SOLUTIONS

PROGRESS TEST-3

RB-1813-1814

RBK-1806

JEE MAIN PATTERN

Test Date: 15-10-2017

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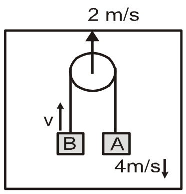
PHYSICS

1. **(D)**



V = (velocity of B w.r.t ground)

$v_{pulley} = \frac{V-4}{2} = 2\text{m/s}$; V is velocity of B w.r.t ground

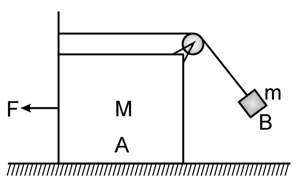
Solving we get

v = 8 m/s

2. **(C)**

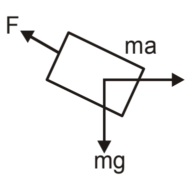
$a_1 = \frac{12}{41} g\ m/s^2 \rightarrow, a_2 = \frac{9}{41} g\ m/s^2 \downarrow$

3. **(A)**



Applying Newton's law on the system in horizontal direction F = (M + m) a.

Now consider the equilibrium of block B w.r.t. block M



$F^2 = (mg)^2 + (ma)^2 = (mg)^2 + \left(m\frac{F}{m+M}\right)^2$

$\therefore\ F^2 = \frac{m^2g^2}{1-\frac{m^2}{(m+M)^2}}$; $F = \frac{mg}{\sqrt{1-\left(\frac{m}{m+M}\right)^2}}$

4. **(B)**

$a_A = 0.4\ m/s^2 \downarrow$;

5. (D)

$F_{min} = 25$ N

6. (B)

$F_{wall} = 50\text{ N}$

7. (B)

If we consider blocks 2 & 1 independently then their accelerations would be for block (1)

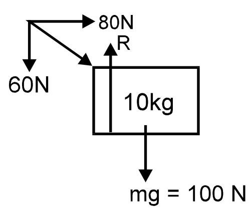
$$a_1 = g\sin\theta - \mu_1 g\cos\theta = g\left[\frac{\sqrt{3}}{2} - \frac{1}{2}\times\frac{1}{2}\right] = \frac{g\left[2\sqrt{3}-1\right]}{4}$$

for block (2)

$$a_2 = g\sin\theta - \mu_2 g\cos\theta = g\left[\frac{\sqrt{3}}{2} - \frac{2}{5}\times\frac{1}{2}\right] = \frac{g}{10}\left[5\sqrt{3}-2\right]$$

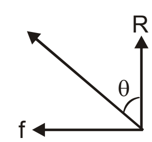
since $a_2 > a_1$ so both blocks will move separately.

8. (D)



R = mg + 60 = 160 N

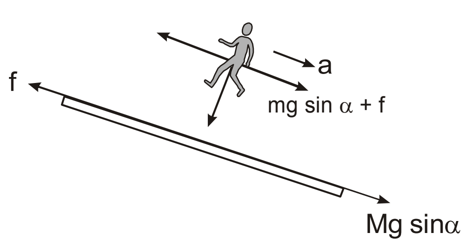
f = 80 N (∵ No sliding)



angle of friction $\theta = \tan^{-1}\frac{f}{R} = \tan^{-1}\frac{80}{160}$ $\boldsymbol{\theta = \tan^{-1}\frac{1}{2}}$ **Ans.**

9. (B)

F.B.D. of man and plank are



For plank to be at rest, applying Newton's second law to plank along the incline

$Mg \sin \alpha = f$ ....(1)

and applying Newton's second law to man along the incline.

$mg \sin \alpha + f = ma$ ....(2)

$a = g \sin \alpha \left(1 + \frac{M}{m}\right)$ down the incline

10. (C)

Let m_A and m_B be the mass of blocks A and B respectively.

As the force F increases from 0 to $\mu_s m_A g$, the frictional force f on block A is such that f = F. When $F = \mu_s m_A g$, the frictional force f attains maximum value $f = \mu_s mg$.

As F is further increased to $\mu_s(m_A+m_B)g$, the block A does not move. In this duration frictional force on block A remains constant at $\mu_s m_A g$.

Hence C is correct choice.

11. (A)

$60 = 40 + T \quad \Rightarrow T = 20N$

$a = \frac{20}{4} = 5 \text{ m/s}^2$

$\Rightarrow \frac{|T|}{|a|} = 4$

12. (C)

$\sqrt{\frac{D\lambda}{2}}$

13. (D)	**14. (C)**	**15. (B)**	**16. (C)**
17. (B)	**18. (B)**	**19. (B)**	**20. (D)**
21. (C)	**22. (D)**	**23. (D)**	

24. (C)

Let us resolve the velocity v imparted to the ball into component parallel with the sides of the table and consider the path of a ball as shown, for example, in the diagram (fig.).

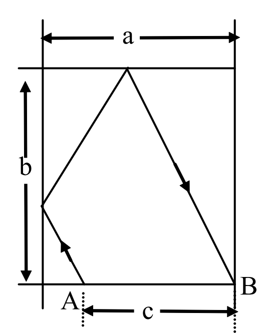
We obtain two equations, evident from the diagram :

$\frac{2a-c}{t} = v \cos \alpha$, $\frac{2b}{t} = v \sin \alpha$,

From these equations we get :

$\cot \alpha = \frac{2a-c}{2b}$,

i.e., we find angle α, at which the ball must be struck. The value for the velocity v which is imparted to the ball plays no part at all.



25. (B)

26. (B)

27. (C)

R = $\sqrt{P^2 + 2PQ\cos\theta + Q^2}$ ...(1)

tan 90º = $\frac{2Q\sin\theta}{P + 2Q\cos\theta} = \frac{1}{0}$

P + 2Q cos θ = 0

from (1)

R = $\sqrt{P(P + 2Q\cos\theta) + Q^2}$

R = Q

28. (B)

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -2\hat{i} + 2\hat{k}$

here $\vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{b}$ unit vector along = $\vec{a} \times \vec{b} = \frac{-2\hat{i} + 2\hat{k}}{\sqrt{(-2)^2 + 2^2}} = \frac{-\hat{i} + \hat{k}}{\sqrt{2}}$

29. (A)

Use R = $\sqrt{a^2 + b^2 + 2ab\cos\theta}$ and

S = $\sqrt{a^2 + b^2 - 2ab\cos\theta}$

also tan α = $\frac{b\sin\theta}{a + b\cos\theta}$

30. (D)

CHEMISTRY

31. (A)

Mole of Nx = $\frac{8}{1}\times\frac{1}{3}\times\frac{1}{1}\times(\text{Moles of M})$

Mole of M = $\frac{206}{103}\times\frac{3}{8}=\frac{3}{4}$

$\therefore$ Mass of M = $\frac{3}{4}\times 56 = 42\,g$

32. (D)

Mass of $BaCl_2$ in 50 ml = $\frac{20.8}{100}\times 50 = 10.4\,g$

$n_{BaCl_2} = \frac{10.4}{208} = 0.05$

Moles of H_2SO_4 = $\frac{9.8}{98} = 0.1$

$\therefore$ $BaCl_2$ is a L.R.

$n_{BaSO_4} = 0.05$

$mass_{BaSO_4} = 0.05 \times 233 = 11.65\,g$

33. (A)

Money to be spent

(A) Rs. 50 x 1

(B) Rs. 56 x 1

(C) Rs. 30 x 2 = Rs. 60

(D) Rs. 27 x 2 = Rs 54

Sample 'A' is most suitable.

34. (D)

n factor is 1

Eq. weight = $\frac{M}{1}$ = 128 g

35. (B)

36. (C)

37. (C)

No. of moles of glucose = $\frac{6}{180}=\frac{1}{30}$

So, no. of moles of C = $\frac{1}{30}\times 6=\frac{1}{5}$

and no. of moles of H = $\frac{1}{30}\times 12=\frac{2}{5}$

mass of O in the compound = $4.4-\left(\frac{1}{5}\times 12+\frac{2}{5}\times 1\right)$ = 1.6g

i.e., moles of O = $\frac{1.6}{16}=\frac{1}{10}$

mole ratio of C : H : O = $\frac{1}{5}:\frac{2}{5}:\frac{1}{10}=2:4:1$

So, empirical formula = C_2H_4O

38. (C)

If mass of one C^{12} atom = 24 a.m.u

then mass of one He atom = 8 a.m.u

39. (B)

$2H_2O_2(aq.)\longrightarrow 2H_2O(\ell)+O_2(g)$

1 litre solution	22.7 litre at S.T.P
2 mole	1 mole at S.T.P

mass of solvent = (1000x – 68)g

So, $\text{molality}=\left(\frac{2\times 1000}{1000x-68}\right)m$

40. (A)

Let x mole $S_2O_8^{-2}$ is oxidised

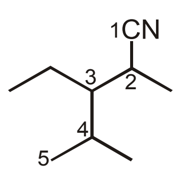
Now, g eq. of $S_2O_8^{-2}$ = g eq. of SO_2

x × 2 = 1 × 2

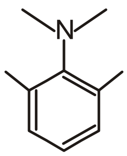
or, x = 1

or, Balance the reaction.

41. (C)



42. (C)



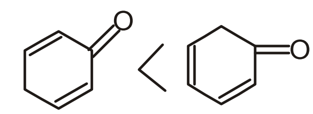
Due to steric hinderence molecule become non planar. So no resonance possible.

43. (D)

Larger number of bond and minimum charge separation make structure most stable.

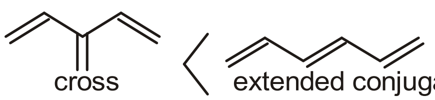
44. (A)

so, Ph—CH_2—Ph > Ph–CH=CH_2



Cross conjugation   external conjugation

$$CH_2 = CH - OH < \underset{\text{more resonance}}{CH_2 = CH - CH = CH - OH}$$



cross conjugation   extended conjugation

45. (C)

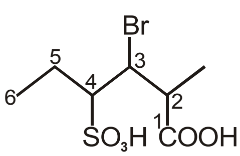
(A) Ph–NH–C(=O)–CH_3 +R

(B) Ph–O–CH_3 +R

(C) Ph–C$(CH_3)_3$ +I

(D) Ph–O^- +R

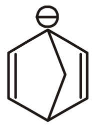
46. (D)



47. (D)

Compound is non-planar due to repulsion between transannular hydrogen.

48. (D)

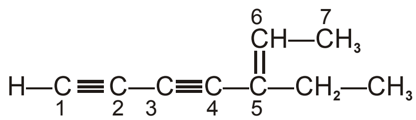


Negative charge not involve in resonance

49. (A)

In structure (iv) lone pair of nitrogen after delocalisation create aromaticity, hence most stable and in structure (iii) lone pair of oxygen after delocalisation create aromaticity, hence slightly less stable than (iv).

50. (A)



51. (B)

52. (A)

53. (D)

54. (B)

55. (B)

$NaOH \rightarrow Na^{+} + :\overset{\ominus}{\ddot{O}}-H$

56. (C)

positive radius $\alpha \dfrac{1}{+\text{ve O.S.}}$

(A) $\overset{+4}{Mn}O_2^{-2}$ (B) $\overset{+1}{K}\overset{+7}{Mn}O_4^{-8}$ (C) $\overset{+2}{Mn}\overset{-2}{O}$ (D) $K_3[\underset{x=+3}{\underset{\downarrow}{Mn}}(CN)_6]$

57. (D)

A Gives equeous solution [PH < 7]

B Reacts with strong acid and alkalis respectively.

C Gives an aqueous solution which is strongly alkaline

A - Acidic – P $(OH)_3$ or H_3PO_4

B - Amphoteric – Al $(OH)_3$, H_3AlO_3

C - Basic – NaOH

x = Phousphorous – Non metal

y = Aluminium - Metal

c = Sodium - Metal

58. (A)

(A) Lattic energy depend upon :

(i) Size of cation and anion both

(ii) Product of charges at cation & anion

(B) $CdCl_2 > CaCl_2$–Both Hydration & Lattice is high than $CaCl_2$

As per (born haber cycle)

(C) $F^- > Cl^- > Br^- > I^-$ (Hydration energy)

so, AgF > AgCl > AgBr > Ag I (Solubility in water)

(D) $Be_3N_2 > Mg_3N_2 > Ca_3N_2$ (Thermal stability)

59. (A)

Lattice α Hardness

(A) Ti > ScN > MgO > NaF – order of lattic energy

(B) NaCl < CsCl – Co-ordinate no. NaCl = 6

CsCl = 8

(C) $BeCl_2 < MgCl_2 < CaCl_2$ – Melting point

60. (B)

$Cs^+I_3^-$ (large cation stabilises by large anion)

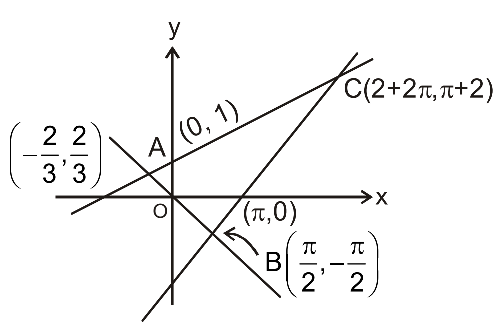
MATHEMATICS

61. (B)

Let ratio be $\lambda : 1 \Rightarrow \frac{6\lambda - 3}{\lambda + 1} = 0,\ \lambda = \frac{1}{2}$

62. (C)

63. (C)



if (a, sina) lie inside the triangle, then $a \in (0, \pi)$

64. (D)

f(– x) = f(x)

An even function hence neither one-one nor onto

65. (B)

$n(A) = n - 1$

$n(A \times A) = (n-1)^2$

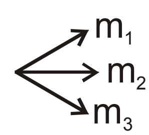
Number of relation on A $= 2^{(n-1)^2}$

66. (B)

$4m^3 - 3am^2 - 8a^2m + 8 = 0$

$m_1m_2m_3 = -2$

$\Rightarrow \quad m_3 = 2 \quad (\because m_1m_2 = -1)$

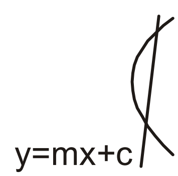


67. (B)

$2x^2 + 3y^2 - 5x\left(\frac{y - mx}{C}\right) = 0$

Coefficient of x^2 + coefficient of y^2 = 0

$5 + \frac{5m}{C} = 0 \Rightarrow m + c = 0$



Then the equation of family of line is y = m(x - 1)

68. (A)

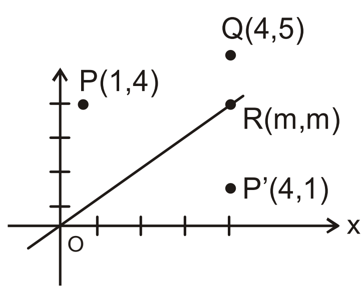


Image of (1, 4) about the line y = x is (4, 1) $\Rightarrow P'(4,1)\,Q(4,5)$ and R(m, m) are collinear.

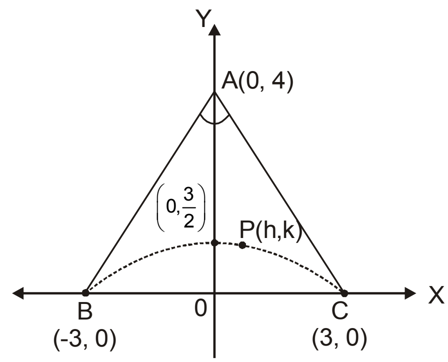
$\Rightarrow \; m = 4$

69. (A)

Let $A \equiv (0,4)$, $B \equiv (-3,0)$ and $C \equiv (0,3)$, then equations of AB, AC and BC are 4x – 3y + 12 = 0, 4x + 3y – 12 = 0 and y = 0 respectively.

Let $P \equiv (h,k)$, then

$$\frac{|4h-3k+12|}{\sqrt{4^2+3^2}}\cdot\frac{|4h+3k-12|}{\sqrt{4^2+3^2}} = |k|^2$$



$\Rightarrow\ (4h-3k+12)(-4h-3k+12)=25k^2$

$\Rightarrow\ h^2+k^2+\frac{9}{2}k-9=0$

$\therefore$ Locus of P is a portion of the circle,

$x^2+y^2+\frac{9}{2}y-9=0$

$x=0 \Rightarrow y=\frac{3}{2}$ or -6

$\therefore$ Minimum distance of P from $A = 4-\frac{3}{2}=\frac{5}{2}$.

70. (A)

$1-3x\ge 0$

$x\le\frac{1}{3}$

And

$-x^2+x+6\ge 0$

$x^2-x-6\le 0$

$x^2-3x+2x-6\le 0$

$(x-3)(x+2)\le 0$

$x\in[-2,3]$

The answer will be $\left[-2,\frac{1}{3}\right]$

71. (A)

$\cos\frac{\pi}{19}+\cos\frac{3\pi}{19}+\cos\frac{5\pi}{19}+\ldots+\cos\frac{17\pi}{19}$

$$=\frac{2\sin\frac{\pi}{19}}{2\sin\frac{\pi}{19}}\left[\cos\frac{\pi}{19}+\cos\frac{3\pi}{19}+\cos\frac{5\pi}{19}+\ldots+\cos\frac{17\pi}{19}\right]$$

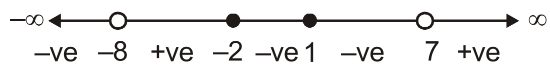
$$= \frac{1}{2\sin\frac{\pi}{19}}\left[\sin\frac{2\pi}{19} + \left(\sin\frac{4\pi}{19} - \sin\frac{2\pi}{19}\right) + \left(\sin\frac{6\pi}{19} - \sin\frac{4\pi}{19}\right) + \ldots + \left(\sin\frac{18\pi}{19} - \sin\frac{16\pi}{19}\right)\right]$$

$$= \frac{\sin\frac{18\pi}{19}}{2\sin\frac{\pi}{19}} = \frac{\sin\left(\pi - \frac{\pi}{19}\right)}{2\sin\frac{\pi}{19}} = \frac{\sin\frac{\pi}{19}}{2\sin\frac{\pi}{19}} = \frac{1}{2}$$

Aliter : Use sum of cosine series

72. (B)

Using wavy curve method :



$\therefore \quad x \in (-\infty, 8) \cup [-2, 1] \cup (1, 7)$

i.e., $x \in (-\infty, 8) \cup [-2, 7)$

73. (C)

$$-2 < x < 0 \text{ or } 0 < x < \frac{1}{2}$$

$$\Rightarrow -\infty < \frac{2}{x} < -1 \text{ or } 4 < \frac{2}{x} < \infty \Rightarrow R - [-1, 4]$$

74. (D)

We have, $\frac{\log_{10} a}{2} = \frac{\log_{10} b}{3} = \frac{\log_{10} c}{5} = \lambda(\text{say})$

$\Rightarrow \quad \log_{10}a = 2\lambda, \log_{10}b = 3\lambda, \log_{10}c = 5\lambda$

$\Rightarrow \quad a = 10^{2\lambda}, b = 10^{3\lambda}, c = 10^{5\lambda} \quad \Rightarrow \quad bc = 10^{8\lambda} = (10^{2\lambda})^4 = a^4$

75. (A)

$$\tan 27x - \tan x = (\tan 27x - \tan 9x) + (\tan 9x - \tan 3x) + (\tan 3x - \tan x)$$

$$= \left(\frac{\sin 27x}{\cos 27x} - \frac{\sin 9x}{\cos 9x}\right) + \left(\frac{\sin 9x}{\cos 9x} - \frac{\sin 3x}{\cos 3x}\right) + \left(\frac{\sin 3x}{\cos 3x} - \frac{\sin x}{\cos x}\right)$$

$$= \frac{\sin 18x}{\cos 27x.\cos 9x} + \frac{\sin 6x}{\cos 9x.\cos 3x} + \frac{\sin 2x}{\cos 3x.\cos x}$$

$$= 2\left[\frac{\sin 9x}{\cos 27x} + \frac{\sin 3x}{\cos 9x} + \frac{\sin x}{\cos 3x}\right]$$

76. (C)

$$\frac{\sin x\sqrt{a+b\tan^2 x}}{\sqrt{a+(b-a)\sin^2 x}}=\frac{\sin x\sqrt{a+b\tan^2 x}}{\sqrt{a\cos^2+b\sin^2 x}}=\frac{\sin x}{|\cos x|}$$

77. (D)

Equation of the lines joining the origin to the points of intersection of the given curves is

$$3x^2+pxy-4x(y+2x)+1.(y+2x)^2=0 \quad \Rightarrow \quad x^2-pxy-y^2=0$$

which are perpendicular for all values of p.

78. (A)

$$2\sin\frac{A+B}{2}\cos\frac{A-B}{2}+2\sin\frac{C+D}{2}\cos\frac{C-D}{2}=4$$

$$\Rightarrow \quad \sin A+\sin B+\sin C+\sin D=4$$

$$\therefore \quad A=B=C=D=90^\circ$$

$$\Rightarrow \quad \sum\cos\frac{A}{2}\cos\frac{B}{2}=6\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}=6\cdot\frac{1}{2}=3$$

79. (A)

minimum value of R.H.S. = 2, at x=1 and maximum value of L.H.S. is 2 at $x=2n\pi, n\in I$

Thus no. of real solution = 0.

80. (C)

sec 40°, sec 80°, sec 160° are the roots of $\frac{8}{t^3}-\frac{6}{t}+1=0$

or $t^3-6t^2+8=0$

$\therefore$ Sum of roots = 6.

81. (A)

$$\tan(\alpha+2\beta)=\frac{\tan\alpha+\tan 2\beta}{1-\tan\alpha.\tan 2\beta}=\frac{\tan\alpha+\frac{2\tan\beta}{1-\tan^2\beta}}{1-\frac{2\tan\alpha.\tan\beta}{1-\tan^2\beta}},\text{ where }\tan\beta=\frac{1}{3}.$$

82. (C)

Let $P=\cos\theta\cos 2\theta\cos 3\theta\ldots\cos 1004\theta$

and $Q=\sin\theta\ \sin 2\theta\ \sin 3\theta\ \ldots\ \sin 1004\theta$

Then $2^{1004}PQ = \sin 2\theta \sin 4\theta \ldots \sin 2008\theta$

$= (\sin 2\theta \sin 4\theta \ldots \sin 1004\theta)\ [\sin(2\pi - 1003\theta)\sin(2\theta - 1001\theta)\ldots\sin(2\pi - \theta)]$

$= (\sin 2\theta \sin 4\theta \ldots \sin 1004\theta)\ [-\sin 1003\theta][-\sin 1001\theta] \ldots [-\sin\theta] = Q$

$\Rightarrow \quad P = \dfrac{1}{2^{1004}}$.

83. (A)

$abc = 6^6$ and $b^2 = ac \Rightarrow b = 36$

Now, a = 27 as it divides b^2. Therefore, c = 48.

84. (D)

Let $x + \dfrac{y}{8} = m$; $x - \dfrac{y}{8} = n$

$x = \dfrac{m+n}{2}$, $y = 4(m - n)$

$f(m,n) = 2(m^2 - n^2)$

Similarly $f(n, m) = 2(n^2 - m^2)$

$= f(m,n) + f(n, m) = 0 \ \forall\ m, n$

85. (D)

$f(x) = \log_{\sqrt{2}}\left(2 - \log_2\left(16\sin^2 x + 1\right)\right)$

$1 \le 16\sin^2 x + 1 \le 17$

$\therefore \quad 0 \le \log_2(16\sin^2 x + 1) \le \log_2 17$

$\therefore \quad 2 - \log_2 17 \le 2 - \log_2(16\sin^2 x + 1) \le 2$

Now consider

$0 < 2 - \log_2(16\sin^2 x + 1) \le 2$

$\therefore \quad -\infty < \log_{\sqrt{2}}[2 - \log_2(16\sin^2 x + 1)] \le \log_{\sqrt{2}} 2 = 2$

$\therefore$ the range is $(-\infty, 2]$

86. (A)

$g(f(x)) = \tan\left(x - \dfrac{\pi}{4}\right) = \dfrac{\tan x - 1}{\tan x + 1} \Rightarrow g(x) = \dfrac{x-1}{x+1}$

$f(g(x)) = \tan\left(\dfrac{x-1}{x+1}\right)$.

87. (C)

$\because 0 \le \{x\} < 1$

$\Rightarrow -1 < -\{x\} \le 0$

$\Rightarrow \cos^{-1}(0) \le \cos^{-1}(-\{x\}) < \cos^{-1}(-1)$........ ($\because \cos^{-1} x$ is a decreasing function)

88. (B)

$h(x) = \ell n(f(x).g(x)) = \ell n\, e^{[K]+\{K\}}; K = e^{|x|}.\text{sgn}(x)$

$$\therefore h(x) = \begin{cases} e^{x} & ; \quad x > 0 \\ 0 & ; \quad x = 0 \\ -e^{-x} & ; \quad x < 0 \end{cases}$$

89. (C)

$x^2 - [x^2] > 0 \Rightarrow \{x^2\} > 0 \Rightarrow \{x^2\} \ne 0$

$\Rightarrow x \ne \pm\sqrt{n}; n \in I^{+} \cup \{0\}$

90. (A)

Domain of f is [–1, 1]

$$f'(x) = \cos x - \sin x + \sec^2 x + \frac{1}{1+x^2} > 0$$

$$(f(x))^{\min} = f(-1) = -\sin 1 + \cos 1 - \tan 1 - \frac{\pi}{2} + \pi - \frac{\pi}{4} = m$$

$$(f(x))^{\max} = f(1) = \sin 1 + \cos 1 + \tan 1 + \frac{\pi}{2} + \frac{\pi}{4} = M$$