SOLUTIONS

MEAITS 2018

UNIT TEST-2

GRA

(MAIN & ADVANCED PATTERN)

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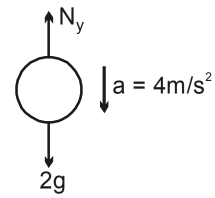
JEE MAIN

PHYSICS

1. (B)

$\vec{v} = \frac{d\vec{r}}{dt} = 4\hat{i} - 4t\hat{j}$

$\vec{a} = \frac{d\vec{v}}{dt} = -4\hat{j}$



F.B.D of base.

$2g - N_y = 2 \times 4$

$N_y = 20 - 8 = 12$ newton

2. (A)

$f_{lim} = \mu N = 0.2 \times 50 = 10$ N.

at t = 0, $F = 4.0 = 4N$

At t = 3 sec., F = 1 N

Hence frictional force = 1 N

3. (A)

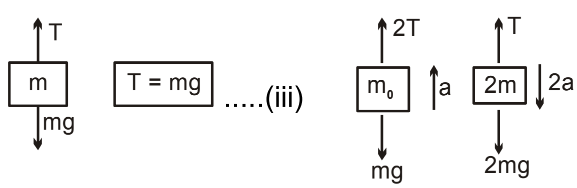
Net force along $F_2 = 30 + \frac{20}{2} = 40$ N

Net force along $F_3 = 10 - 10 = ON$

$\therefore\ F_{net} = \sqrt{(40)^2 + (0)^2} = 40$ N

Hence, frictional force = 40 N

4. (A)



$2T - mg = m_0 a$ (ii)

$2mg - T = 2m.2a$

$2mg - T = 4ma$ – (iii)

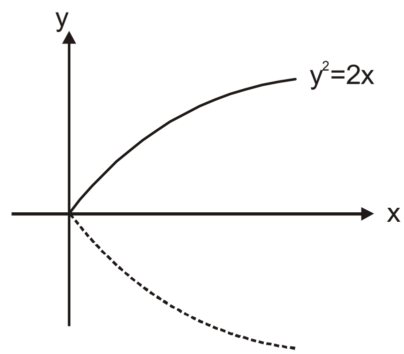
From (i), (ii) and (iii)

$\boxed{m_0 = \frac{8}{5}m}$

5. **(B)**

$$\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2=4 \qquad ...(1)$$

$$\frac{dy}{dt}=\frac{1}{y}\cdot\frac{dx}{dt}=\frac{1}{\sqrt{2x}}\cdot\frac{dx}{dt} \qquad ...(2)$$



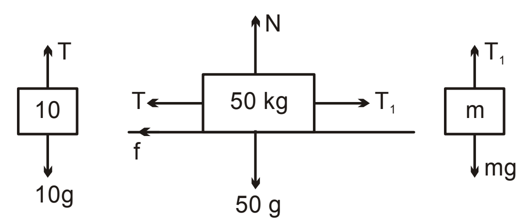
From (1) & (2)

$$a=\frac{4}{5\sqrt{5}}$$

thus, $F=\frac{4}{5}$

6. **(C)**

If slipping tendency is towards rightward. For 50 k.g block.



N = 500 newton. $\boxed{T_1=mg}$

$f_{lim}=\mu\cdot 500$

$=50$ newton.

For Eqb. of 50 k.g. block

$T_1=100+50=150$ newton

Hence $m=\frac{T_1}{g}=15$ k.g

Similarly for slipping tendency towards leftward.

$m=5$k.g

Hence required range

$5\le m\le 15$

7. (B)

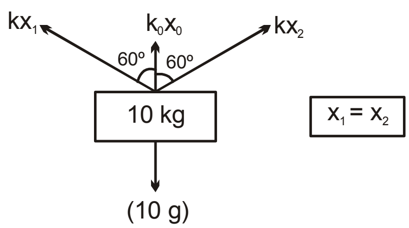
After equilibrium spring force = $m_1 g$

Acceleration of m_3 just after cutting of string $a_3 = g$

Acceleration of $m_2 = \left(\frac{m_1 + m_2}{m_2}\right)g$

Ratio of acceleration = $\left(\frac{m_1 + m_2}{m_2}\right)$

8. (B)



$$k_0 x_0 + 2kx_1 \cos 60^\circ = Mg$$

$$2kx_1 \cos 60^\circ = Mg - k_0 x_0$$

$$= 100 - \frac{5}{100} \times 1000$$

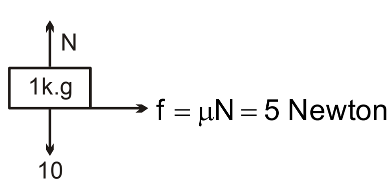
$$= 50\ N$$

$$a_x = 0\ m/s^2$$

$$a_y = \frac{50}{10} = 5\ m/s^2$$

9. (A)

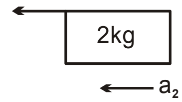
$\rightarrow a_1$



$$a_1 = \frac{5}{1} = 5\ m/s^2$$

$$v_1 = -2 + 5t$$

f = 5N



$$a_2 = \frac{5}{2} = 2.5 m/s^2$$

$$v_2 = 8 - \frac{5}{2}t.$$

At

$$v_1 = v_2$$

$$-2 + 5t = 8 - \frac{5}{2}t$$

$$5t + \frac{5}{2}t = 10$$

$$\frac{15}{2}t = 10$$

$$t = \frac{20}{15} = \frac{4}{3} \text{ sec.}$$

$$V_{common} = -2 + 5 \cdot \frac{4}{3}$$

$$= \frac{-6+20}{3} = \frac{14}{3} m/s.$$

10. (A)

Tension in rubber band

$$T = k \cdot 2\pi (R - r)$$

Consider a small section of rubber band subtands an angle $\Delta\theta$ at the centre

$$N = 2 \cdot T \cdot \sin\frac{\Delta\theta}{2} = T\Delta\theta$$

$\Rightarrow$ Frictional force on rubber band

$$f = \int_0^{2\pi} \mu T d\theta = 2\pi\mu T$$

$$= 2\pi\mu \cdot 2\pi k (R - r)$$

$$= 4\pi^2 \mu k (R - r)$$

11. (B)

$$I = k\sin^2\theta$$

$$\frac{dI}{d\theta} = k\cdot 2\sin\theta\cdot\cos\theta$$

$$dI = 2k\sin\theta\cdot\cos\theta\cdot d\theta$$

$$\rightarrow \frac{dI}{I} = \frac{2k\sin\theta\cdot\cos\theta\cdot d\theta}{k\sin^2\theta}$$

$$\rightarrow \frac{dI}{I} = 2\cot\theta\cdot d\theta$$

$\Rightarrow$ % error in angle

$$= \frac{d\theta}{\theta}\cdot 100 = \frac{dI}{2\cdot I\cdot\cot\theta}\cdot\frac{1}{\theta}\cdot 100 = \frac{0.005}{2\cdot 2}\cdot\frac{1}{\cot 45^\circ}\cdot\frac{4}{\pi}\cdot 100$$

$$= \frac{5}{1000}\cdot\frac{100}{\pi} = \frac{1}{2\pi}\%$$

12. (A)

$$\frac{I_{max}}{I_{min}} = \frac{16}{I}$$

$$\frac{(a_1+a_2)^2}{(a_1-a_2)^2} = \frac{16}{1}$$

$$\Rightarrow a_1 + a_2 = 4a_1 - 4a_2$$

$$\Rightarrow 5a_2 = 3a_1$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{5}{3}$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{25}{9} = \frac{b_1}{b_2} \rightarrow \text{width Ratio}$$

13. (A)

$$\beta = \frac{\lambda D}{d} = \frac{600\times 10^{-9}}{5\times 10^{-4}}\times\frac{1}{2}$$

$$= 6\times 10^{-4}$$

$$= 0.6\,\text{mm}$$

Required distance = $4\beta + \frac{5}{2}\beta$

$= 6.5\beta$

$= \frac{13}{2} \times 0.6 = \frac{7.8}{2}\text{mm}$

$= 3.9\,\text{mm}$

14. (C)

Angular stringe width

$\theta = \lambda / d$

$\frac{\theta_{air}}{\theta_{liquid}} = \frac{\lambda_{air}}{\lambda_{liquid}}$

$\frac{\theta_{air}}{\theta_{liquid}} = \frac{5}{3}$

$\theta_{liquid} = 0.30^{o}$

15. (C)

$\tan\theta_B = \mu = \frac{4}{3}$

$\theta_B = 53°$

$r = 90° - \theta_B = 90° - 53°$

$= 37°$

16. (B)

Conceptual.

17. (B)

For maxima

$d\sin\theta = \pm n\lambda$

$\Rightarrow \sin\theta = \pm\frac{n}{3}$

$n = 0, 1, 2, 3$

But n = 3 is not possible.

Hence. Total no. of maxima = 5

for minima

$$d\sin\theta = \pm(2n-1)\frac{\lambda}{2}$$

$$\sin\theta = \pm(2n-1)\frac{\lambda}{2d}$$

$$\sin\theta = \pm\frac{(2n-1)}{6}$$

Here. n = 1, 2, 3

$\Rightarrow$ Total no. of minima = 6

sum = 5 + 6 = 11

18. (A)

19. (B)

$\Rightarrow x^2 + y^2 = c$ is a circle

$$\frac{dy}{dx} = -x / y$$

force is along tangent to the path of the block

$$F = \sqrt{F_x^2 + F_y^2} = k$$

$$\omega = \int f ds = k\int ds = 2\pi RK$$

20. (A)

Work done by spring force does not depend on the frame of reference.

$$w_{ext} = -\left(-\frac{1}{2}kx_0^2\right) = \frac{1}{2}kx_0^2.$$

21. (D)

$$W = \text{change in} K \cdot E$$

$$= \frac{1}{2}m\nu^2 - \frac{1}{2}mu^2$$

$$v(t = 2s) = 3t^2 = 12 m / s.$$

u (t = 1s) = $3t^2$ = 3. (1) 2 = 3m/s.

$$W = \frac{1}{2}\cdot 1\cdot 144 - \frac{1}{2}\cdot 1\cdot 9$$

= 67.5 J

22. (B)

Conceptual.

23. (A)

$$\langle p \rangle = \frac{mg \cdot \frac{1}{2} + \frac{1}{2} m\nu^2}{\ell / v_0}$$

$$= \frac{1 \cdot 10 \cdot \frac{1}{2} + \frac{1}{2} \cdot (1) \cdot (1)^2}{\frac{1}{1}} = 5.5 \text{ W}$$

24. (A)

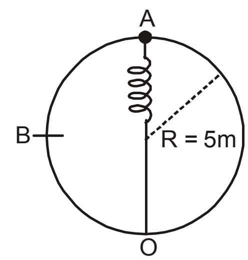
F is conservative

$$W_{AB} = \int_0^2 x^2 dx = \frac{8}{3}$$

25. (D)

26. (D)

From conservation of energy at point A & B



$$m\nu^2 = 2mgR + k(2 - \sqrt{2})^2 R^2$$

$$N = \frac{m\nu^2}{R} = 26N$$

27. (B)

For dark band

$$2\mu t \cos r = n\lambda$$

for λ = 4000 Å

Let n = n_1

$$n_1 = \frac{2\mu t \cos r}{\lambda_1}$$

cosr = 0.86

$\Rightarrow r_1 \approx 60$

Similary $r_2 \approx 48$

Hence No. of dark band = 12

28. (B)

$$Y_n = \frac{n\lambda D}{d}$$

$$\lambda = \frac{Y_n d}{nD} = \frac{10^{-3} \times 2 \times 10^{-3}}{n \times 2.5}$$

$$= \frac{200000}{25} \times \frac{10^{-10}}{n} m$$

$\Rightarrow n = 1 \Rightarrow \lambda = 8000\,\text{Å}$ Infrared

$n = 2 \Rightarrow \lambda = 4000\,\text{Å}$

29. (C)

$2\mu t = (2n - 1)\lambda / 2$

$$t = (2n - 1)\frac{\lambda}{4\mu}$$

μ = 1.25

$$t = (2n - 1)\frac{6000}{4.125}$$

t = (2n – 1) 1200 Å

t = 1200 Å

30. (C)

$\boxed{3kg} \rightarrow 3T$ ($\rightarrow a_1$) $4T \leftarrow \boxed{4kg}$ ($\leftarrow a_2$)

a_1 = T   (at point P)

a_2 = T   (F = T = 1 N)

Now, $a_{PB} = 3a_{AB}$ (As a_{AB} = 2 m/s²)

$\therefore \quad a_P = a_B + 3a_{AB} = 1 + 6 = 7\ m/s^2$

CHEMISTRY

31. (B)

$$Z = \frac{PM}{dRT} = \frac{1 \times 78}{1.56 \times 0.082 \times 500} = \frac{50}{41}$$

32. (D)

Let, there be (240x)g. of each CH_4 & C_2H_6

$$\therefore n_{CH_4} = \left(\frac{240x}{16}\right)\text{mol} = (15x)\text{mol}$$

$$\& \therefore n_{C_2H_6} = \left(\frac{240x}{30}\right)\text{mol} = (8x)\text{mol}$$

$$\therefore \frac{KE_{CH_4}}{KE_{CH_4}} = \frac{(15x) \times \frac{3}{2}RT_{CH_4}}{(8x) \times \frac{3}{2}RT_{C_2H_6}} = \frac{5}{2}$$

$$\Rightarrow \frac{15}{8} \times \frac{T_{CH_4}}{T_{C_2H_6}} = \frac{5}{2}$$

$$\Rightarrow \frac{T_{CH_4}}{T_{C_2H_6}} = \frac{5 \times 8}{2 \times 15} = \frac{4}{3} = 4:3$$

33. (A)

$$P = \frac{1}{3V}.Nm\overline{v^2}$$

$$= \left(\frac{1}{3 \times 10^{-3}} \times 10^{24} \times 10^{-26} \times 10^{6}\right)\text{Pa}$$

$= 3.33 \times 10^{6}$ Pa

34. (B)

The maximum value of pressure, $P_{CO_2} = 0.8$ atm

$\therefore$ Using $P_1V_1 = P_2V_2$

$\Rightarrow$ 0.2 atm × 10 L = 0.8 atm × V_2

$\therefore \quad V_2 = 2.5\,L$.

35. (C)

If He (g) fills the bag completely by same mole of it & at same temp. Then pressure of He be

P_{He} & is given as $P_{He} = \frac{0.3\ \text{atm} \times 1\text{L}}{3\text{L}} = 0.1\ \text{atm}$

$\therefore$ Partial pr. of $\text{Ne} = (0.4 - 0.1)\ \text{atm} = 0.3\ \text{atm}$.

$\therefore$ mole fraction of Ne, $x_{Ne} = \frac{P_{Ne}}{P_{total}} = \frac{0.3\text{atm}}{0.4\text{atm}} = \frac{3}{4}$

36. (C)

Let moles of N_2 injected = x then

$$\frac{32 \times 1 + 2x}{1 + x} = \frac{88}{3} \Rightarrow x = 2$$

Now, let moles of gaseous H_2O in container = y then

$$\frac{32 \times 1 + 28 \times 2 + 18y}{1 + 2 + y} = 28.2 \Rightarrow y = \frac{1}{3}$$

So, mass of H_2O in gaseous form $= \frac{1}{3} \times 18\ \text{g} = 6\ \text{g}$

i.e., mass of liquid water in container = (20 – 6) = 14 g

So, volume of liquid water in container = 14 ml [$\because d_{H_2O(l)} = 1\ \text{g/ml}$]

37. (D)

32°F = 0°C and 10^5 Pa = 1 bar

So, condition is S.T.P.

Volume = 0.681 dm^3 = 0.681 litre.

So, $\frac{3.24}{28 + 16x} = \frac{0.681}{22.7} \Rightarrow \frac{324}{28 + 16x} = 3$

$$\Rightarrow 324 = 84 + 48x \Rightarrow 48x = 324 - 84 \Rightarrow x = \frac{240}{48} = 5$$

Let oxidation of N be y, then

$$2y - 5 \times 2 = 0 \Rightarrow y = +5$$

38. (C)

Balanced reaction is

$$\underset{x\ \text{moles}}{3I_2} + 3H_2O \longrightarrow \underset{10\ \text{moles}}{IO_3^-} + 5I^- + 6H^+$$

So, $\frac{3}{1} = \frac{x}{10} \Rightarrow x = 30$ moles

[OR]

n-factor of I_2 is $\frac{5}{3}$ and IO_3^- is 5.

So, $x \times \frac{5}{3} = 5 \times 10 \Rightarrow x = 30$ moles

39. (B)

Volume of 2 mole gas at 1 atm and 0°C = 44800 cm^3

According to Charle's law,

$$\frac{1}{273.15} \times 44800(T - 300) = 4480$$

$$T = 327.315K \simeq 327K$$

40. (C)

$P_{gas\,f} + 15 = 76 + 25$

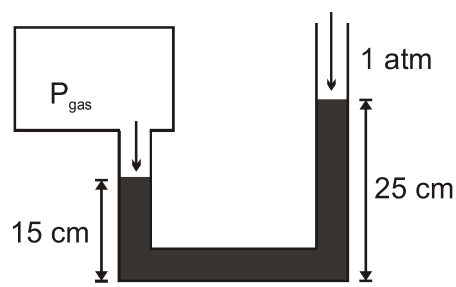
$P_{gas\,f} = 76 + 25 - 15 = 86$ cm of Hg

From Boyle's Law

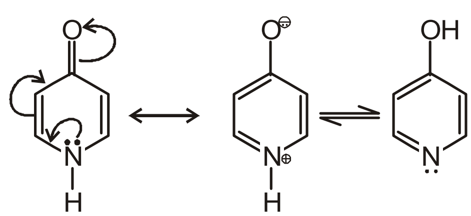
$P_i V_i = P_f V_f$

$P_i V = 86 \times V \times \frac{110}{100}$

$P_i = 86 \times 1.1 = 94.6$ cm of Hg



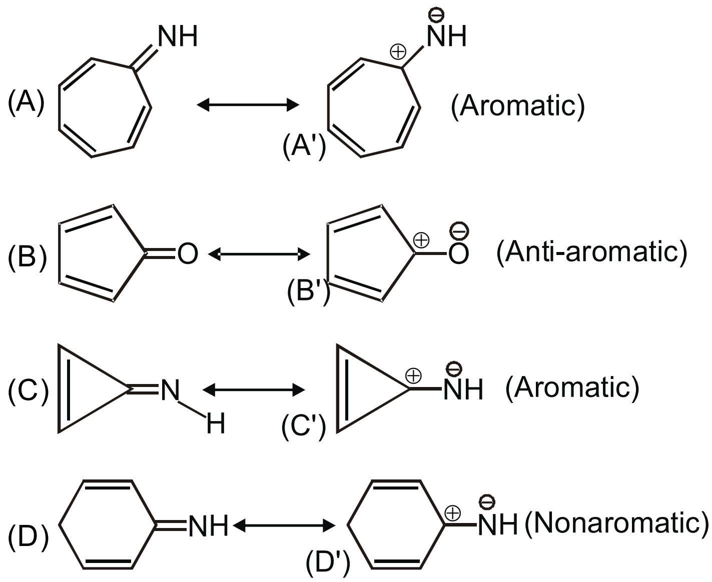
41. (B)



42. (D)

Compound D is a cummulative double bonded molecule having odd no. of cumulated double bond (hence shows geometrical isomerism) with chiral carbon (hence shows optical isomerism).

43. (A)



stability of : A' > C' > D' > B'

∴ compound in option A is most polar.

44. (B)

Due to the rigid geometry of compound (B), it has only two optically active isomer allthough it has two chiral carbons.

45. (D)

Compound haing no plane of symmetry or centre of symmetry, so it is optically active, and also having two fold axis of symmetry.

46. (D)

47. (B)

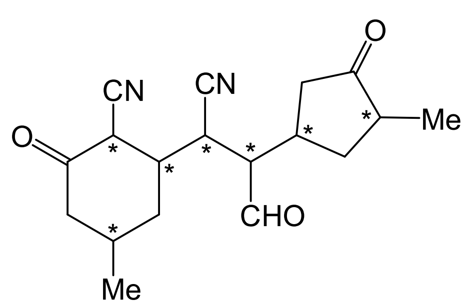
Compound 'B' having a chiral carbon and one geometrical centre, so it can show geometrical as well as optical isomerism.

48. (B)

Compound (B) having a chiral carbon so, it can show optical isomerism.

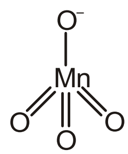
49. (B)

50. (A)

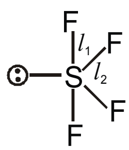


* = Chiral carbons

51. (A)



52. (A)



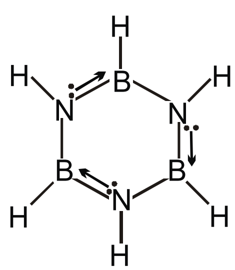
$l_1 > l_2$

Hint : $PCl_5(s) \longrightarrow PCl_4^+ + PCl_6^-$

sp^3 sp^3d^2

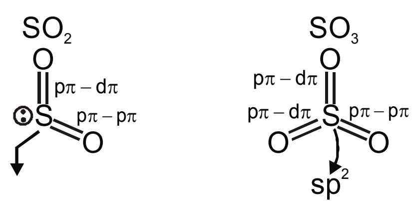
all bonds are identical bond length

53. (C)



sp^2, $\mu = 0$ } Planar as well as non-polar

54. (D)

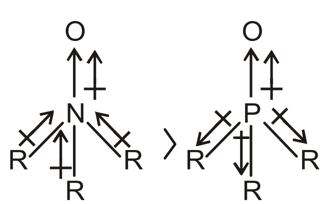


sp^2 (one p-orbital = unhybridized)

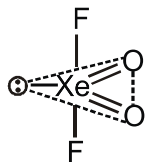
↓

π – bond

55. (B)



56. (A)

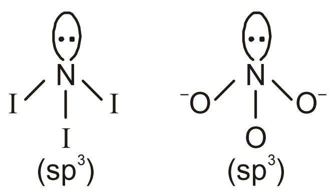


57. (D)

$H - C \equiv N: \rightarrow$ L.P. = 1, σ-bonds = 2, π bonds = 2

$H - \overset{+}{N} \equiv \overset{-}{C}: \rightarrow$ L.P. = 2, σ-bonds = 2, π bonds = 2

58. (B)



59. (C)

Electronegativity of S < O < F.

60. (A)

$$\underset{sp^2}{\underset{\uparrow}{AlH_3}} \longrightarrow \underset{sp^3}{\underset{\uparrow}{AlH_4^-}}$$

MATHEMATICS

61. (C)

$$f'(x+y)(1+y') = g'(x+y)(1+y')$$

$$\Rightarrow \{f'(x+y) - g'(x+y)\}(1+y') = 0 \Rightarrow y' = -1$$

62. (C)

For $x^2 + y^2 - 9 = 0$

Center : $C_1 \equiv (0,0)$; radius: $r_1 = 3$

For $x^2 + y^2 + 2ax + 2y + 1 = 0$

Centre : $C_2 :\equiv (-a, -1)$; radius: $r_2 = |a|$

$\text{Now}, C_1C_2 = r_1 + r_2 \text{ or } |r_1 - r_2|$

$\Rightarrow |a| = \frac{4}{3}$

63. (C)

$$f'(x) = 3\left(\frac{\sqrt{a+1}}{a-1} - 1\right)x^2 - 1 < 0,\ \forall x \in R$$

If $\frac{\sqrt{a+1}}{a-1} - 1 < 0$

64. (B)

The slope of chord is $m = -\frac{8}{y} \Rightarrow y \in \{-8, -4, -2, -1, 1, 2, 4, 8\}$

but (8, y) must also lie inside the circle $x^2 + y^2 = 125$

$\Rightarrow$ y can be equal to $\pm 1, \pm 2, \pm 4 \Rightarrow 6$ values

65. (C)

$f(x) = \ln x - (x^2 - 1)$

$$f'(x) = \frac{1}{x} - 2x < 0\ \forall x > 1$$

66. (B)

Since SP, 4, SQ are in H.P.

$$4 = \frac{2SP \cdot SQ}{SP + SQ}$$

SQ = 3

Hence PQ = 6 + 3 = 9

67. (C)

Let f(x) = (x – 3) logx

f(1) = 0 & f(3) = 0, As (x –3) and logx are continuous and differentiable in (1, 3) so by Rolle's theorem, there exists at least one value of x in (1, 3) such that

$$f'(x) = 0 \Rightarrow \log x + (x-3)\frac{1}{x} = 0 \Rightarrow x\log x = 3 - x$$

68. (B)

$$P'(x) = 2a_1x + 4a_2x^3 + ... + 2na_nx^{2n-1}$$

$= 2x\left(a_1 + 2a_2x^2 + \ldots + na_nx^{2n-2}\right)$

$P'(x) = 0$ only at x = 0,

Now, $P''(x) = 2a_1 + 12a_2x^2 + \ldots + 2n(2n-1)a_nx^{2n-2}$

$P''(0) = 2a_1 > 0, \Rightarrow$ x = 0 is point of minima

69. **(A)**

$$x = \frac{\sin y}{\cos(a+y)}$$

$$\frac{dx}{dy} = \frac{\cos a}{\cos^2(a+y)}$$

70. **(C)**

$$f'(x) = \frac{-2}{3(x+1)^{2/3}}$$

$f'(-1)$ does not exist

also, $f'(-1^-) < 0$ & $f'(-1^+) < 0$

71. **(C)**

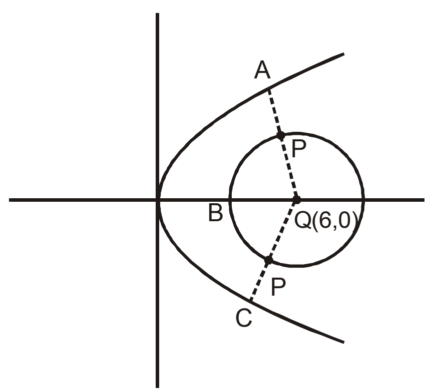
Shortest distance will take place along the common normal for $y^2 = 4x$ at $\left(t^2, 2t\right)$

is $y = -tx + 2t + t^3$ if passes through (6,0), then

$t^3 - 4t = 0 \Rightarrow t = 0, t = \pm 2$

$A \equiv (4,4), C \equiv (4,-4)$

$PA = PC = \sqrt{5}$



72. **(B)**

$f'(x) = (x-1)^{m-1}(x-2)^m\left[(2m+1)x - (3m+1)\right]$

If m is odd then (m –1) is even, at x = 1 is point of inflection

If m is even then x = 2 is point of inflection

73. **(C)**

Orthocentre of the triangle formed by three tangents of a parabola lies on the directirix

74. **(D)**

P = (6, 6)

The equation of circle touching y = x at (6,6) is

$(x-6)^2 + (y-6)^2 + \lambda(x-y) = 0$

$x^2 + y^2 + x(\lambda - 12) - y(\lambda + 12) + 72 = 0$

Hence c = 72

75. **(B)**

Focus (1, 3) and directrix 5x – 12 y + 17 = 0

$\text{Latus rectum} = 4a = 2x\left|\frac{5-36+17}{\sqrt{(5)^2+(12)^2}}\right| = \frac{2\times 14}{13} = \frac{28}{13}$

76. **(D)**

Minimum value of $a\tan^2 x + b\cot^2 x$ is $2\sqrt{ab}$ and maximum value of $a\sin^2\theta + b\cos^2\theta$ is 'a'

So a = $2\sqrt{ab}$ $\Rightarrow$ a = 4b

77. **(A)**

f is increasing & g is decreasing

$x < x + 1$

$g(x) > g(x+1)$

$f(g(x)) > f(g(x+1))$

also $x > x - 1$

$\Rightarrow f(x) > f(x-1)$

$\Rightarrow g(f(x)) < g(f(x-1))$

$x > x - 1$

$\Rightarrow g(x) < g(x-1)$

$\Rightarrow f(g(x)) < f(g(x-1))$

78. **(D)**

Minimum length of focal chord is length of latus rectum

79. **(B)**

$f'(x) = -\pi\sin\pi x + 10 + 6x + 3x^2$

$= 3(x+1)^2 + 7 - \pi\sin x > 0 \quad \forall x \in R$

$\Rightarrow$ f(x) is increasing, hence the required value is

f(–2) = 1 – 20 + 12 – 8 = –15

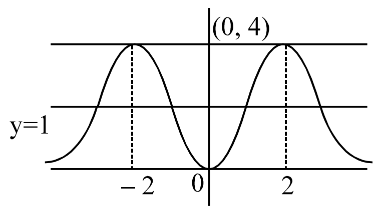
80. (A)

$$\frac{dy}{dx} = \frac{\sin\theta}{1+\cos\theta}\bigg|_{\frac{\pi}{2}} = 1$$

$$= y\sqrt{1+\left(\frac{dy}{dx}\right)^2} = a\sqrt{2}$$

81. (B)

draw graphs of $y = x^2 . e^{2-|x|}$ and $y = 1$



$$y = \left[\begin{array}{ll} x^2 \cdot e^{2-x} & \text{if } x \geq 0 \\ x^2 \cdot e^{2+x} & \text{if } x < 0 \end{array}\right]$$

82. (B)

When x = 1/2, y = 1/e

as; $x = \frac{1}{2} > 0$; $y = e^{-2x} \Rightarrow \frac{dy}{dx} = -2e^{-2x}$

$$\frac{dy}{dx} \text{ at } \left(\frac{1}{2}, \frac{1}{e}\right) = -\frac{2}{e}$$

$\therefore$ Equation of normal

$$y - \frac{1}{e} = \frac{e}{2}\left(x - \frac{1}{2}\right)$$

$$\Rightarrow 2e(ex - 2y) = e^2 - 4$$

83. (C)

$2a^2 + a + 1 > 3a^2 - 4a + 1$

$\Rightarrow a^2 - 5a < 0 \quad \Rightarrow a \in (0, 5)$ ... (i)

further $2a^2 + a + 1 > 0$
and $3a^2 - 4a + 1 > 0$ $\Rightarrow a < \frac{1}{3}$ or $a > 1$ ... (ii)

From (i) and (ii)

$$a \in \left(0, \frac{1}{3}\right) \cup (1,5)$$

84. **(C)**

Let

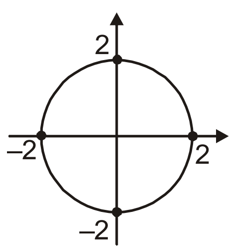
$$f(x) = ax^4 + bx^3 + cx^2 + dx + e$$

f(3) = e, f(0) = e

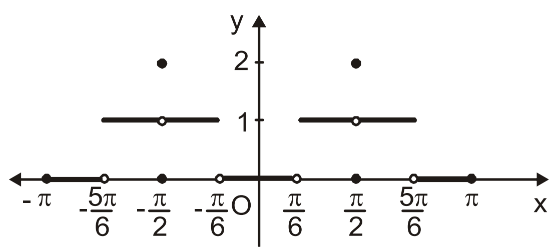
$\therefore f'(x) = 0$ has at least one root between 0 and 3

85. **(C)**

$x^2 + y^2 = 4$



y = [|sinx| + sin|x|]

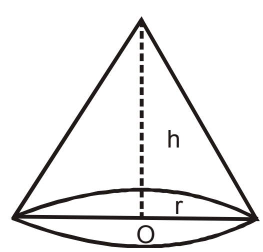


Points of intersection are : $(\sqrt{3},1)$ and $(-\sqrt{3},1)$

Angle of intersection is : $\theta = 60^\circ$

86. **(C)**

Given that $h = \frac{1}{6}r$,



volume, $V = \frac{1}{3}\pi r^2 h$

$$V = \frac{1}{3}\pi(6h)^2 h = 12\pi h^3$$

$$\frac{dV}{dt} = 36\pi h^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{3\pi \times 16} = \frac{1}{48\pi}$$

87. (A)

Let $\cos(\sin x) = \sin(\cos x) \; = \cos\left(\frac{\pi}{2} - \cos x\right)$

$\Rightarrow \quad \sin x + \cos x = \frac{\pi}{2} > \sqrt{2}$ or $\sin x - \cos x = -\frac{\pi}{2} < -\sqrt{2}$, which is not possible.

$\Rightarrow$ either, cos (sinx) > sin (cosx) $\forall\ x \in R$

or, cos (sinx) < sin (cosx) $\forall\ x \in R$

At x = 0 :

cos(sin0) > sin (cos0)

$\Rightarrow$ cos (sinx) > sin (cosx) $\forall\ x \in R$

88. (B)

f (xy) = 0

$$f'(xy)\left\{y + x\frac{dy}{dx}\right\} = 0$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

89. (D)

Image of focus in the tangent always lies on the directrix of parabola.

i.e. (2, –3) and (–2, 3) will lie on the directrix

slope of directrix $= \frac{6}{-4} = \frac{-3}{2}$

hence slope of the axis of parabola is $\frac{2}{3}$

90. (D)

Area of quadrilateral $= \sqrt{c} \times \sqrt{9 + 25 - c} = 15$

c = 9 , 25

JEE ADVANCED

PHYSICS

1. (D)

At origin, $\frac{dU}{dx} = +ve$ and hence force is –ve, as a result, it will move in –ve x-axis.

When the particle is released at x = +2, it will reach the point of least potential energy (–15J), where it will have maximum kinetic energy,

$\frac{1}{2}mv_{max}^2 = 25$ $\Rightarrow v_{max} = 5m/s$

The particle will now perform oscillatory motion with x = 5, as mean position.

It option (C), $E_i = U_i + K_i = 15 + 6 = 21J$

At x = 10, $U_f = 10 \Rightarrow K_f = 1$

So particle will crosses x = 10

2. (B)

$$a = g\sin\theta - \mu g\cos\theta = g\left(\sin\theta - \frac{x}{2}\cos\theta\right) = \frac{g(2-x)}{2\sqrt{2}} \quad (\because \theta = 45^\circ)$$

$$v\frac{dv}{dx} = \frac{g(2-x)}{2\sqrt{2}}$$

$$\int_0^v vdv = \frac{g}{2\sqrt{2}} \int_0^2 (2-x)dx$$

$$\Rightarrow \frac{v^2}{2} = \frac{g}{2\sqrt{2}}\left[2x - \frac{x^2}{2}\right]_0^2$$

$$\Rightarrow v = \sqrt{g\sqrt{2}}\ m/s$$

Also it is Clear that for x < 2, the body accelerates; at x = 2, acceleration is zero and for x > 2, the body retards till it comes to rest.

3. (C)

Direction of light is given by normal vector $\vec{n} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

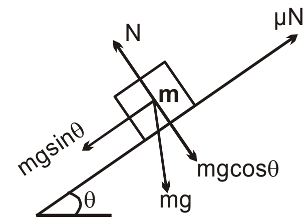
$\therefore$ Angle made by $\vec{n}$ with z-axis is given by $\cos\gamma = \frac{5}{\sqrt{3^2+4^2+5^2}} = \frac{1}{\sqrt{2}}$

4. (A)

$a = g\sin\theta - \mu g\cos\theta$

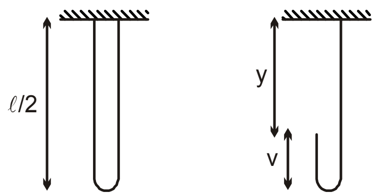
$\because \quad \mu_1 > \mu_2 \quad \Rightarrow a_1 < a_2$

Hence T = 0 & $N \neq 0$



5. (A)

From energy conservation



$K_i + U_i = K_f + U_f$

$$\Rightarrow 0 + \left[-\lambda\ell g\left(\frac{\ell}{4}\right)\right] = \frac{1}{2}\lambda\left(\frac{\ell - y}{2}\right)v^2 - \lambda(L-y)g\left(y + \frac{L-y}{4}\right)$$

$$\Rightarrow v = \sqrt{2gy\left(\frac{L - y/2}{L-y}\right)}$$

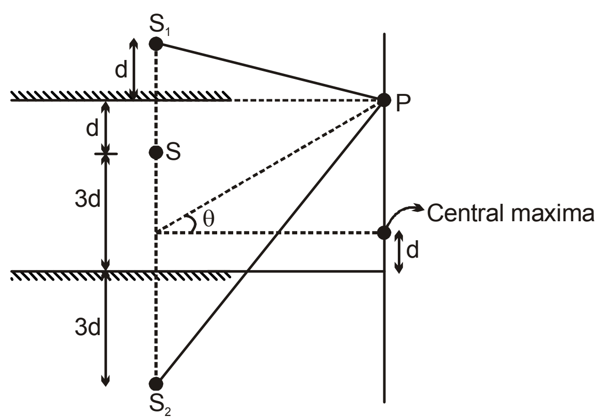
6. (A)

$\because \quad \Delta x = 2\lambda$

$8d\sin\theta = 2\lambda$

$$\Rightarrow 8d \times \frac{3d}{D} = 2\lambda$$

$$\Rightarrow \lambda = \frac{12d^2}{D}$$



7. (C)

Conceptual

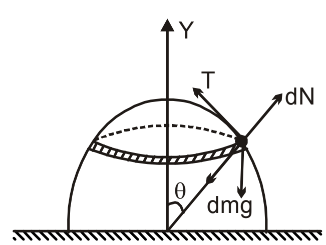
8. (D)

$$2T\sin\frac{d\phi}{2} - dN\sin\theta = dm\frac{R}{\sqrt{2}}\omega^2 \qquad ...(i)$$

$$dN\cos\theta = dmg \qquad ...(ii)$$

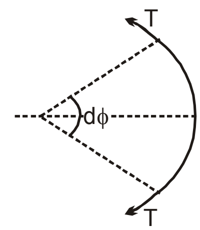
From (i) and (ii)

$$2T\frac{d\phi}{2} - dmg\tan\theta = dm\frac{R}{\sqrt{2}}\omega^2$$



$$\Rightarrow \quad T\int_0^{2\pi} d\phi - g\tan\theta\int_0^m dm = \frac{R}{\sqrt{2}}\omega^2\int_0^m dm$$

$$\Rightarrow \quad T = \frac{m}{2\pi}\left(g\tan\theta + \frac{R}{\sqrt{2}}\omega^2\right)$$



According to questions, $\theta = \frac{\pi}{4}$ & $\omega = \sqrt{\frac{g}{R}}$

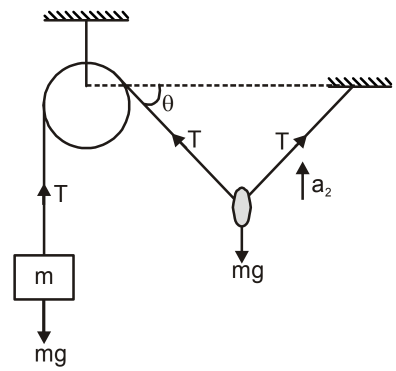
$$T = mg\frac{\left(\sqrt{2}+1\right)}{2\sqrt{2}\,\pi}$$

Solution for (9 & 10)

$2T\sin\theta - mg = ma_2$ ...(i)

$mg - T = ma_1$ ...(ii)

$a_1 = 2a_2\sin\theta$ ...(iii)



from equation (i), (ii) and (iii)

$$a_1 = \frac{2g(2\sin\theta - 1)}{1+4\sin^2\theta}\sin\theta$$

$$a_2 = \frac{(2\sin\theta - 1)g}{1+4\sin^2\theta}$$

For both to be in equilibrium

$$a_1 = a_2 = 0$$

$$\Rightarrow \sin\theta = \frac{1}{2}$$

$$\Rightarrow \theta = 30^o$$

9. (A)

10. (B)

Solution for (11 & 12)

$0 \le t \le 0.4, \;\; F = 800N, \; v = \frac{20}{0.5} = 40t$

$P = \vec{F}.\vec{v} = 800 \times 40t = (32t)KJ$

For $0.4 \le t \le 0.5$

$F = 4000 - 8000t,\ v = 40t$

$P = \vec{F}.\vec{v} = (4 - 8t) \times 10^3 \times 40t = (160t - 320t^2)KJ$

$\frac{dP}{dt} = 160 - 640t = 0$

$\Rightarrow t = \frac{160}{640} = 0.25$

$\because$ $0.25 < 0.4$, hence maximum P at t = 0.4 s

$w = \int Pdt = \int_0^{0.4} (32t)dt + \int_{0.4}^{0.5} (160t - 320t^2)dt$

$= 16\left[t^2\right]_0^{0.4} + \left[80\left[t^2\right]_{0.4}^{0.5} - \frac{320}{3}\left[t^3\right]_{0.4}^{0.5}\right]$

$= 2.56 + [7.2 - 6.5]$

$= 2.56 + 0.7 = 3.26$ KJ

11. (B)

12. (C)

Solution for (13 & 14)

$OS_3 = \frac{\lambda D}{d} = \frac{6 \times 10^{-7} \times 1}{3 \times 10^{-3}} = 2 \times 10^{-4} m$

Given intensity at S_4 is I_0

$\because$ $I = 4I_0 \cos^2 \frac{\phi}{2}$

$\Rightarrow$ $I_0 = 4I_0 \cos^2 \frac{\phi}{2}$

$\Rightarrow$ $\phi = \frac{2\pi}{3}$

$\therefore$ $\Delta x = \frac{\lambda}{2\pi} \times \Delta\phi = \frac{\lambda}{3}$

$\because$ $y = \Delta x \frac{D}{d} = \frac{\lambda D}{3d}$

$$\therefore \quad S_3S_4 = \frac{\lambda D}{d} + \frac{\lambda D}{3d} = \frac{4}{3}\frac{\lambda D}{d} = \frac{4}{3} \times 2 \times 10^{-4} = \frac{8}{3} \times 10^{-4} m$$

At S_3, $\Delta\phi_1 = 2\pi$

At S_4, $\Delta\phi_2 = \frac{2\pi}{3}$

At O', phase difference $= 2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$

$$\therefore \quad I' = I_0 + 4I_0 + 2\sqrt{I_0 \times 4I_0}\cos\frac{4\pi}{3} = 3I_0$$

13. (A) **14. (B)**

15. (B, C)

At A,

$$KR - N = \frac{mv_0^2}{R}$$

$$N = KR - \frac{mv_0^2}{R}$$

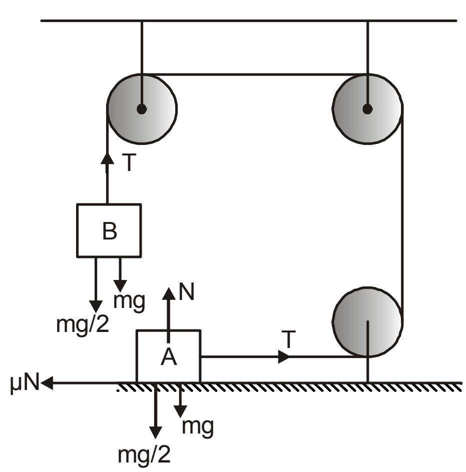
Apply work-energy theorem from A to lowest point,

$$mgR + \frac{1}{2}K\left[R^2 - \left(\sqrt{2}R - R\right)^2\right] = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$\Rightarrow \quad v = \sqrt{v_0^2 + 2gR + \frac{2KR^2}{m}\left(\sqrt{2} - 1\right)}$$

16. (B, C, D)

With respect to lift



$$N = \frac{3mg}{2}$$

$$f_\ell = \mu N = \frac{mg}{2}$$

$$\frac{3mg}{2} - T = ma \qquad ...(i)$$

$$T - \frac{mg}{2} = ma \qquad ...(ii)$$

from equation (i) and (ii)

$$a = \frac{g}{2} = 5\,m/s^2 \quad \text{and} \quad T = 1\,N$$

17. (A, C, D)

$$y = \frac{D}{d}(\mu - 1)p$$

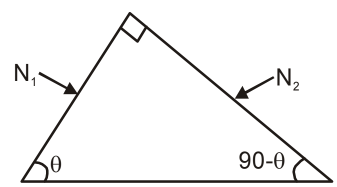
$$= \frac{D}{d}(\mu_0 + Kt - 1)p$$

$$v = \frac{dy}{dt} = \frac{D}{d}Kp$$

$$a = \frac{dv}{dt} = 0$$

18. (A, D)

For wedge to be stationary



$$N_1 \sin\theta = N_2 \sin(90 - \theta)$$

$$\Rightarrow \quad (m_1 g\cos\theta)\sin\theta = (m_2 g\sin\theta)\cos\theta$$

$$\Rightarrow m_1 = m_2$$

If $m_1 > m_2$

$$\Rightarrow \quad N_1 \sin\theta > N_2 \cos\theta$$

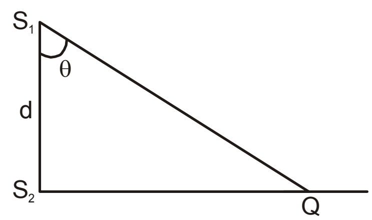
$\Rightarrow$ Wedge will move towards right

19. (B, C)

Along y-axis

$$d\sin\theta = n\lambda \qquad \Rightarrow n = \frac{d}{\lambda}\sin\theta$$

$\Rightarrow$ n is maximum, when $\sin\theta$ is maximum



$$\Rightarrow \quad n = \frac{d}{\lambda} = \frac{3\lambda}{\lambda} = 3$$

Hence number of maxima is 5 and minima 6.

Along x-axis :

$$\Delta x = S_1Q - S_2Q$$

$$= \frac{d}{\cos\theta}(1-\sin\theta)$$

Condition for maxima is,

$$\frac{d}{\cos\theta}(1-\sin\theta) = n\lambda \quad \Rightarrow n = \frac{3(1-\sin\theta)}{\cos\theta}$$

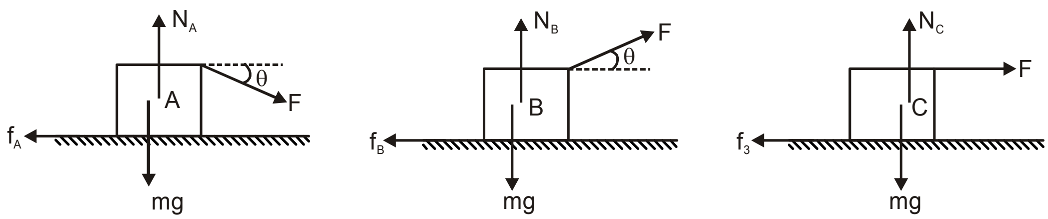
At S_2, $\theta = 0 \quad \Rightarrow n = 3$

i.e., 3rd order maxima

At $x \to \infty, \theta = \frac{\pi}{2} \quad \Rightarrow n = 0$

Hence along x-axis 3 minima will be observed

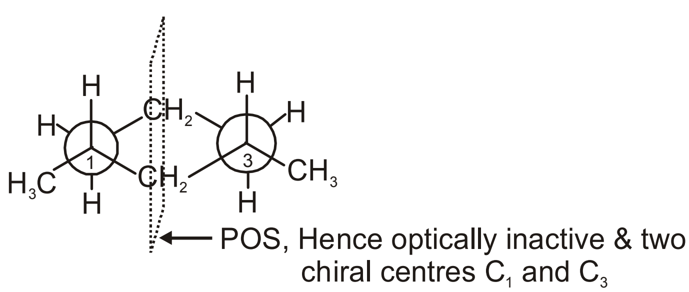
20. (A, B, C)



	$N_A = F\sin\theta + mg$	$N_B = mg - F\cos\theta$	$N_C = mg$
Rest :	$f_A = F\cos\theta$	$f_B = F\cos\theta$	$f_3 = F$
Motion :	$f_A = \mu N_A$	$f_B = \mu N_B$	$f_C = \mu N_C$

CHEMISTRY

21. (D)



22. (B)

$$PBr_5(s) \longrightarrow \underset{sp^3}{PBr_4^+} + Br^-$$

23. (D)

Across C=C bond, I & II are geometrical isomers and both having chiral carbon, hence they are optically active and also these two are not mirror images of each other hence they are also diastereomers.

24. (A)

$$r = 10^{-8}\text{cm},\ b = 4\times\frac{4}{3}\pi r^3 N_A$$

$$\text{or, } b = 4\times\frac{4}{3}\frac{3\times10^{-24}\times6\times10^{23}}{1000} = 96\times10^{-4}\ \text{lit/mol}$$

$$\text{Now, } T_b = \frac{a}{Rb} = \frac{48\times10^4}{125\times0.08\times96} = 500\text{K}$$

Clearly $T > T_b$ and P is very high.

$$\text{So, } P + \frac{a}{v^2} \simeq P$$

$$\text{i.e., } z = 1 + \frac{Pb}{RT} = 1 + \frac{1000\times96\times10^{-4}}{0.08\times800} = 1.15$$

25. (C)

→ compound (I) and (IV) are diastereomers

→ compound (II) and (III) are enantiomers.

→ compound (I) and (II) are constitutional isomer

→ compound (II) and (IV) are constitutional isomers.

26. (C) $z = \frac{PV_m}{RT}$

Here $T = T_B = \frac{a}{Rb}$ and $z = 1$

$$\therefore\quad z = 1 = \frac{PV_m}{R\times\frac{a}{Rb}} = \frac{PV_m b}{a}$$

$$P = \frac{a}{bV_m}$$

27. (A)

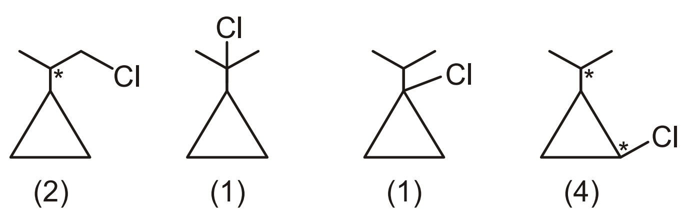
As the size of surrounding atom increases the electronic repulsion increases bond angle increases.

28. (B)

$$\frac{\lambda_{H_2}}{\lambda_{O_2}} = \frac{\dfrac{1}{\sqrt{2}\pi\sigma_{H_2}^2 N_{H_2}^*}}{\dfrac{1}{\sqrt{2}\pi\sigma_{o_2}^2 N_{o_2}^*}} = \frac{N_{o_2}^*}{N_{H_2}^*}\left(\frac{\sigma_{o_2}^2}{\sigma_{H_2}^2}\right)^2 = \frac{\dfrac{P_{O_2}}{KT_{O_2}}}{\dfrac{P_{H_2}}{KT_{H_2}}} \times \left(\frac{\sigma_{O_2}}{\sigma_{H_2}}\right)^2$$

$$\frac{\lambda_{H_2}}{\lambda_{O_2}} = \frac{4}{2} \times \frac{300}{800} \times (2)^2 = 3:1$$

29. (D)



Total 8 monochlorinated product will be obtained.

30. (A)

There are 6 resolvable products.

31. (B)

(A) Temperature decreases A to B, so RMS speed decreases $(\because V_{RMS} \propto \sqrt{T})$

(B) $d_{gas} = \dfrac{PM}{RT}$. Upon moving from A to B, K increases and temperature decreases. So, density of gas increases.

(C) This graph is possible if during the process: P $\propto \dfrac{1}{V^{1/2}}$.

$\because$ PT = Constant

$\therefore$ $P\left(\dfrac{PV}{nR}\right)$ = Constant

$\therefore$ P^2V = Constant or $PV^{1/2}$ = Constant

(D) If $P_B = 4P_A$, then $V_A = 16V_B$ (according P_2V = Constant for process).

32. (B)

Equation of straight line

$$(y - y_1) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$\therefore \quad (P - 16) = \left(\frac{4-16}{15-6}\right)(V-6)$$

$3P + 4V = 72$

$$T_{max} = \frac{(PV)_{max}}{nR}$$

For $(PV_{max}$, $3P = \frac{72}{2}$ and $4V = \frac{72}{2}$

$P = 12, V = 9$

$$T_{max} = \frac{12 \times 9}{1 \times (1/12)} = 648K.$$

33. (A)

$$O = \overset{\overset{\displaystyle O}{\|}}{\underset{\underset{\displaystyle O}{\|}}{Cl}} - O^{\ominus}$$ sp^3 [No p-orbital is unhybridized.]

So, all double bond contain $P\pi - d\pi$ bond.

34. (B)

The relative strength of π- bonds increases when the intermolecular distance decreases.

35. (B, D)

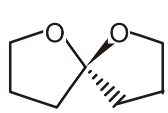
(A) $V_C = 3b$ (C) $\frac{P}{T} = \text{constant at constant V}$ (D) $Z_{II} \propto \bar{u}N^{*2}$

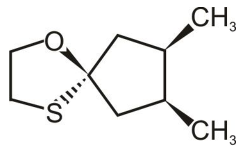
36. (A, B, C, D)

Factual

37. (A, B, D)

38. (A, C, D)

(A)  absence of POS & COS

(B)  POS is present hence optically inactive

(C) Absence of POS & COS hence optically active

(D) Absence of POS & COS, hence optically active.

39. (A, B, D)

As per concept.

40. (B)

P(V – nb) = nRT

PV = n(RT + Pb)

$$P = d\left[\frac{RT}{M} + \frac{Pb}{M}\right]$$

$$\frac{d}{P} = \frac{M}{RT}\left[1 + \frac{bP}{RT}\right]^{-1} \qquad \left[1 + \frac{bP}{RT}\right]^{-1} = 1 - \frac{bP}{RT}$$

$$0.4 = \frac{M}{RT} - \frac{Mb}{RT^2} \times 300$$

$$0.5 = \frac{M}{RT} - \frac{Mb}{RT^2} \times 125$$

$$\frac{M}{RT} = \frac{4}{7} \qquad M = \frac{4}{7} \times 0.821 \times 700 = 32.84\,g$$

MATHEMATICS

41. (D)

$$f(x) = \sin^{-1}\left\{[3x+2] - \{3x + (x - \{2x\})\}\right\}$$

$$\because x \in \left(0, \frac{\pi}{12}\right) \Rightarrow 2x \in \left(0, \frac{\pi}{6}\right) \therefore \{2x\} = 2x \text{ and } 3x \in \left(0, \frac{\pi}{4}\right)$$

$$f(x) = \sin^{-1}\{2 - \{3x - x\}\} = \sin^{-1}\{2 - 2x\}\left(\text{As } \{2-2x\} = 1 - 2x\right)$$

$$f(x) = \sin^{-1}(1-2x) = y \Rightarrow 1 - 2x = \sin y \Rightarrow x = \frac{1 - \sin y}{2} = f^{-1}(y)$$

$$\therefore f^{-1}(x) = \frac{1 - \sin x}{2} = g(x)$$

$$g'(x) = -\frac{1}{2}\cos x$$

$$g'\left(\frac{\pi}{6}\right) = \frac{-1}{2}\left(\frac{\sqrt{3}}{2}\right) = \frac{-\sqrt{3}}{4}$$

42. (B)

$$2x + 2yy' = 0$$

$$x + yy' = 0 \Rightarrow y' = -\frac{x}{y}$$

$$1 + yy'' + (y')^2 = 0$$

$$y'' = -\frac{1+(y')^2}{y}$$

$$\text{Now, } k = \frac{y''}{\left(1+(y')^2\right)^{3/2}} = -\frac{1+(y')^2}{y\left(1+(y')^2\right)^{3/2}}$$

$$= -\frac{1}{y\sqrt{1+(y')^2}} = -\frac{1}{y\sqrt{1+\frac{x^2}{y^2}}} = -\frac{1}{\sqrt{y^2+x^2}} = -\frac{1}{R}$$

43. (A)

Here h(x) = f (g(x))

and $h'(x) = f'(g(x))g'(x) < 0 \ \forall x \in [0, \infty)$

Since $g'(x) < 0 \forall x \in [0, \infty)$ and $f'(g(x)) > 0 \forall x \in [0, \infty)$

Also, h(0) = 0

Therefore, $h(x) < 0 \forall x \in [0, \infty)$

Now, $P(x) = h(x^3 - 2x^2 + 2x) - h(4)$

or $P'(x) = h'\left(x^3 - 2x^2 + 2x\right)\left(3x^2 - 4x + 2\right) < 0 \forall x \in (0, 2)$

since $h'\left(x^3 - 2x^2 + 2x\right) < 0 \forall x \in (0, \infty)$ and $3x^2 - 4x + 2 > \forall x \in R$

$\Rightarrow$ P(x) is decreasing function

P(2) < P(x) < P(0) for all $x \in (0,2)$

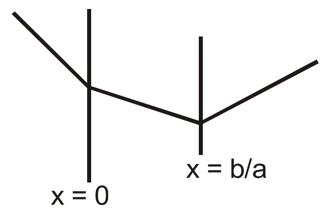
h(4) – h(4) < P(x) < h(0) – h(4)

$\Rightarrow$ 0 < P(x) < – h(4)

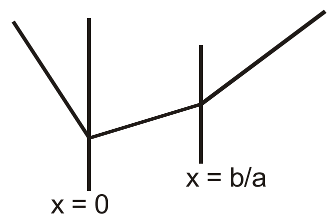
44. (B)

$$f(x) = \begin{cases} b-(a+c)x \quad , & x<0 \\ b+(c-a)x \quad , & 0 \le x < \dfrac{b}{a} \\ (a+c)x+b \quad , & x \ge \dfrac{b}{a} \end{cases}$$

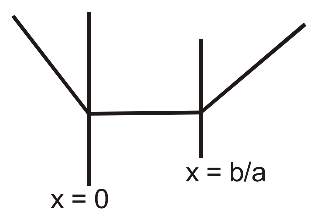
Case-I: When c < a



Case-II: When c > a



Case-III: When c = a



45. (B)

Centre of circle $'c' = \left(\dfrac{a}{2}, \dfrac{b}{2}\right)$, slope of OC = b/a

Slope of tangent = $-\dfrac{a}{b}$, equation of tangent $y = -\dfrac{a}{b}x$

by + ax = 0, $n = \dfrac{b^2}{\sqrt{a^2+b^2}}, m = \dfrac{a^2}{\sqrt{a^2+b^2}}$

$m+n=\sqrt{(a^2+b^2)} \Rightarrow (m+n)^2 = a^2+b^2 = AB^2$

AB = m + n

46. (A)

$x^2+y^2+8x-10y-40=0$

$(x+4)^2+(y-5)^2=1$

$x=\cos\theta-4,\ y=\sin\theta+5$

$(x+2)^2+(y-3)^2$

$=(\cos\theta-2)^2+(\sin\theta+2)^2=9+4(\sin\theta-\cos\theta)$

$a=\max(9+4(\sin\theta-\cos\theta))=9+4\sqrt{2}$

$b=\min(9+4(\sin\theta-\cos\theta))=9-4\sqrt{2}$

a + b = 18, a – b = $8\sqrt{2}$ $\quad\therefore\ a\cdot b=49$

47. (C)

Here $y=(x-3)^2-3 \Rightarrow y+3=(x-3)^2$

Hence the vertex is (3, –3)

Slope of the line joining origin (0, 0) and vertex (3, –3) is –1

Hence the slope of the normal is 1

$\frac{dy}{dx}=2x-6$ and $-\frac{dx}{dy}=1$

$\Rightarrow -1=\frac{1}{2x-6} \Rightarrow 2x-6=-1 \Rightarrow x=\frac{5}{2}$

and $y=\left(\frac{5}{2}\right)^2-6\times\left(\frac{5}{2}\right)+6=\frac{25}{4}-15+6=\frac{-11}{4}$

Hence the normal $y+\frac{11}{4}=1\left(x-\frac{5}{2}\right) \Rightarrow 4x-4y-21=0$

48. (B)

Tangent to $y^2=4x$ in terms of m is $y=mx+\frac{1}{m}$ and normal to $x^2=4by$ in terms of ‘m’ is

$$y = mx + 2b + \frac{b}{m^2}$$

If these are same lines then $\frac{1}{m} = 2b + \frac{b}{m^2} \Rightarrow 2bm^2 - m + b = 0$

For two different tangents $1 - 8b^2 > 0$

$$\Rightarrow |b| < \frac{1}{\sqrt{8}} \Rightarrow |b| < \frac{1}{2\sqrt{2}}$$

49. (A)

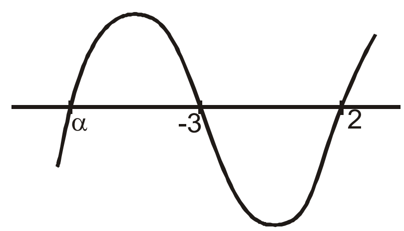
The point of touching has to be rational

$\Rightarrow$ the two roots of f(x) = 0 are rational

$\Rightarrow$ third root is also rational

50. (B)

The graph of f(x) is shown



The third root α is lesser than –3

$\Rightarrow c < -18$

51. (D)

$$S_2 - S_1 = 6x + 9 = 0 \Rightarrow x = -\frac{3}{2}$$

$$S_2 - S_3 = 10x + 8y + 45 = 0$$

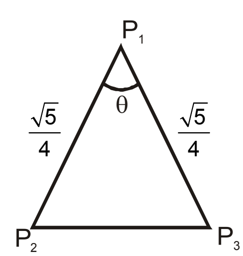
$$8y = -45 + 15 \Rightarrow P\left(-\frac{3}{2}, -\frac{15}{4}\right)$$

52. (D)

$\Delta P_1P_2P_3$

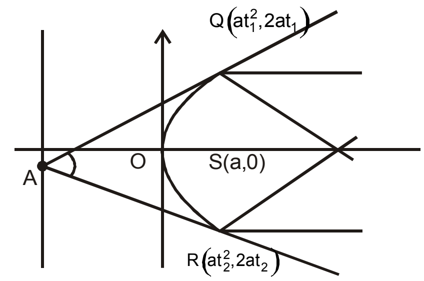
$P_1P_2 = P_1P_3$ = Length of tangent with respect to point P to any of the circle

Length of tangent $= \sqrt{S_1} = \frac{\sqrt{5}}{4}$



$$\text{Area} = \frac{1}{2} \times \left(\frac{\sqrt{5}}{4}\right)^2 \sin\theta = \frac{5}{32}\sin\theta < \frac{5}{32} \Rightarrow [D]$$

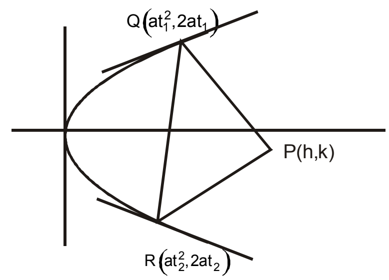
53. (D)



$-a = at_1 t_2 \Rightarrow t_1 t_2 = -1$

$y = a(t_1 + t_2)$

54. (C)



Slope of $PQ = \dfrac{k - 2at_1}{h - at_1^2}$

Slope of $RP = \dfrac{k - 2at_2}{h - at_2^2}$

55. (A, C)

$f(x) = (x^2 + bx + c)e^x$

$\therefore f'(x) = \left(x^2 + (b+2)x + (b+c)\right)e^x$

$f(x) > 0$ if $D = b^2 - 4c < 0$

Now, $f'(x) > 0$ If $D' = (b+2)^2 - 4(b+c) = D + 4 < 0$

Thus for $f'(x) > 0, D + 4 < 0$ holds $\Rightarrow d < 0 \Rightarrow f(x) > 0$

Note that the converse need not be true,

e.g. b = c = 1, f(x) > 0 but $f'(-1) = 0$

56. (A)

P(x) = $x^3 + ax^2 + bx + c$

$P'(x) = 3x^2 + 2ax + b$

$P(-3) = 0 \Rightarrow -27 + 9a - 3b + c = 0$ ....(i)

$P(2) = 0 \Rightarrow 8 + 4a + 2b + c = 0$ ....(ii)

Hence (i) – (ii) gives

5a – 5b – 35 = 0

a – b = 7 .....(iii)

$P'(-3) = 27 - 6a + b < 0 \Rightarrow 27 - 6(a - b) - 5b < 0$

$\Rightarrow 27 - 42 - 5b < 0 \Rightarrow -15 - 5b < 0 \Rightarrow 3 + b > 0 \Rightarrow b > -3$, hence

$a - 7 = b > -3$

$a - 7 > -3$ [from (iii)]

$a > 4$ ......(iv)

$\therefore$ from (ii)

$8 + 16 - 6 + c_{max} = 0$; $c_{max} = -18$ $\Rightarrow c < -18$

57. (A, B)

Given that $\frac{x^2+x+2}{x^2+5x+6} < 0 \Rightarrow x \in (-3, -2)$

We have to find the extrema of the function $f(x) = 1 + a^2x - x^3$

For max or min, $f'(x) = 0$

$\Rightarrow a^2 - 3x^2 = 0$ or $x = \pm\frac{a}{\sqrt{3}}$ and $f''(x) = -6x$ is +ve

when x is negative

If a is positive then point of minima is $\frac{-a}{\sqrt{3}}$

i.e. $-3 < \frac{-a}{\sqrt{3}} < -2$ or $2\sqrt{3} < a < 3\sqrt{3}$

And if a is negative then point of minima is $\frac{a}{\sqrt{3}}$

i.e. $-3 < \frac{a}{\sqrt{3}} < -2$ or $-3\sqrt{3} < a < -2\sqrt{3}$

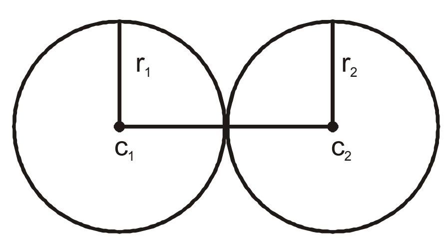
then $a \in \left(-3\sqrt{3}, -2\sqrt{3}\right) \cup \left(2\sqrt{3}, 3\sqrt{3}\right)$

58. (A,B,C,D)

$x^2 + y^2 - 8x + 2y + 8 = 0$

$x^2 + y^2 - 2x - 6y + 10 - a^2 = 0$

$|r_1 - r_2| < c_1c_2 < r_1 + r_2$



$c_1(4, -1), c_2(1, 3)$

$r_1 = \sqrt{16 + 1 - 8} = 3$

$r_2 = \sqrt{1+9-10+a^2} = |a|$

$c_1.c_2 = \sqrt{9+16} = 5$

$|3-|a|| < 5 < 3+|a|$

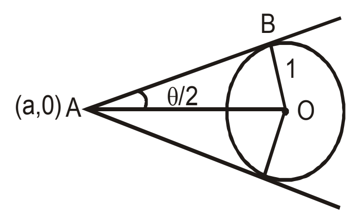
$|a| > 2$

$a \in (-\infty, -2)$ or $(2, \infty)$

59. (B,C)

$x^2 + y^2 = 1$

$\frac{\pi}{2} < \theta < \pi, \ \frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{2}$



$\tan\frac{\theta}{2} > 1, \ AB = \sqrt{a^2-1}, \ OB = 1$

$\frac{1}{\sqrt{a^2-1}} > 1$

$a^2 - 1 > 0$ and $a^2 - 1 < 1$

$a^2 > 1, a^2 < 2$

$a \in (1, \sqrt{2})$ and $(-\sqrt{2}, -1)$

60. (B,C)

The directrix of the parabola is x + y = 0 and focus is (1,1)

The length of the perpendicular from (1, 1)

to the directrix is $\sqrt{2}$

$\Rightarrow$ focal length is $\frac{1}{\sqrt{2}} \Rightarrow$ the point on the axis which is at a distance equal to focal length from the focus towards the right is (3/2, 3/2). All the points to the right of this point, lying on the axis will satisfy the required condition.