forall\, x, y \in R,$

which explains that all the points that divide the line joining P(y, f(y)) and Q(x, f(x)) externally in the ratio 5 : 3 lies on the curve y = f(x). Therefore it is a linear function

$\Rightarrow\ f(x) = ax + b \quad \Rightarrow\ f(0) = b \quad \Rightarrow\ b = 3$

$f'(x) = a \quad \Rightarrow\ a = f'(0) = 2$

$\Rightarrow\ f(x) = 2x + 3$

$\Rightarrow$ Period of $\sin(f(\pi x))$ is 1.

46. (3)

$f\left(x + \frac{7}{4}\right) = f\left(\frac{7}{4} - x\right)$

$\Rightarrow b = \frac{-7a}{2} \Rightarrow f(x) = ax^2 - \frac{7a}{2}x + a = 7x + a$

$\Rightarrow ax^2 - \frac{7(a+2)}{2}x = 0$ has only one solution

$\Rightarrow a = -2 \Rightarrow b = 7 \qquad \therefore\ b - a^2 = 3$

47. (8)

$$\lim_{x\to 0} \frac{\tan 3x - n\sin 2x}{\left(\frac{\sin^{-1}x}{x}\right)^3 x^3}$$

$$= \lim_{x\to 0} \frac{\sin 3x - n\sin 2x\cos 3x}{x^3\cos 3x}$$

$$= \lim_{x\to 0} \frac{\sin x}{x}\left(\frac{3 - 4\sin^2 x - 2n\cos x\cos 3x}{x^2\cos 3x}\right)$$

Here 3 – 2n = 0 $\Rightarrow n = 3/2$

Also for $n = 3/2$

$$L = \lim_{x \to 0}\left(\frac{3 - 3\cos x \cos 3x}{x^2} - 4\frac{\sin^2 x}{x^2}\right)$$

$$= \lim_{x \to 0} \frac{6 - 3\cos 4x - 3\cos 2x}{2x^2} - 4$$

$$= \lim_{x \to 0} \frac{12\sin 4x + 6\sin 2x}{4x} - 4$$

$= 12 + 3 - 4 = 11$

48. (1)

$$\lim_{x \to \infty}\left\{x^{3c}\left(1 + \frac{4}{x} + \frac{1}{x^3}\right)^c - x\right\}$$

$$\lim_{x \to \infty} x\left[x^{3c-1}\left\{1 + \left(\frac{4}{x} + \frac{1}{x^3}\right)\right\}^c - 1\right]$$

$$\lim_{x \to \infty} x\left[x^{3c-1}\left\{1 + c\left(\frac{4}{x} + \frac{1}{x^3}\right)\right\} + ... - 1\right]$$

$3c = 1$

49. (C, D)

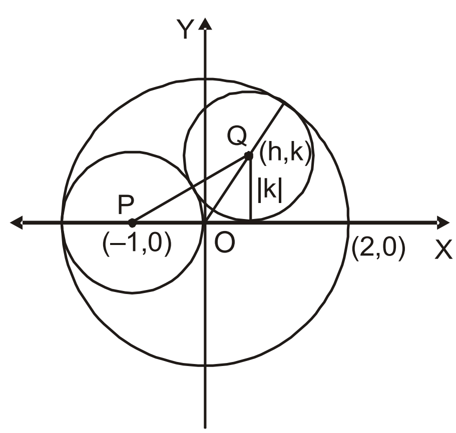
Let centre of C be Q(h, k), then its radius is | k |.

$\therefore \quad PQ = \sqrt{(h+1)^2 + k^2} = 1 + |k|$

$\Rightarrow h^2 + 2h = 2|k| \quad$ ... (i)

Also $\quad OQ = \sqrt{h^2 + k^2} = 2 - |k|$

$\Rightarrow h^2 = 4 - 4|k| \quad$ ... (ii)



From (i) and (ii),

$2(h^2 + 2h) + h^2 = 4$

$\Rightarrow 3h^2 + 4h - 4 = 0$

$\Rightarrow h = \frac{2}{3} \quad (\because h \neq -2)$

$\therefore \; k = \pm\frac{8}{9}$

∴ Equation of circle C is

$$x^2 + y^2 - \frac{4}{3}x \mp \frac{16}{9}y + \frac{4}{9} = 0$$

50. (A, B, C,D)

Let us take $P(3t_1^2, 6t_1)$ and $Q(3t_2^2, 6t_2)$ on the parabola $y^2 = 12x$

Here, $6t_2 = 2 \times 6t_1 \Rightarrow t_2 = 2t_1$

Points of intersection of tangents at P and Q are $(3t_1t_2, 3(t_1 + t_2))$

For locus let us take $h = 3t_1t_2 = 6t_1^2$ and $k = 3(t_1 + t_2) = 9t_1$

eleminating 't_1' we have $h = 6\left(\frac{k}{9}\right)^2$

$\Rightarrow \quad k^2 = \frac{27}{2}h \Rightarrow y^2 = \frac{27}{2}x$

points of intesection of normals at P & Q is $(6 + 3(t_1^2 + t_2^2 + t_1t_2), -3t_1t_2(t_1+t_2))$

For locus let us take

$h = 6 + 3(t_1^2 + t_2^2 + t_1t_2) = 6 + 21t_1^2 \qquad k = -3t_1t_2(t_1 + t_2) = -18t_1^3$

Eliminating t_1 we have $\left(\frac{h-6}{21}\right)^3 = \left(\frac{k}{-18}\right)^2$

$\Rightarrow \quad 12(x-6)^3 = 343y^2$

mid-points of P and Q $\left(3\frac{(t_1^2 + t_2^2}{2}, 3(t_1 + t_2)\right)$

For locus let us take $2h = 3(t_1^2 + t_2^2)$

$\Rightarrow 2h = 15t_1^2$ & $k = 3(t_1 + t_2) = 9t_1$

Eliminating t_1 we have $2h = 15\left(\frac{k}{9}\right)^2 \Rightarrow 5y^2 = 54x$

when P is $(1, 2\sqrt{3})$ then Q is $(4, 4\sqrt{3})$

Hence, $PQ = \sqrt{9 + 12} = \sqrt{21}$

51. (A, C)

The focus of the parabola is at (p/2, 0) & directrix is x = – p/2

centre of the circle is (p/2, 0) & radius = $\frac{p}{2}-\left(-\frac{p}{2}\right)=p$

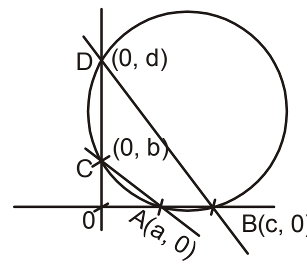
Equation of the circle is

$$\left(x-\frac{p}{2}\right)^2+(y-0)^2=p^2 \Rightarrow x^2+y^2-px-\frac{3p^2}{4}=0$$

solving this equation with $y^2=2px$ we get $x=\frac{p}{2}, y=\pm p$

$\therefore$ The points of intersection are $\left(\frac{p}{2},p\right)\&\left(\frac{p}{2},-p\right)$

52. (A, B, C, D)



If points A, B, C, D are concyclic then ac = bd

Also the points of intersection of lines are

$$\left(\frac{ac(b-d)}{bc-ad}, \frac{bd(c-a)}{bc-ad}\right)$$

Let (h, k) be the point of intersection :

since $c^2 + a^2 = b^2 + d^2$

and ac = bd

$(c-a)^2 = (b-d)^2 \Rightarrow h = \pm k$

hence locus of pt of intersection is y = $\pm$x

53. (B, C)

$$\sin^{-1}\left(a^2x^2+b^2y^2\right)+\cos^{-1}|ax+by|=\pi$$

$$\Rightarrow a^2x^2+b^2y^2=1 \text{ and } ax+by=0$$

$$\Rightarrow 2axby=-1$$

54. (A,B,D)

We have f(x) = $\cos^{-1}(-\{-x\})$

D_f = R

As $0 \le \{-x\} < 1 \ \forall x \in R$

$\Rightarrow -1 < -\{-x\} \le 0$

So $R_f = \left[\frac{\pi}{2},\pi\right)$

Clearly, f is neither even nor odd.

But f (x + 1) = f (x) $\Rightarrow$ f is periodic with period 1.

55. (A, C, D)

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\tan^2\{x\}}{(x^2-[x]^2)} = \lim_{x\to 0^+} \frac{\tan^2 x}{x^2} = 1$$

Also, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \sqrt{\{x\}\cot\{x\}} = \sqrt{\cot 1} \quad \Rightarrow \quad \cot^{-1}\left(\lim_{x\to 0^-} f(x)\right)^2 = 1$

56. (A, C)

$$\ell = \lim_{x\to 0} \frac{(1+qx)-(1+px)\sqrt{1+x}}{x^3\sqrt{1+x}(1+qx)}$$

$$= \lim_{x\to 0} \frac{(1+qx)-(1+px)\sqrt{1+x}}{x^3} = \lim_{x\to 0} \frac{(1+qx)-(1+px)\left(1+\frac{x}{2}-\frac{x^2}{8}+\frac{x^3}{16}+...\right)}{x^3}$$

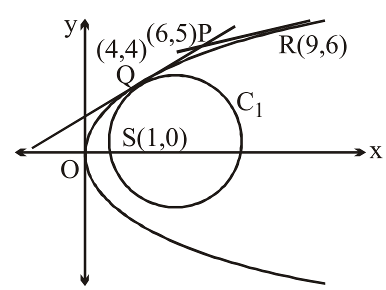
$$\lim_{x\to 0} \frac{qx-\frac{x}{2}+\frac{x^2}{8}-\frac{x^3}{16}-px-\frac{px^2}{2}+\frac{px^3}{8}}{x^3}$$

Now cofficient of x and x^2 must be 0 $\Rightarrow q-p=\frac{1}{2}$ & $\frac{p}{2}=\frac{1}{8} \Rightarrow p=\frac{1}{4}, q=\frac{3}{4}$

$$\therefore \ell = -\frac{1}{32}$$

57. (B)

Equation of tangent of slope m to $y^2 = 4x$ is



$y = mx + \frac{1}{m}$ ....(1)

As (1) passes through P(6, 5), so

$5 = 6m + \frac{1}{m}$

$\Rightarrow 6m^2 - 5m + 1 = 0 \Rightarrow m = \frac{1}{2}$ or $m = \frac{1}{3}$

Points of contact are $\left(\frac{1}{m_1^2}, \frac{2}{m_1}\right)$ and $\left(\frac{1}{m_2^2}, \frac{2}{m_2}\right)$

Hence P (4, 4) and Q (9, 6)

Area of $\Delta PQR = \frac{1}{2}\begin{vmatrix} 6 & 5 & 1 \\ 4 & 4 & 1 \\ 9 & 6 & 1 \end{vmatrix} = \frac{1}{2}$

58. (C)

$y = \frac{1}{2}x + 2 \quad \Rightarrow \quad x - 2y + 4 = 0$ ....(2)

and $\quad y = \frac{1}{3}x + 3 \Rightarrow x - 3y + 9 = 0$

Now equation of circle C_2 touching $x - 3y + 9 = 0$ at (9, 6), is

$(x - 9)^2 + (y - 6)^2 + \lambda(x - 3y + 9) = 0$

As above circle passes through (1, 0), so

$64 + 36 + 10\lambda = 0 \Rightarrow \lambda = -10$

Circle C_2 is $x^2 + y^2 - 28x + 18y + 27 = 0$ .....(3)

Radius of C_2 is

$r_2^2 = 196 + 81 - 27 = 277 - 27 = 250 \quad \Rightarrow \quad r_2 = 5\sqrt{10}$

59. (B)

$A = (\tan^{-1}x + \cot^{-1}x)^3 - 3\tan^{-1}x \cot^{-1}x (\tan^{-1}x + \cot^{-1}x)$

$\Rightarrow \quad A = \left(\frac{\pi}{2}\right)^3 - \frac{3\pi}{2}(\tan^{-1}x \cot^{-1}x)$

$\Rightarrow \quad A = \frac{\pi^3}{8} - \frac{3\pi}{2}(\tan^{-1}x)\left(\frac{\pi}{2} - \tan^{-1}x\right) \Rightarrow \quad A = \frac{\pi^3}{32} + \frac{3\pi}{2}\left(\tan^{-1}x - \frac{\pi}{4}\right)^2$

$\therefore \quad \frac{\pi^3}{32} \le A < \frac{\pi^3}{8}$

60. (A)

$B = (\sin^{-1} t + \cos^{-1}t)^2 - 2\sin^{-1}t \cos^{-1}t$

$\Rightarrow B = \frac{\pi^2}{4} - 2\sin^{-1}t\left(\frac{\pi}{2} - \sin^{-1}t\right) \Rightarrow B = \frac{\pi^2}{8} + 2\left(\sin^{-1}t - \frac{\pi}{4}\right)^2$

$\therefore$ maximum value of $B = \frac{\pi^2}{8} + \frac{2\pi^2}{16} = \frac{\pi^2}{4}$

Now $\quad \lambda = \frac{\pi^3}{32}$ and $\mu = \frac{\pi^2}{4}$

$\therefore \quad \frac{\lambda - \mu\pi}{\mu} = -\frac{7\pi}{8} \qquad \therefore \quad \cot^{-1}\cot\left(\frac{\lambda - \mu\pi}{\mu}\right) = \frac{\pi}{8}$