SOLUTIONS

MEAITS 2018

PART TEST-1

(ADVANCED PATTERN)

(PAPER - I & II)

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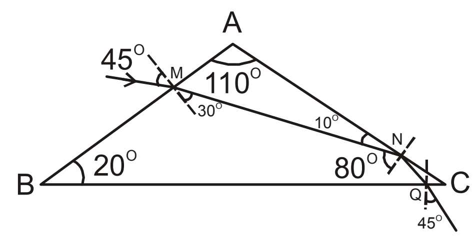
JEE ADVANCED PAPER-I

PHYSICS

1. (C)

Critical Angle



$$\theta_c = \sin^{-1}\left(\frac{1}{\mu}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^o$$

Here $A > 2\theta_c$

Therefore, total internal reflection will take place at AC face and the ray of light emerges from the prism as shown in the figure.

Applyling, $\mu = \frac{\sin i}{\sin r}$ at M and Q , we get the respective angles as shown in the figure.

Now, deviation at M is clockwise, deviation at N is clockwise but deviation at Q is anti-clockwise.

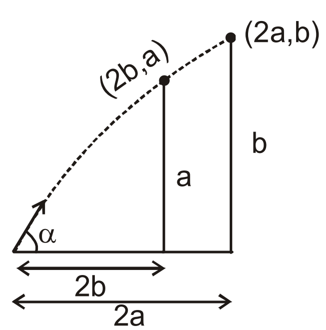
$\therefore \ \delta_{total} = \delta_M + \delta_N - \delta_Q$

$= (45^o - 30^o) + (180^o - 2\times 80^o) - (45^o - 30^o)$

$= 20^o$

So correct option is (C) 20^o

2. (A)



The co-ordinates of the two walls are (2b, a) and (2a, b).

So, from equation of trajectory,

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}; \ b = 2a\tan\alpha - \frac{g(2a)^2}{2u^2\cos^2\alpha}$$

and $a = 2b\tan\alpha - \dfrac{g(2b)^2}{2u^2\cos^2\alpha}$

$$\therefore \frac{4ga^2}{2u^2\cos^2\alpha} = 2a\tan\alpha - b \text{ and } \frac{4gb^2}{2u^2\cos^2\alpha} = 2b\tan\alpha - a$$

$$\therefore \frac{4ga^2}{2u^2\cos^2\alpha} \times \frac{2u^2\cos^2\alpha}{4gb^2} = \frac{2a\tan\alpha - b}{2b\tan\alpha - a}$$

$$\frac{a^2}{b^2} = \frac{2a\tan\alpha - b}{2b\tan\alpha - a}$$

$$\Rightarrow 2a^2b\tan\alpha - a^3 = 2ab^2\tan\alpha - b^3$$

$$\Rightarrow b^3 - a^3 = 2ab(b-a)\tan\alpha$$

$$\therefore \tan\alpha = \frac{\left(a^2 + b^2 + ab\right)}{2ab}$$

$$\therefore \tan\alpha = \frac{(a-b)^2 + 3ab}{2ab}$$

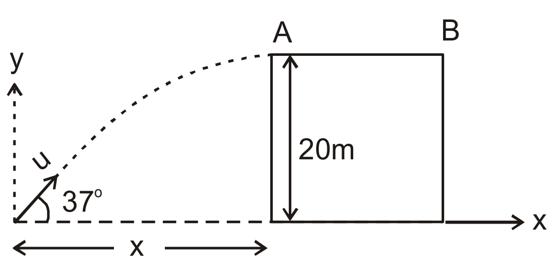
$$= \frac{(a-b)^2}{2ab} + \frac{3ab}{2ab}$$

$$\tan\alpha = \frac{(a-b)^2}{2ab} + \frac{3}{2}$$

As $(a-b)^2$ is positive, hence $\tan\alpha > \dfrac{3}{2}$

$$\therefore \alpha > \tan^{-1}\left(\frac{3}{2}\right)$$

3. **(A)**



Let us assume that person throws the ball from distance x. Taking point of projection as origin.

By equation of trajectory

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}; \qquad \theta = 37^o$$

$$20 = x \times \frac{3}{4} - \frac{1}{2}\frac{gx^2}{u^2} \times \frac{25}{16}$$

$$20 = \frac{3x}{4} - \frac{25gx^2}{32u^2}$$

$$\frac{25gx^2}{32u^2} = \frac{3x}{4} - 20$$

$$u^2 = \frac{25gx^2}{8(3x-80)}$$

For u to be minimum, $\frac{du}{dx} = 0$

On differentiating wrt x, we get,

$$2u\frac{du}{dx} = \frac{25g}{8}\left[\frac{(3x-80)2x - x^2(3)}{(3x-80)^2}\right]$$

For $\frac{du}{dx} = 0$

$$(3x-80)2x - 3x^2 = 0$$

$$6x^2 - 160x - 3x^2 = 0$$

$$x(3x-160) = 0$$

$$\therefore \; x = \frac{160}{3}m$$

For $x > \frac{160}{3}m$, slope is positive and for

$x < \frac{160}{3}m$, slope is negative

So, at $x = \frac{160}{3}\,m$, there is minima,

$\therefore$ Required minimum velocity;

$$u_{min}^2 = \frac{25}{8}\frac{gx^2}{(3x-80)}$$

$$u_{min}^2 = \frac{25}{8}\times 10\times\frac{160}{3}\times\frac{160}{3\left(3\times\frac{160}{3}-80\right)}$$

$$\therefore\ u_{min} = \frac{100}{3}\,m/s$$

From same position and same angle of projection,

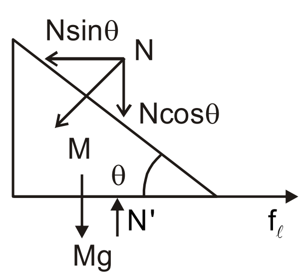
$$u^2_{max} = \frac{25}{8}g\frac{\left(\frac{160}{3}+\frac{80}{3}\right)^2}{(3\times 80-80)} = \frac{25}{8}\times\frac{10\times 80\times 80}{160}$$

$$\therefore\ u_{max} = 25\sqrt{2}\,m/s$$

Hence the range of speed is $\frac{100}{3}\,m/s$ to $25\sqrt{2}\,m/s$

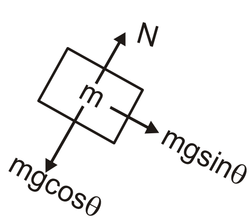
4. (A)

F.B.D of triangular block;



$N' = Mg + N\cos\theta$

F.B.D. of mass m;



$N = mg\cos\theta$

For triangular block at rest;

$f_\ell \geq N\sin\theta$

$f_\ell = \mu\left(Mg + mg\cos^2\theta\right)$

$\therefore\ \mu g\left(M + m\cos^2\theta\right) \geq mg\sin\theta\cos\theta$

for minimum μ;

$\mu g\left(M + m\cos^2\theta\right) = mg\sin\theta\cos\theta$

$$\therefore\ \mu = \frac{m\sin\theta\cos\theta}{M + m\cos^2\theta}$$

$$\frac{d\mu}{d\theta} = 0$$

$$\frac{d\mu}{d\theta} = \frac{\left(M + m\cos^2\theta\right)m\left[\cos^2\theta - \sin^2\theta\right] - m\sin\theta\cos\theta\left[0 - 2m\cos\theta\sin\theta\right]}{\left(M + m\cos^2\theta\right)^2}$$

For $\frac{d\mu}{d\theta} = 0$

$\left(M + m\cos^2\theta\right)m\left(\cos^2\theta - \sin^2\theta\right) = -2m^2\sin^2\theta\cos^2\theta$

$Mm\cos^2\theta - Mm\sin^2\theta + m^2\cos^4\theta - m^2\cos^2\theta\sin^2\theta + 2m^2\sin^2\theta\cos^2\theta = 0$

$Mm\cos^2\theta - Mm\sin^2\theta + m^2\cos^4\theta + m^2\sin^2\theta\cos^2\theta = 0$

$m^2\cos^2\theta\left(\cos^2\theta + \sin^2\theta\right) + Mm\cos^2\theta - Mm\sin^2\theta = 0$

$m^2\cos^2\theta + Mm\cos^2\theta = Mm\sin^2\theta$

$m\cos^2\theta\left(M + m\right) = Mm\sin^2\theta$

$$\tan^2\theta = \left(\frac{M + m}{M}\right)$$

$$\tan\theta = \frac{\sqrt{M + m}}{\sqrt{M}},\ \sin\theta = \frac{\sqrt{M + m}}{\sqrt{2M + m}}, \cos\theta = \frac{\sqrt{M}}{\sqrt{2M + m}}$$

$$\therefore\ \mu_{min} = \frac{m\sin\theta\cos\theta}{M + m\cos^2\theta}$$

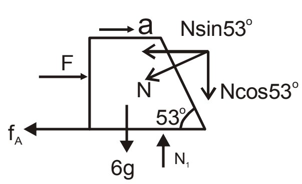
$$=\frac{\left(\frac{m\cdot\sqrt{M+m}}{\sqrt{2M+m}}\cdot\frac{\sqrt{M}}{\sqrt{2M+m}}\right)}{M+m\left(\frac{M}{2M+m}\right)}=\frac{m\sqrt{M}\sqrt{M+m}}{(2M+m)}\times\frac{(2M+m)}{2M^2+2Mm}$$

$$=\frac{m\sqrt{M}\sqrt{M+m}}{2M(M+m)}=\frac{m}{2\sqrt{M}}\times\frac{1}{\sqrt{M+m}}$$

$$\therefore\ \mu_{min}=\frac{m}{2\sqrt{M(M+m)}}$$

5. (B)

From FBD of block A;



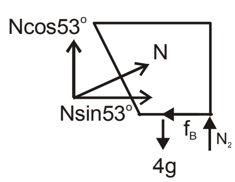
$$F-f_A-N\sin 53^o=6a$$

$$N_1=6g+N\cos 53^o$$

$$f_A=\mu_A N_1=0.5\left(60+\frac{3N}{5}\right)$$

$$\therefore\ F-30-\frac{3N}{10}-\frac{4N}{5}=6a \qquad \dots (i)$$

From FBD of block B;



$$N\sin 53^o-f_B=4a$$

$$N_2=4g-N\cos 53^o$$

$$f_B=\mu N_2=\frac{1}{4}\left(4g-\frac{3N}{5}\right)$$

$$= g - \frac{3N}{20} = 10 - \frac{3N}{20}$$

$$\therefore \ \frac{4N}{5} - 10 + \frac{3N}{20} = 4a \qquad \ldots \text{(ii)}$$

From (i) + (ii), we get

$$F - 30 - \frac{3N}{10} - \frac{4N}{5} = 6a$$

$$\frac{4N}{5} - 10 + \frac{3N}{20} = 4a$$

$$\overline{F - 40 - \frac{3N}{20} = 10a}$$

$$F - 40 - \frac{3N}{20} = 10a \qquad \ldots \text{(iii)}$$

Now from equation (i)

$$F - 30 - \frac{3N}{10} - \frac{4N}{5} = 6a$$

$$F - 30 - N\left(\frac{3}{10} + \frac{4}{5}\right) = 6a$$

$$F - 30 - \frac{11N}{10} = 6a$$

$$\therefore \ N = \frac{(F - 30 - 6a)10}{11}$$

By putting this value of N in equation (iii), we get

$$F - 40 - \frac{3}{20}\left(\frac{F - 30 - 6a}{11}\right) \times 10 = 10a$$

$$F - 40 - \frac{3F}{22} + \frac{45}{11} + \frac{9a}{11} = 10a$$

$$\frac{19F}{22} - \frac{395}{11} = \left(10 - \frac{9}{11}\right)a$$

$$\frac{19F - 790}{22} = \frac{101}{11}a$$

$$a = \frac{19F - 790}{202}$$

for acceleration to be positive;

$$19F - 790 > 0$$

$$\therefore\ F > \frac{790}{19}N$$

So the minimum force F to just start the motion is $\frac{790}{19}N$.

Now, maximum F will be when N_2 just becomes zero. So for N_2 to be zero, $N = \frac{200}{3}N$

Now form equation (ii);

$$\frac{19N}{20} - 10 = 4a$$

$$\frac{\left(\frac{19}{20} \times \frac{200}{3} - 10\right)}{4} = a$$

$$a = \frac{40}{3}ms^{-2}.$$

Now from equation (i)

$$F - 30 - \frac{3N}{10} - \frac{4N}{5} = 6a$$

$$F = \frac{6 \times 40}{3} + 30 + \left(\frac{19}{20} \times \frac{200}{3}\right)$$

$$= \frac{240}{3} + 30 + \frac{190}{3} = \frac{520}{3}N.$$

If we apply $F > \frac{520}{3}N$, then B will start sliding upon A. So the maximum force is $F = \frac{520}{3}N$

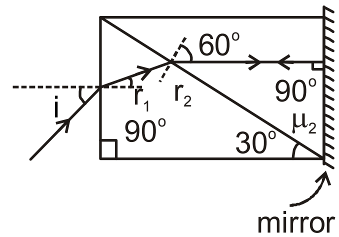
6. **(B)**

Since incident ray retraces its path it must strike the plane mirror perpendicularly.

From Snell's law,

$\sin i = \mu_1 \sin r_1$

and $\mu_1 \sin r_2 = \mu_2 \sin 60^o$



$\Rightarrow \mu_1 \sin r_2 = \frac{\sqrt{3}}{2}\mu_2$

$\Rightarrow r_2 = \sin^{-1}\left(\frac{\sqrt{3}\mu_2}{2\mu_1}\right)$

Here, $r_1 + r_2 = 60^o$

$\therefore i = \sin^{-1}\left[\mu_1 \sin\left(\frac{\pi}{3} - \sin^{-1}\left(\frac{\sqrt{3}\mu_2}{2\mu_1}\right)\right)\right]$

7. (A)

From constraint relation;

$2a_1 + 3a_2 = 0$ … (i)

From NLM, equation;

$2T - 4mg = 4ma_1 \Rightarrow \frac{T}{m} - 2g = 2a_1$ … (ii)

and $3T - mg = ma_2 \Rightarrow \frac{3T}{m} - g = a_2$ … (iii)

So, from (i), (ii) and (iii);

$\frac{T}{m} - 2g + \frac{9T}{m} - 3g = 0$

$\frac{10T}{m} = 5g \Rightarrow T = \frac{mg}{2}$

$\therefore a_2 = \frac{3mg}{2m} - g = \frac{g}{2}\uparrow$

From constraint relation;

$2h + 3h_2 = 0 \therefore h_2 = \frac{2h}{3}\uparrow = 2m\uparrow$

At the moment body 1 touches the ground floor, the body 2 will have velocity;

$$v_2 = \sqrt{2a_2h_2} = \sqrt{2\times\frac{g}{2}\times 2} = \sqrt{2g}\ m/s$$

After the body 1, touches the floor the thread becomes slack and thus body 2 goes up under gravity.

$$\therefore\ h' = \frac{v_2^2}{2g} = \frac{2g}{2g} = 1\,m$$

hence the maximum height attained by the body 2,

$$H = h_2 + h' = 2 + 1 = 3m.$$

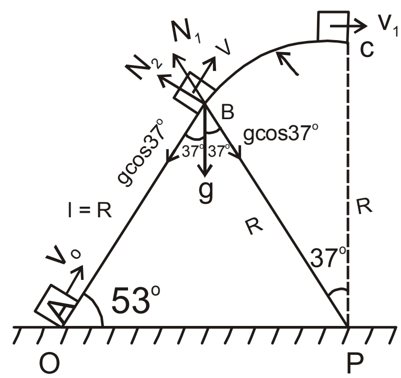
8. (A)

In triangle OBP;

Length OB = R

$$\therefore\ \ell = R$$

For contact not to be lost at B,



$$mg\cos 37^o - N = \frac{mv^2}{R}$$

$$N = mg\cos 37^o - \frac{mv^2}{R} \ge 0$$

$$\frac{4mg}{5} - \frac{mv^2}{R} \ge 0$$

$$\frac{4mg}{5} \ge \frac{mv^2}{R}$$

$$\therefore\ v \le \sqrt{\frac{4Rg}{5}} \qquad \dots \text{(i)}$$

from B to C;

$$\Delta K + \Delta U = 0$$

$$\left(\frac{1}{2}mv_1^2 - \frac{1}{2}mv^2\right) + mgR\left(1-\cos 37^o\right) = 0$$

$$v^2 = v_1^2 + 2gR\left(1-\frac{4}{5}\right)$$

$$v^2 = v_1^2 + \frac{2gR}{5} \qquad \dots \text{(ii)}$$

from A to B;

$$a = g\cos 37^o + \mu g \sin 37^o$$

$$= \frac{4g}{5} + \left(\frac{3}{4}g \times \frac{3}{5}\right)$$

$$= \frac{4g}{5} + \frac{9g}{20} = \frac{16g + 9g}{20}$$

$$a = \frac{5g}{4}$$

$$v^2 = v_o^2 - 2a\ell$$

$$v^2 = v_o^2 - 2 \times \frac{5g}{4} \times R$$

$$v^2 = v_0^2 - \frac{5g}{2}R \qquad \dots \text{(iii)}$$

From (i) & (iii);

$$v_o^2 - \frac{5gR}{2} \le \frac{4Rg}{5}$$

$$v_o^2 \le \frac{4Rg}{5} + \frac{5Rg}{2}$$

$$v_o \le \sqrt{\frac{33}{10}Rg} \qquad \dots \text{(iv)}$$

From equation (ii) and (iii)

$$v_1^2 + \frac{2gR}{5} = v_o^2 - \frac{5Rg}{2}$$

$$v_1^2 = v_o^2 - \left(\frac{5}{2} + \frac{2}{5}\right)Rg$$

$$v_1^2 = v_o^2 - \frac{29Rg}{10}$$

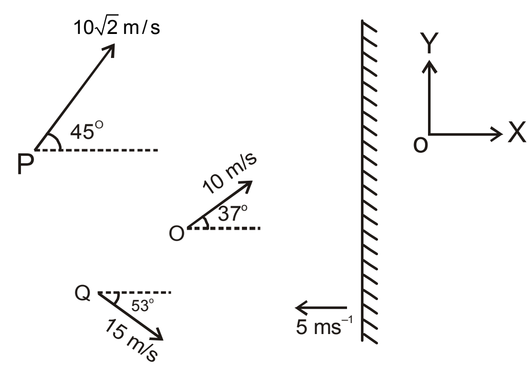
Now to reach at C; $v_1 \geq 0$

$$\Rightarrow v_o^2 - \frac{29Rg}{10} \geq 0$$

$$\Rightarrow v_o \geq \sqrt{\frac{29Rg}{10}}$$

$\therefore$ Range of velocity is $\sqrt{\frac{29Rg}{10}} \leq v_o \leq \sqrt{\frac{33}{10}Rg}$

9. (C)



$$\overrightarrow{V_p} = 10\sqrt{2}\cos 45^o\,\hat{i} + 10\sqrt{2}\sin 45^o\,\hat{j}$$

$$= \left(10\sqrt{2} \times \frac{1}{\sqrt{2}}\right)\hat{i} + \left(10\sqrt{2} \times \frac{1}{\sqrt{2}}\right)\hat{j} = 10\hat{i} + 10\hat{j}$$

$$\overrightarrow{V_m} = -5\hat{i}$$

$$\vec{V}_{pm} = \vec{V}_p - \vec{V}_m = 10\hat{i} + 10\hat{j} - (-5\hat{i})$$

$$= 15\hat{i} + 10\hat{j}$$

$\vec{V}_{I_pm}$= velocity of image of object p wrt mirror

$$= -15\hat{i} + 10\hat{j}$$

$$\vec{V}_{I_pO} = \vec{V}_{I_pm} - \vec{V}_{om} = \left(-15\hat{i} + 10\hat{j}\right) - \left(\vec{V}_0 - \vec{V}_m\right)$$

$$= \left(-15\hat{i} + 10\hat{j}\right) - 6\hat{i} - 8\hat{j} + (-5\hat{i})$$

$$= -15\hat{i} - 11\hat{i} + 2\hat{j}$$

$$= -26\hat{i} + 2\hat{j}$$

$$\vec{V}_Q = 9\hat{i} - 12\hat{j}$$

$$\vec{V}_m = -5\hat{i}$$

$$\vec{V}_{Qm} = -14\hat{i} - 12\hat{j}$$

$$\vec{V}_{I_Qm} = -14\hat{i} - 12\hat{j}$$

$$\vec{V}_{I_Qo} = \vec{V}_{I_Qm} - \vec{V}_{om}$$

$$= \left(-14\hat{i} - 12\hat{j}\right) - \left(\vec{V}_o - \vec{V}_m\right)$$

$$= -14\hat{i} - 12\hat{j} - 6\hat{i} - 8\hat{j} - 5\hat{i}$$

$$= -25\hat{i} - 20\hat{j}$$

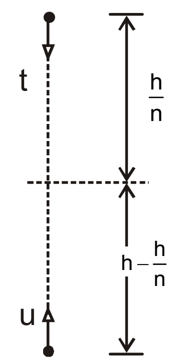
$$\therefore\ \vec{V}_{I_pI_Q} = \vec{V}_{I_po} - \vec{V}_{I_Qo}$$

$$= -26\hat{i} + 2\hat{j} + 25\hat{i} + 20\hat{j}$$

$$\therefore \vec{V}_{I_pI_Q} = -\hat{i} + 22\hat{j}$$

10. (C)

Time taken by particle to descend a distance $\frac{h}{n}$,



$$t = \sqrt{\frac{2h}{ng}} \qquad \ldots \text{(i)}$$

In the same time, another particle goes up by $h - \frac{h}{n}$ distance,

So, $h - \frac{h}{n} = ut - \frac{1}{2}gt^2 \qquad \ldots \text{(ii)}$

From equation (i); $h = \frac{ngt^2}{2}$

By putting this value in equation (ii) we get,

$$\frac{ngt^2}{2} - \frac{ngt^2}{2n} = ut - \frac{1}{2}gt^2$$

$$\therefore\ u = \frac{ngt}{2} = \sqrt{\frac{n^2g^2}{4} \times \frac{2h}{ng}}$$

$$\therefore\ u = \sqrt{\frac{ngh}{2}}$$

and $\nu_1 = gt, \nu_2 = u - gt$

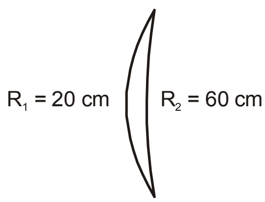
$$\frac{\nu_2}{\nu_1} = \frac{u - gt}{gt} = \frac{u}{gt} - 1$$

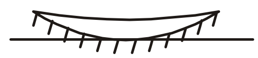
$$\Rightarrow \frac{\nu_2}{\nu_1} = \frac{ngt}{2gt} - 1 = \frac{n}{2} - 1 \qquad \Rightarrow \frac{\nu_1}{\nu_2} = \frac{2}{n-2}$$

11. (A, B, C)

$$\frac{1}{f_\ell} = (1.5-1)\left[\frac{1}{20} - \frac{1}{60}\right] = \frac{1}{60}$$

$$f_\ell = 60 \text{ cm}$$





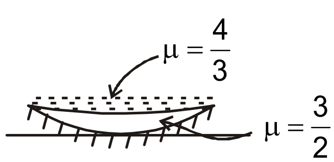
When convex side is silvered and kept on horizontal surface,

$$\frac{1}{F} = \frac{2}{f_\ell} - \frac{1}{f_m}, \quad f_m = -\frac{R_1}{2} = -10 \text{ cm}$$

$$\therefore \frac{1}{F} = -\frac{1}{10} - 2\left(\frac{1}{60}\right)$$

$$F = -\frac{30}{4} = -7.5 \text{ cm}$$

After water is filled :



Let f_{ℓ_1} is the focal length of water lens,

$$\frac{1}{f_{\ell_1}} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{60}\right) = \frac{1}{180}$$

$$\frac{1}{F} = \frac{2}{f_{\ell_1}} - \frac{2}{f_{\ell_2}} - \frac{1}{f_m}$$

By solving we get, $F = -\frac{90}{13}$ cm.

12. (A, B, C & D)

(A) $3mgx_m = \frac{1}{2}kx_m^2$

$\Rightarrow x_m = \frac{6mg}{k}$

(B) $x = \frac{x_m}{2} = \frac{3mg}{k}$

$m_B gx = \frac{1}{2}kx^2 + \frac{1}{2}(m_A + m_B)v^2$

$3mgx = \frac{1}{2}k\left(\frac{3mg}{k}\right)^2 + \frac{1}{2}5\,mv^2$

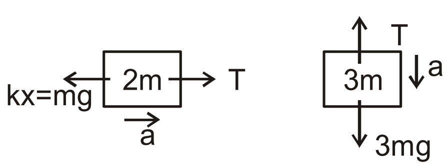
$3mg \times \frac{3mg}{k} = \frac{1}{2}k\frac{9m^2g^2}{k^2} + \frac{1}{2}5mv^2$

$\frac{9m^2g^2}{k} - \frac{9}{2}\frac{m^2g^2}{k} = \frac{1}{2}5\,mv^2$

$\frac{9}{2}\frac{m^2g^2}{k} = \frac{1}{2}5\,mv^2$

$\Rightarrow v = \sqrt{\frac{9m^2g^2}{5mk}} \Rightarrow v = 3g\sqrt{\frac{m}{5k}}$

(C) At $x = \frac{x_m}{6} = \frac{mg}{k}$

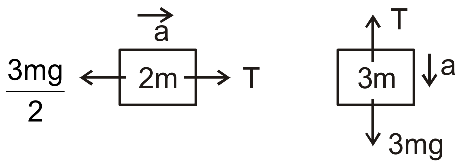


$T - mg = 2ma$
$3mg - T = 3ma$

$2mg = 5ma$

$\therefore a = \frac{2g}{5}ms^{-2} \downarrow$

(D) At $x = \frac{x_m}{4} \Rightarrow x = \frac{3mg}{2k}$



$$3mg - T = 3ma$$

$$T - \frac{3mg}{2} = 2ma$$

$$\frac{3mg}{2} = 5\,ma$$

$$\therefore\ a = \frac{3g}{10}\,ms^{-2} \downarrow$$

13. (B, C)

If the image is real and magnified means object is between f and 2f.

When immersed in liquid, then focal length,

$$f_\ell = \frac{(\mu - 1)}{\left(\frac{\mu}{\mu_\ell} - 1\right)} f = \frac{\left(\frac{3}{2} - 1\right)}{\left(\frac{3}{2} \times \frac{4}{5} - 1\right)} f$$

$$= \frac{1}{2} \times \frac{10}{2} f$$

$$f_\ell = \frac{5}{2} f$$

Now object is between pole and focus, so image is virtual and magnified.

$\therefore$ (B) and (C)

14. (A, B, C, D)

For total internal reflection to take place angle of incidence, $i >$ critical angle θ_c

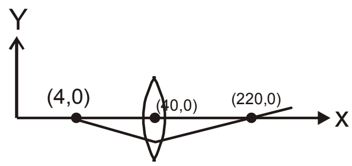
Or, $\sin i > \sin\theta_c$

$\Rightarrow \sin 60^{o} > \frac{1}{\mu} \qquad \Rightarrow \frac{\sqrt{3}}{2} > \frac{1}{\mu} \qquad \Rightarrow \mu > \frac{2}{\sqrt{3}}$

$\Rightarrow \mu > 1.155$

Therefore possible values of μ can be all in the given options.

15. (B, C)



The ray is intersecting y = 0 line at x = 4 and x = 40 line at y = –1

$\therefore\ u = 36$ cm

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow v = 180$ cm

$\therefore$ Equation of refracted ray is

$180y = x - 220$

If space on the right of the lens is filled with liquid of $\mu = \frac{4}{3}$, then $\frac{1.5}{v_1} + \frac{1}{36} = \frac{0.5}{30}$

$$\Rightarrow \frac{1.5}{v_1} = \frac{1}{60} - \frac{1}{36} \Rightarrow v_1 = -135\,\text{cm}$$

$$\frac{4}{3v} - \frac{1.5}{v_1} = \frac{\left(\frac{4}{3} - 1.5\right)}{-30} \Rightarrow v = -240\text{ cm}$$

So co-ordinate of final image with respect to origin $= (-200, 0)$

$\therefore$ Equation of refracted ray is

$y = \frac{1}{-240}(x + 200)$

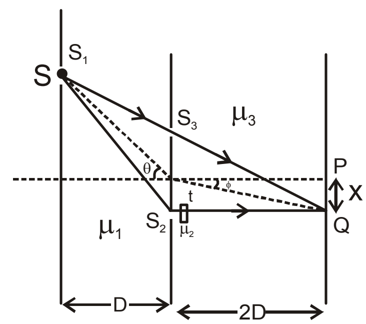
So, $x + 240y + 200 = 0$

This is the equation of required refracted ray.

Hence correct option is (B) and (C)

16. (5)

For the central bright to be formed at Q,



$$(S_1S_3)\mu_1 + (S_3Q)\mu_3 = (S_1S_2)\mu_1 + (S_2Q - t)\mu_3 + \mu_2 t$$

$$(S_3Q - S_2Q)\mu_3 = (S_1S_2 - S_1S_3)\mu_1 + (\mu_2 - \mu_3)t$$

$$(d\sin\phi)\mu_3 = (d\sin\theta)\mu_1 + (\mu_2 - \mu_3)t$$

$$d \times \frac{x}{2D}\mu_3 = d \times \frac{d}{D}\mu_1 + (\mu_2 - \mu_3)t$$

By putting all the values, we get,

$$10^{-3} \times \frac{x}{2} \times 2 = \left(\frac{(10^{-3})^2}{1} \times \frac{5}{4}\right) + \left(\frac{3}{2} - 2\right) \times 10^{-6}$$

$$10^{-3} \times x = \left(\frac{5}{4} - \frac{1}{2}\right) \times 10^{-6}$$

$$x = \left(\frac{5-2}{4}\right) \times 10^{-3}$$

$$x = \frac{3}{4} \times 10^{-3} m$$

So, $\sqrt{m^2 + n^2} = \sqrt{3^2 + 4^2} = 5$

17. (1)

Let, $AS = h$

Now, $\beta = \dfrac{\lambda D}{d}$

In the first care, $d = 2h$

$$\therefore\ \beta = \frac{\lambda D}{2h} \qquad \ldots \text{(i)}$$

In the second care, $d = 2(h + \Delta x)$, where $\Delta x =$ shift in the source away from the mirror along AB.

$$\therefore\ \beta' = \frac{\lambda D}{2(h + \Delta x)} \qquad \ldots \text{(ii)}$$

From (i)/ (ii), we get

$$\frac{\beta}{\beta'} = \frac{h+\Delta x}{h} = 1 + \frac{\Delta x}{h}$$

$$\Rightarrow \frac{\beta - \beta'}{\beta'} = \frac{\Delta x}{h} \qquad \Rightarrow h = \left(\frac{\Delta x \beta'}{\beta - \beta'}\right) = \frac{\left(0.5 \times \frac{1}{5}\right)}{\left(\frac{1}{4} - \frac{1}{5}\right)}$$

$$= 0.1 \times 20\text{ mm} = 2\text{ mm}$$

From equation (i);

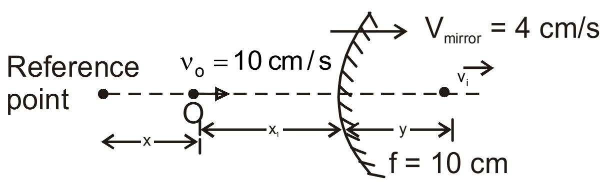
$$\lambda = \frac{2\beta h}{D} = 2 \times \frac{1}{4} \times 10^{-3} \times 2 \times 10^{-3}$$

$$= 1 \times 10^{-6}\text{ m}$$

So, n = 1

18. (4)

We have to find the location of image for the given situation.



$$\frac{1}{y} + \frac{1}{-x_1} = \frac{1}{10} \qquad \ldots \text{(i)}$$

$$\frac{1}{y} - \frac{1}{10} = \frac{1}{10} \Rightarrow \frac{1}{y} = \frac{2}{10} = \frac{1}{5}$$

$$\Rightarrow y = 5\text{ cm}$$

$$\frac{dx}{dt} = 10\text{ cm/s}, \quad \frac{d(x+x_1)}{dt} = 4\text{ cm/s}$$

$$\frac{dx}{dt} + \frac{d(x_1)}{dt} = 4\text{ cm/s}$$

$$10 + \frac{d(x_1)}{dt} = 4 \quad \Rightarrow \frac{d(x_1)}{dt} = -6\text{ cm/s}$$

$$\frac{dy}{dt} = v_i - 4$$

From equation (i), we get,

$$\frac{dy}{dt} = \left(\frac{y}{x_1}\right)^2 \frac{dx_1}{dt}$$

$$\Rightarrow v_i - 4 = \left(\frac{5}{10}\right)^2 (-6)$$

$$\Rightarrow v_i - 4 = -\frac{6}{4}$$

$$\Rightarrow \; v_i = 4 - \frac{6}{4} = \frac{16-6}{4}$$

$$v_i = \frac{10}{4} \text{cm/s}$$

So value of n = 4

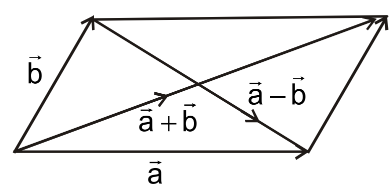
19. (2)

Let $\vec{a} = x\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{b} = 2x\hat{i} + x\hat{j} - \hat{k}$ be the adjacent sides of the parallelogram.

Now, angle between $\vec{a}$ and $\vec{b}$ is acute i.e.

$$|\vec{a} + \vec{b}| > |\vec{a} - \vec{b}|$$

$$\Rightarrow \; |3x\hat{i} + (x-3)\hat{j} - 2\hat{k}|^2 > |-x\hat{i} - (x+3)\hat{j}|^2$$



$$9x^2 + (x-3)^2 + 4 > x^2 + (x+3)^2$$

$$\Rightarrow 8x^2 - 12x + 4 > 0$$

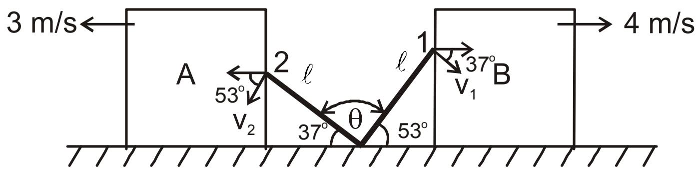
$$\Rightarrow \; 2x^2 - 3x + 1 > 0$$

$$\Rightarrow \; (2x-1)(x-1) > 0$$

$$\Rightarrow \; x < \frac{1}{2} \text{ or } x > 1$$

Hence the least positive integral value is 2.

20. (5)



From figure; $\nu_1 \cos 37^o = 4$

$\therefore \nu_1 = 5 m/s$

and $v_2 \cos 53^o = 3$

$v_2 = 5\, m/s$

so, $\omega_1 = \frac{5}{2} rad/s$

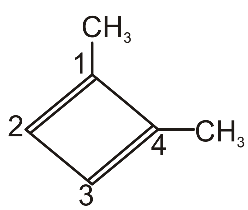
and $\omega_2 = \frac{5}{2} rad/s$

hence $\frac{d\theta}{dt} = \omega = \omega_1 + \omega_2 = 5\, rad/s.$

CHEMISTRY

21. (D)

22. (D)



23. (D)

S > O > P > N

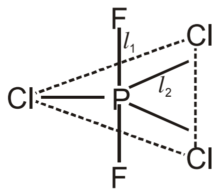
24. (A)

$$\frac{C_{av}}{C_{RMS}} = \sqrt{\frac{8}{3\pi}} = \frac{2}{\sqrt{1.5\pi}}$$

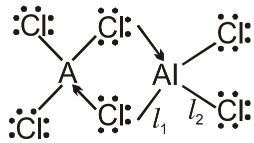
$$C_{av} = \frac{2x}{\sqrt{1.5\pi}}$$

$$Z_1 = \frac{C_{av}}{\lambda} = \frac{2x}{\sqrt{1.5\pi}y}$$

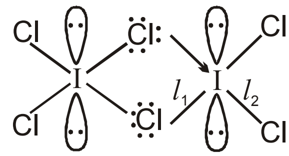
25. (D)



$l_2 > l_1$



$l_1 > l_2$



$l_1 > l_2$

26. (C)

Volume of 1 molecule of N_2

$$= \frac{6.4\pi}{6 \times 10^{23}} \text{Cm}^3$$

$$\frac{4}{3}\pi r^3 = \frac{6.4\pi}{6} \times 10^{-23}$$

$r^3 = 8 \times 10^{-24}$

$r = 2 \times 10^{-8} = 200$ A°

$$\sigma = 2r = 400 \text{ A}^{o}$$

27. (C)

Hg $\rightarrow$ max I.E among d-Block metals

28. (A)

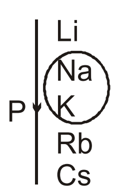
$$\frac{dN}{N} = 4\pi \left[\frac{M}{2\pi RT}\right]^{3/2} U^2 \cdot e^{\frac{-M}{2RT}U^2} \cdot du$$

$$du = (1.01) \times C_{rms}$$

$$C_{rms} = \sqrt{\frac{3RT}{M}}$$

29. (C)

Li < K < Na < Rb < Cs



P = Cs > Rb > Na > K > Li

30. (A)

$$\underset{x}{C_2H_6} + \frac{7}{2}O_2 \longrightarrow \underset{2x}{2CO_2} + 3H_2O$$

$$\begin{array}{llll} C_3H_8 + SO_2 \longrightarrow & 3CO_2 & + & 4H_2O \\ 200 - x & 3(200 - x) & & \end{array}$$

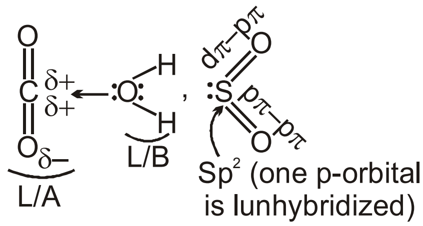
$2x + 3\,(200 - x) = 450$

$\therefore$ $x = 150 = P$ of C_2H_6

$\therefore$ P of $C_3H_8 = 50$

$\therefore$ Required ratio is $\frac{150}{50} = 3$

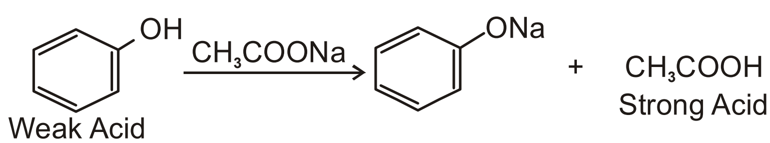
31. (A,C)



$S_2 \rightarrow$ Paramagnetic like O_2

$n = 2(\pi^*\text{A.B.M.O})$

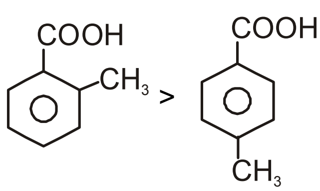
32. (B,C,D)

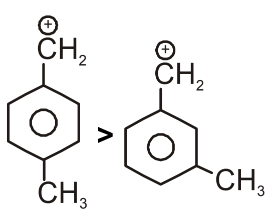


Equilibrium favour strong acid $\rightarrow$ weak acid

In which backward reaction is favourable.

33. (C,D)

(C)  acidity due to ortho effect

(D)  due to Hyperconjucation power of —CH_3 group

34. (A,B,C)

T_C is maximum a/b is highest for most ideal behaviour 1/b is maximum.

35. (A,B,D)

$$(NH_4)_2CO_3 \longrightarrow NH_2-\overset{O}{\overset{\|}{C}}-NH_2 + 2H_2O$$

$$\frac{11.52}{96} = 0.12 \text{mole}$$

$NH_2-\underset{O}{\underset{\|}{C}}-NH_2$ mole = 0.12 mole

$$MnO_4^- + (NH_4)_2C_2O_4 \longrightarrow Mn^2 + CO_2 + NH_4^+$$

$$\text{Mole of } (NH_4)_2C_2O_4 = 20 \times 1.6 \times \frac{5}{2} \times 10^{-3} = 0.08$$

Total mole of $(CN)_2$ = 0.08 + 0.12 = 0.20

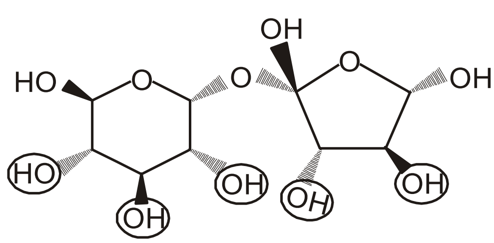
$\therefore$ Mass of $(CN)_2$ = 0.20 × 52 = 10.4 gm

$$\therefore \ \% \text{purity} = \frac{10.4}{104} \times 100 = 10\%$$

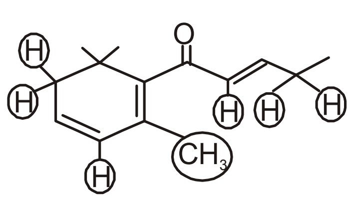
For I $\dfrac{0.08}{0.12+0.08} \times 100 = 40\%$

II = 60 %

36. (5)



37. (9)



38. **(8)**

Non-existing molecule/ion;

$K[HCl_2], PbI_4, SI_6, KH_3, PBr_6^-, PI_5$, $K[HBr_2]$, $K[HI_2]$

39. **(2)**

Given compound and its enantiomer.

40. **(8)**

NH_3 = V ml

H_2 = (50 –V) ml

$$2NH_3 \longrightarrow N_2 + 3H_2$$

$$V_{ml} \qquad \frac{3}{2}v\,ml$$

	$2H_2(g)$	+	$O_2(g)$	→	$2H_2O(\ell)$
vol. initially	$\left(50+\frac{v}{2}\right)$		40		
vol. finally	0		6		

$$50 + \frac{V}{2} = 68$$

V = 36 ml

$\therefore$ % of NH_3 = 72 %

$\therefore$ x = 8

MATHEMATICS

41. **(D)**

At x = 2n, $n \in I$

$$\lim_{x\to 2n^-} f(x) = \lim_{x\to 2n^+} f(x) = f(2n) \quad \Rightarrow a_n + 2 = b_n + 2n = b_n + 2n \qquad .....(i)$$

At x = 2n –1, $n \in I$,

$$\lim_{x\to (2n-1)^-} f(x) = \lim_{x\to (2n-1)^+} f(x) = f(2n-1) \Rightarrow b_{n-1} + 2(n-1) = a_n + 1 = a_n + 1 \qquad .....(ii)$$

From (i) & (ii), $b_n + 2n - 1 = b_{n-1} + 2(n-1)$

$\Rightarrow b_n = b_{n-1} - 1$

$\therefore b_n = b_0 - n = 2 - n \quad (\because \{b_n\}$ is an A.P.)

$\therefore a_n = b_n + 2n - 2 = n$

42. (D)

$y = 3e^2 - x$

Let $f(x) = x^y = x^{3e^2-x}$, then $f'(x) = x^{3e^2-x}\left(\dfrac{3e^2-x}{x} - \ell nx\right)$

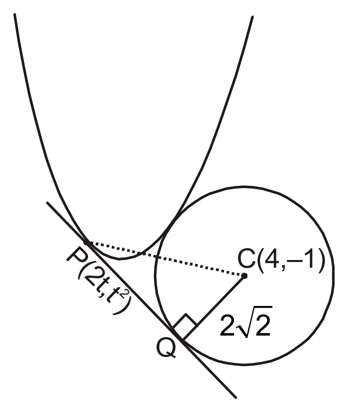
Let $g(x) = \dfrac{3e^2}{x} - 1 - \ell nx$, then $g'(x) = -\dfrac{3e^2}{x} - \dfrac{1}{x} < 0\ \forall x > 0$

Also $f'(e^2) = 0$

$\therefore f'(x) > 0$ in $(0, e^2)$ and $f'(x) < 0$ in (e^2, ∞)

$\therefore$ f(x) is maximum at x = e^2 and maximum value $= f(e^2) = e^{4e^2}$

43. (B)



Tangent at P(2t, t^2) to the parabola is xt = y + t^2.

The line touches the circle $(x-4)^2 + (y+1)^2 = (2\sqrt{2})^2$

$\therefore \dfrac{|4t+1-t^2|}{\sqrt{t^2+1}} = 2\sqrt{2}$

$\Rightarrow t^4 - 8t^3 + 6t^2 + 8t - 7 = 0$

$\Rightarrow (t-1)^2(t+1)(t-7) = 0 \ \Rightarrow t = 1, -1$ or 7.

For t = 1, circle and parabola touches each other.

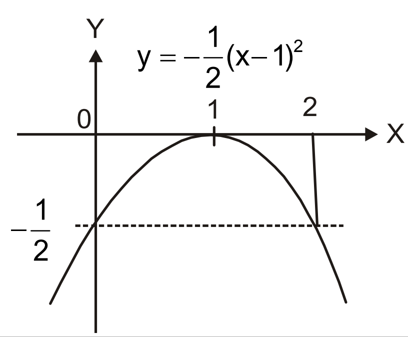
For t = –1, P = (–2,1) and for t = 7, P = (14, 49).

Now $PQ = \sqrt{PC^2 - CQ^2} = \sqrt{PC^2 - 8}$

So least value of PQ is obtained for t = –1.

$\therefore$ least value of $PQ = \sqrt{6^2 + 2^2 - 8} = \sqrt{32} = 4\sqrt{2}.$

44. (D)



$\log(ax+1) = \log(x-a) + \log(2-x) \quad \Rightarrow ax + 1 = (x-a)(2-x)$

$\Rightarrow a = -\dfrac{1}{2}(x-1)^2 \le 0$

For a = 0, x = 1 is the unique solution

For a < 0, ax + 1 > 0 $\Rightarrow -\dfrac{1}{2}x(x-1)^2 + 1 > 0$

$\Rightarrow (x^2+1)(x-2) < 0 \Rightarrow x < 2$

$x - a > 0 \Rightarrow x + \frac{1}{2}(x-1)^2 > 0 \Rightarrow x^2 + 1 > 0$ (always true) and $2 - x > 0 \Rightarrow x < 2$

$\therefore$ For unique solution in $(-\infty, 2)$, we must have $a \le \frac{-1}{2}$

$\therefore a \in \left(-\infty, -\frac{1}{2}\right] \cup \{0\}$

45. (C)

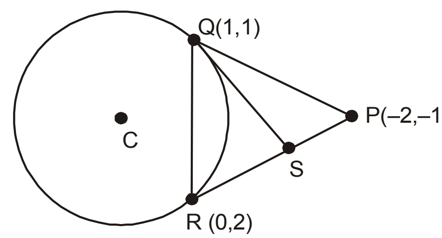
Let Q = (x, y), then $x^2 + y^2 - 4x - 6y + 8 = 0$ ....(i)

and $(x + 2)^2 + (y + 1)^2 = 13$ ....(ii)

Solving (i) and (ii), we get (x,y) = (1,1) or (0, 2).

Tangent at Q (1,1) is $x \cdot 1 + y \cdot 1 - 2(x+1) - 3(y+1) + 8 = 0$

i.e. x + 2y = 3.

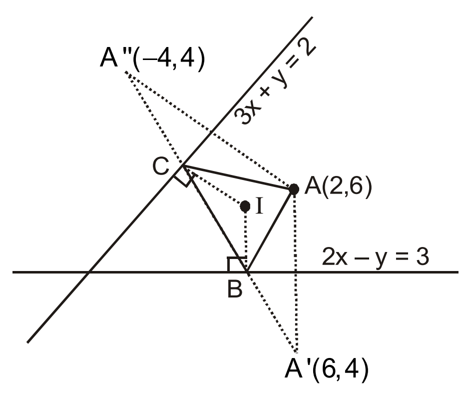


Equation of PR is $y = \frac{3}{2}x + 2$, $\therefore S = \left(-\frac{1}{4}, \frac{13}{8}\right)$

$\therefore$ $ar(\Delta QRS) = \frac{5}{16}$ sq. units

46. (A)

Image of A(2,6) about 2x – y = 3 is $A'(6,4)$ and about 3x + y = 2 is $A''(-4,4)$. B and C are points of intersection of the line $A'A''$ with the lines 2x – y = 3 and 3x + y = 2 respectively.



$\therefore$ $B = \left(\frac{7}{2}, 4\right)$ and $C = \left(-\frac{2}{3}, 4\right)$

Incentre is the point of intersection of normals to 2x – y = 3 at B and 3x + y = 2 at C respectively.

Equation of BI is 2x + 4y = 23 and that of CI is 3x – 9y + 38 = 0

$\therefore I = \left(\frac{11}{6}, \frac{29}{6}\right)$

47. (D)

There exists $\alpha \in \left(0, \frac{1}{2}\right)$ and $\beta \in \left(\frac{1}{2}, 1\right)$ such that

$$f'(\alpha) = \frac{f\left(\frac{1}{2}\right) - f(0)}{\frac{1}{2} - 0} = 2 \text{ and } f'(\beta) = \frac{f(1) - f\left(\frac{1}{2}\right)}{1 - \frac{1}{2}} = 2$$

$\therefore$ There exist $\gamma \in (\alpha, \beta)$ such that $f''(\gamma) = \frac{f'(\beta) - f'(\alpha)}{\beta - \alpha} = 0$

But $f'''(x) > 0 \Rightarrow f''(x)$ is increasing

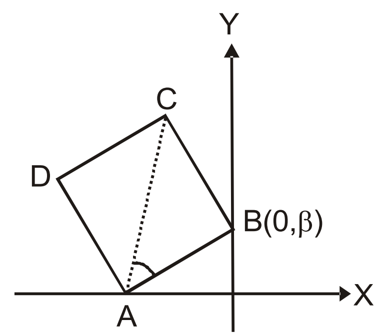
$\therefore$ $f''(x) < 0$ in $(-\infty, \gamma)$ and $f''(x) > 0$ in (γ, ∞)

$f'(x) > f'(\alpha)\ \forall x \in (-\infty, \alpha)$ and $f'(x) > f'(\beta)\ \forall x \in (\beta, \infty)$

$\therefore f'(x) > 2\ \ \forall x \in (-\infty, \alpha) \cup (\beta, \infty)$

$\therefore f(x) < 2x + 1\ \forall x \in (-\infty, 0)$ and $f(x) > 2x + 1\ \forall x \in (1, \infty)$

48. (A)



Let $B = (0, \beta)$. $A = (-2, 0)$.

slope of AC = 2 and slope of $AB = \frac{\beta}{2}$

$$\therefore \tan 45° = \frac{2 - \frac{\beta}{2}}{1 + 2.\frac{\beta}{2}} \Rightarrow 1 + \beta = 2 - \frac{\beta}{2} \Rightarrow \beta = \frac{2}{3}$$

Now image of $B\left(0, \frac{2}{3}\right)$ about AC is given by

$$\frac{x - 0}{2} = \frac{y - \frac{2}{3}}{-1} = -2\frac{\left(0 - \frac{2}{3} + 4\right)}{2^2 + 1^2} \Rightarrow D = \left(-\frac{8}{3}, 2\right)$$

49. (C)

$12^x - 4^x - 3^x + 1 = (4^x - 1)(3^x - 1) \geq 0\ \forall x \in R$

Also $\cos(-\theta) = \cos\theta$

$\therefore f(x) = 12^x - 4^x - 3^x + 1 + \cos^{-1}\left|x - \frac{1}{2}\right| + \cos(x-2)$

$\cos^{-1}|x - \frac{1}{2}|$ is non-differentiable at $x = -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$.

$\therefore$ f is non-differentiable at three points.

50. (C)

Put $x = \cos\theta, \theta \in (0, \pi]$

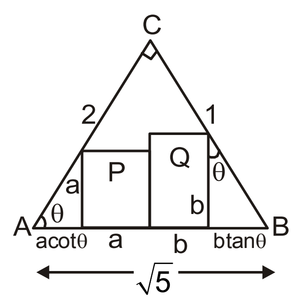
$\therefore f(x) = \cos^{-1}\left|\sin\frac{\theta}{2}\right| + \tan^{-1}\left|\cot\frac{\theta}{2}\right|$

$= \cos^{-1}\cos\left(\frac{\pi}{2} - \frac{\theta}{2}\right) + \tan^{-1}\tan\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = \pi - \theta = \pi - \cos^{-1}x, \; x \in [-1, 1)$

51. (A, C)

Clearly, AB = $a\cot\theta + a + b + b\tan\theta = \sqrt{5}, \tan\theta = 1/2$,

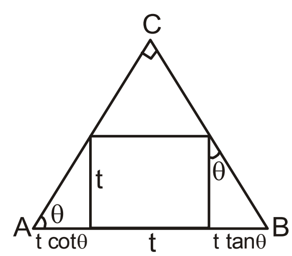
$\Rightarrow \quad 2a + b = \frac{2\sqrt{5}}{3}$ ... (i)



Again in the second figure,

$t\cot\theta + t + t\tan\theta = \sqrt{5} \quad \Rightarrow t = \frac{2\sqrt{5}}{7}$ ... (ii)

$\therefore \quad \frac{4\sqrt{5}}{21} \le a \le \frac{2\sqrt{5}}{7}$ (using (i) & (ii))



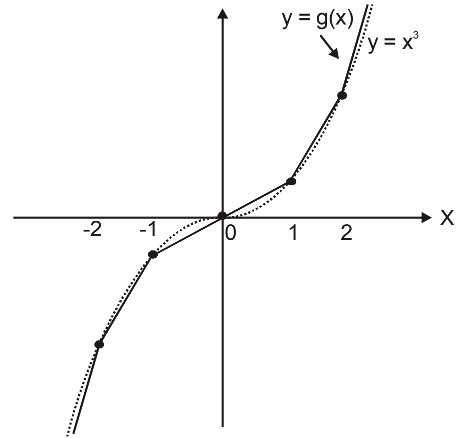
Sum of areas of P and Q

$= \quad a^2 + b^2 = a^2 + \left(\frac{2\sqrt{5}}{3} - 2a\right)^2$ (using (i))

$= \quad 5\left(a - \frac{4}{3\sqrt{5}}\right)^2 + \frac{4}{9} = f(a)$ (say)

$\therefore$ least value m = $f\left(\frac{4}{3\sqrt{5}}\right) = \frac{4}{9}$ and greatest value m = $f\left(\frac{4\sqrt{5}}{21}\right) = \frac{260}{441}$

52. (A,B,C)



For each $n \in I$,

g(n) = (1 – 0) f(n) + 0. f(n + 1) = f(n)

also for $x \in [n, n+1)$,

g(x) = (1 – (x – n)) f(n) + (x – n) f(n + 1)

$= (f(n+1) - f(n))x + (n+1)f(n) - nf(n+1)$

which is equation of a straight line.

From the graph of y = g(x),

options (A), (B) and (C) are clearly true.

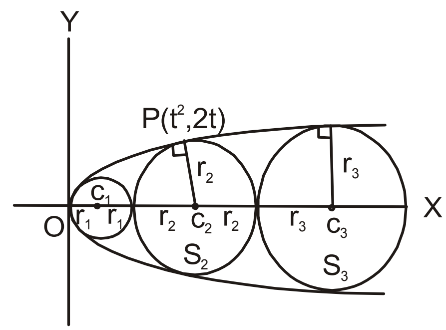
Option (D) is false as g is differentiable at x = 0.

53. (A, C)

$C_1 = (r_1, 0), C_2 = (2r_1 + r_2, 0)$ and $C_3 = (2r_1 + 2r_2 + r_3, 0)$

Normal at $P(t^2, 2t)$ is $y = -tx + 2t + t^3$

this passes through C_2



$\therefore 0 = -t(2r_1 + r_2) + 2t + t^3 \Rightarrow t^2 = 2r_1 + r_2 - 2$

$\therefore r_2 = PC_2 = \sqrt{(2r_1 + r_2 - t^2)^2 + (2t - 0)^2}$

$\Rightarrow r_2^2 = 4 + 4(2r_1 + r_2 - 2) \Rightarrow r_2 = 2(1 \pm \sqrt{2r_1})$

$\therefore r_2 = 2(1 + \sqrt{2r_1}) = 2(1 + 2) = 6 \ (\because r_2 > r_1 = 2)$

similarly, $r_3 = 2(1 + \sqrt{2(r_1 + r_2)})$ = 2 (1 + 4) = 10.

length of direct common tangent to S_1 and $S_2 = 2\sqrt{r_1 r_2} = 4\sqrt{3}$

and that to S_2 and $S_3 = 2\sqrt{r_2 r_3} = 4\sqrt{15}$

54. (A, C)

$$\ell = \lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{xf(x)}{x}$$

Applying L'Hospital's rule

$$\ell = \lim_{x\to\infty} \frac{xf'(x) + f(x)}{1} = \lim_{x\to\infty} xf'(x) + \ell \qquad \Rightarrow \lim_{x\to\infty} xf'(x) = 0$$

Again $0 = \lim_{x\to\infty} \dfrac{x^2 f'(x)}{x} = \lim_{x\to\infty} \dfrac{2xf'(x) + x^2 f''(x)}{1}$

$= 0 + \lim_{x\to\infty} x^2 f''(x) \qquad \therefore \lim_{x\to\infty} x^2 f''(x) = 0$

55. (B, D)

Here L is the line $S_1 - S_2 = 0$

$\therefore$ Options (B) and (D) are obviously true.

Also (D) $\Rightarrow$ (A) is false.

Now line joining centres (4,3) & (1,–1) of S_1 and S_2 is 4x – 3y = 7 which intersects L at $\left(\frac{11}{5}, \frac{3}{5}\right)$

Let $x^2 + y^2 + 2gx + 2fy + c = 0$ be the circle with centre $\left(\frac{11}{5}, \frac{3}{5}\right)$ and cutting S_1 and S_2 orthogonally, then $g(-2) + f \cdot 2 = c + 0$

$$\frac{-11}{5}(-2) + \left(\frac{-3}{5}\right) \cdot 2 = c + 0 \Rightarrow c = \frac{16}{5}$$

$\therefore$ radius $= \sqrt{g^2 + f^2 - c} = \sqrt{2}$

$\therefore$ only one circle with radius $\sqrt{2}$ is possible.

56. (7)

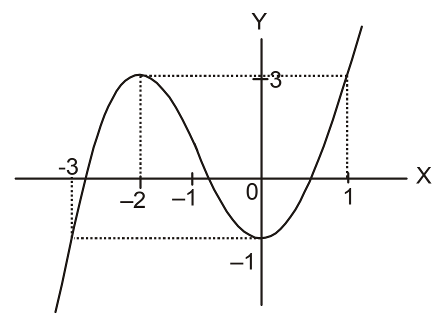
$f'(x) = 3x^2 + 6x = 3x(x+2)$

$f(-3) = -1, f(-2) = 3, f(-1) = 1,$

f (0) = –1, f(1) = 3

$\therefore$ f(x) = 0 has three real roots α, β, γ

such that $-3 < \alpha < -2, -1 < \beta < 0$ and $0 < \gamma < 1$.



$\therefore$ $f(x) = \alpha, f(x) = \beta$ and $f(x) = \gamma$ have 1, 3 and 3 roots respectively.

$\therefore$ f(f(x)) = 0 has 7 distinct real roots

57. (5)

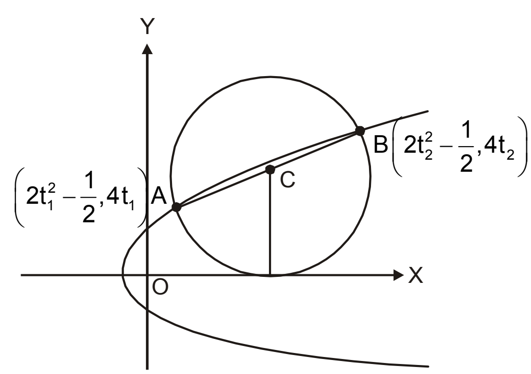
y-co-ordinate of centre C $= 2(t_1 + t_2) = 2$

$\Rightarrow t_1 + t_2 = 1$

Now length of chord AB

$= a\,|t_1 - t_2|\,\sqrt{(t_1 + t_2)^2 + 4} = 2\sqrt{5}\,|t_1 - t_2| = 4$

$\Rightarrow |t_1 - t_2| = \frac{2}{\sqrt{5}} \Rightarrow t_1 t_2 = \frac{1}{4}\left\{1^2 - \frac{4}{5}\right\} = \frac{1}{20}$



$\therefore$ Point of intersection of tangents at A and B $= \left(at_1t_2 - \frac{1}{2}, a(t_1 + t_2)\right) = \left(-\frac{2}{5}, 2\right)$

58. (4)

Domain of f is [–1, 1].

$\because$ both $\tan^{-1}x$ and $\sin^{-1}x$ are increasing

$\therefore$ f is also an increasing function

$\therefore f_{min} = f(-1) = \left[-\left(\tan^{-1}\frac{\pi}{2} + \sin^{-1}\frac{\pi}{4}\right)\right]$ and $f_{max} = f(1) = \left[\tan^{-1}\frac{\pi}{2} + \sin^{-1}\frac{\pi}{4}\right]$

Now $\sin^{-1}\frac{\pi}{4} < \sin^{-1}0.79 < \sin^{-1}\frac{\sqrt{5}+1}{4} = \frac{3\pi}{10}$ and $\tan^{-1}\frac{\pi}{2} < \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

$\therefore \tan^{-1}\frac{\pi}{2} + \sin^{-1}\frac{\pi}{4} < \frac{3\pi}{10} + \frac{\pi}{3} = \frac{19\pi}{30} < 2$

$\therefore$ Range of f = {–2, –1, 0,1}

59. (6)

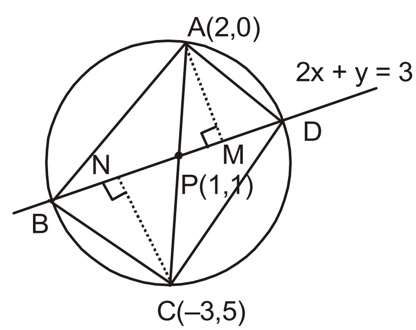
$\ell = e^{\lim\limits_{x\to 0}\left(\tan^{-1}\frac{1}{x^2} + x^2\sin\frac{1}{x^4}\right)}$ $(\because 1^{\infty}$ form$)$

$= e^{\frac{\pi}{2}}$

60. (8)

Equation of AC is x + y = 2.

$\therefore P = (1,1)$ is the point of intersection of AC and BD



$\because PB \cdot PD = PA \cdot PC = 8$

$\therefore \frac{PB + PD}{2} \geq \sqrt{PB \cdot PD} = 2\sqrt{2}$

$\Rightarrow BD \geq 4\sqrt{2}$

Now area of quad. ABCD

$= \frac{1}{2} \times BD \times AM + \frac{1}{2} \times BD \times CN$

$= \frac{1}{2} \times BD \times \left(\frac{1}{\sqrt{5}} + \frac{4}{\sqrt{5}}\right) = \frac{\sqrt{5}}{2} \times BD \geq \frac{\sqrt{5}}{2} \times 4\sqrt{2} = 2\sqrt{10} = A$

JEE ADVANCED PAPER-II

PHYSICS

1. (B, C)

Normal Reaction $N = 20N + 15N = 35N$

Kinetic friction acting $= \frac{1}{7} \times 35N = 5N$

In case of moving object friction acts opposite to the direction of velocity so,

the friction force $= -5\left(\frac{20\hat{i} + 15\hat{j}}{25}\right) = \left(-4\hat{i} - 3\hat{j}\right)N$

The net force acting of the block

$$= 20\hat{i} + 15\hat{j} + \left(-4\hat{i} - 3\hat{j}\right)$$

$$= 16\hat{i} + 12\hat{j}$$

So, acceleration $= \frac{16\hat{i} + 12\hat{j}}{2} = \left(8\hat{i} + 6\hat{j}\right)m/s^2$

As, $\vec{F}_{net} \cdot \vec{v} = 16 \times 20 + 12 \times 15$ is positive the speed increases

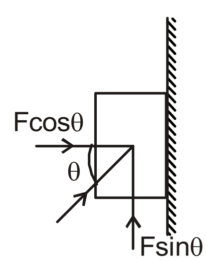
2. (A, D)

If the wall is smooth then 50 N = mg

$\Rightarrow m = 5kg$

$F\sin\theta + \mu F\cos\theta = mg$

$\Rightarrow F = \frac{mg}{\sin\theta + \mu\cos\theta}$ ....(i)



$\Rightarrow F = \frac{\frac{\sqrt{3}}{2} mg}{\sin\left(\theta + 30^\circ\right)}$

$\Rightarrow F_{min}$ from $\theta = 60^\circ$

From equation (i)

$mg = F\left(\sin\theta + \mu\cos\theta\right)$

clearly $m > 5kg$

3. **(B, C)**

The focal length given is for yellow -green light, which is mean of the visible light.

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad ....(i)$$

$$\frac{-df}{f^2}=\left(\frac{1}{R_1}-\frac{1}{R_2}\right)dn \qquad ....(ii)$$

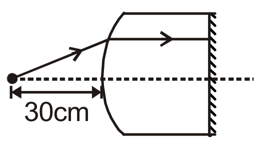
Dividing (ii) by (i)

$$\Rightarrow \frac{-df}{f}=\frac{dn}{n-1} \Rightarrow df=-\left(\frac{dn}{n-1}\right)f$$

$$\Rightarrow f_v - f_R = -\omega f = -\omega f$$

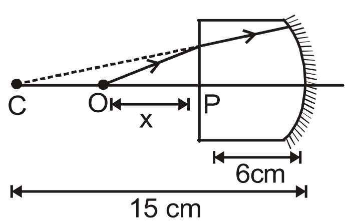
$$\Rightarrow f_R - f_v = \omega f = .04\times 10 = 0.4\,cm$$

4. **(A, C)**

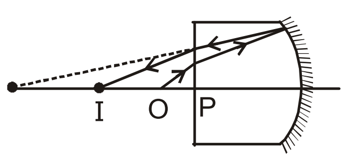


$$\frac{1.5}{\infty}-\frac{1}{-30}=\frac{1.5-1}{R}$$

$$\Rightarrow R = 15\,cm$$



When curved surface is silvered the ray should incident on curved surface normally. Hence,



$$\frac{OP}{PC}=\frac{1}{3/2} \Rightarrow OP=\frac{2}{3}(PC)=\frac{2}{3}(15-6)\,cm = 6\,cm$$

When the object is at 2 cm from plane surface. Then image in plane surface form at $2\times\frac{3}{2}=3\text{cm}$ from the plane surface. Now the ray reaches to the mirror.

So, the object distance for mirror $=6\,\text{cm}+3\,\text{cm}=9\,\text{cm}$

Using, mirror formula $\frac{1}{v}+\frac{1}{-9}=\frac{1}{-15/2}$

$\Rightarrow\ \frac{1}{v}=\frac{1}{15}+\frac{1}{9}=\frac{-6+5}{45}$

$\Rightarrow\ v=-45\,\text{cm}$

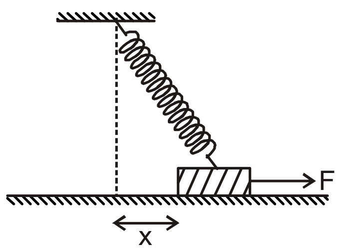
Now, as the come out of plane surface. If again deviated so, that

$PI=\frac{45-6}{3/2}=26\,\text{cm}$

So, distance from object $=26-2=24\,\text{cm}$

5. (B, C)

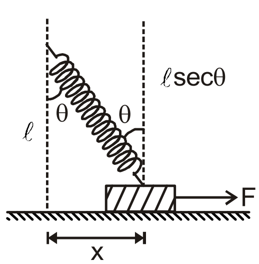
If normal reaction is constant and equal to mg. Then work doen by friction is



for $\mu=\frac{1}{2}$

$w_f=\frac{1}{2}\times 20\times 0.3=3J$

but as the friction is μN and nomral decreases as the spring get elongated. So, the negative work down by friction will be less than 3 J.



$\tan\theta = \frac{x}{\ell} \Rightarrow dx = \ell\sec^2\theta.d\theta$

friction force acting at the position shown in figure $= \mu\left(mg - k\ell(\sec\theta - 1)\cos\theta\right)$

So, negative work done by friction in moving a small distance dx

$= \mu\left(mg - k\ell(1-\cos\theta)\right)dx$

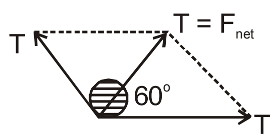
For whole path

$$= \mu(mg - k\ell)x + \mu k\ell\int_{0^\circ}^{37^\circ}\cos\theta.\ell\sec^2\theta\, d\theta$$

$$= \frac{1}{2}\left\{-18 + 32\,\ell n\left(\sec 37^\circ + \tan 37^\circ\right)\right\} = \frac{1}{2}(-18+22) = 2\,J$$

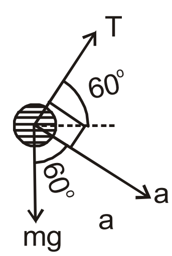
6. (A, D)

Net force acting on the pulley due to string is as shown.



As both ends are fixed and only pulley is moving. As work done by string is always zero the acceleration must be perpendicular to the F_{net} due to string.

So, FBDof pulley is as shown



$mg\cos 60^\circ = ma$

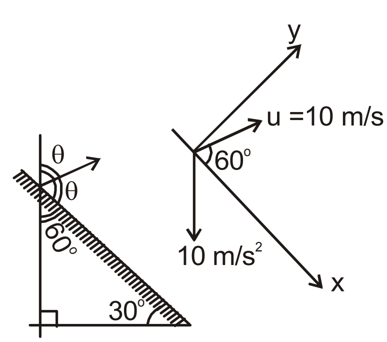
$\Rightarrow a = g/2$

and $mg\sin 60^\circ = T$

$\Rightarrow T = 10\sqrt{3}\,N$

7. (B, C)

To throw the ball down the incline the angle at which the ball should be projected with minimum speed is shown in figure .



$\theta + \theta + 60^\circ = 180^\circ$

$\Rightarrow \theta = 60^\circ$

As, $\theta = 60^\circ$,

$u_x = 10\cos 60^\circ = 5\,m/s$

$u_y = 5\sqrt{3}\,m/s$

$a_x = 5\,m/s^2$

$a_y = -5\sqrt{3}\,m/s^2$

From y - direction

$$s = ut + \frac{1}{2}at^2$$

$$0 = 5\sqrt{3} \times t - \frac{1}{2} \times 5\sqrt{3} \times t^2 \qquad \Rightarrow t = 0, 2 \text{ sec}$$

From x - direction

$$5 \times 2 + \frac{1}{2} \times 5 \times 2^2 = 20\,m$$

When Shyam has to throw the ball to Ram with minimum speed then

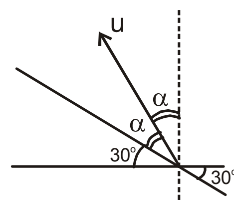
$$\alpha + \alpha + 30^\circ = 90^\circ \qquad \Rightarrow \alpha = 30^\circ$$

$u_x = u\cos 30^\circ$

$u_y = u\sin 30^\circ$

$a_x = -5$

$a_y = -5\sqrt{3}$



$$20 = u\cos 30^\circ . t - \frac{1}{2} 5t^2 \qquad ......(i)$$

$$0 = u\sin 30^\circ . t - \frac{1}{2} \times 5\sqrt{3} \times t^2 \qquad ....(ii)$$

$$\Rightarrow t = 0, \frac{4}{5\sqrt{3}}$$

Putting in equation (i)

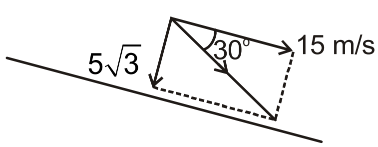
$$20 = \frac{u \times \sqrt{3}}{2} \times \frac{u}{5\sqrt{3}} - \frac{5}{2}\ \frac{u^2}{25 \times 3} = \frac{u^2}{10} - \frac{u^2}{30} = \frac{2u^2}{30} = \frac{u^2}{15}$$

$$\Rightarrow u^2 = 300 \qquad \Rightarrow u = 10\sqrt{3}\,m/s$$

When Ram throws the ball to Shyam with minimum speed. The ball reaches to Shyam with speed

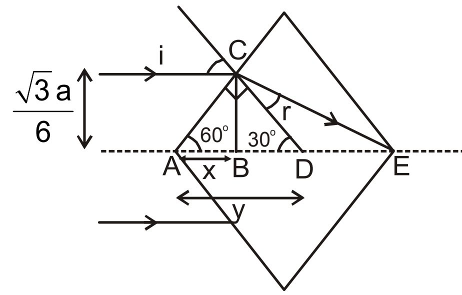
$$u_x = 5 + 552 = 15\,m/s$$

$$u_y = 5\sqrt{2} - 2 \times 5\sqrt{3} = -5\sqrt{3}\,m/s$$



Means 30° below the incline so, path is exactly same as when Shyam throw the ball to Ram.

8. (B, D)



$$\tan 60^\circ = \frac{BC}{AB} = \frac{\sqrt{3}\frac{a}{6}}{AB}$$

$$\Rightarrow AB = \frac{a}{6}$$

CD is $\perp \ell r$ to AC So,

$$\frac{AC}{AD} = \cos 60^\circ$$

$$\Rightarrow AD = \frac{\sqrt{\left(\frac{a}{6}\right)^2 + \left(\frac{\sqrt{3}a}{6}\right)^2}}{\cos 60^\circ}$$

$$\Rightarrow AD = \frac{a/3}{1/2} = \frac{2a}{3}$$

$$CE^2 = (BC)^2 + (BE)^2$$

$$= \left(\frac{\sqrt{3}a}{6}\right)^2 + \left(a - \frac{a}{6}\right)^2$$

$$= \frac{28a^2}{36}$$

$$\Rightarrow CE = \frac{2\sqrt{7}a}{6}$$

Now, apply sine rule in the trinagle CDE

$$\frac{\sin r}{DE} = \frac{\sin 150^\circ}{CE}$$

$$\Rightarrow \sin r = \frac{1}{2\sqrt{7}}$$

Now, apply snell's law to the face AC

$$1 \cdot \sin 30^\circ = \mu \cdot \sin r$$

$$\Rightarrow \frac{1}{2} = \mu \times \frac{1}{2\sqrt{7}} \Rightarrow \mu = \sqrt{7}$$

9. (B)

10. (A)

When a velocity 'u' is given vertically upward then maximum height above point of projection that the particle goes is $\frac{u^2}{2g}$. As the speed be half so height becomes one forth.

so, total distance moved

$$=(21-9)+\frac{1}{4}(21-9)\times 2+9=27\,m$$

The time of flight is maximum in the case hitted at lowest level.

11. (A)

12. (D)

$$T_A=\lambda\ell g+\int_0^{\pi/6}\lambda R\cos\theta g\,d\theta$$

$$=\lambda\ell g+\lambda Rg\frac{1}{2}$$

$$T_B=\lambda\ell g+\int_0^{\pi/2}\lambda R\cos\theta g\,d\theta$$

$$=\lambda\ell g+\lambda Rg$$

Given,

$$T_B=\frac{3}{2}T_A$$

$$\Rightarrow \lambda\ell g+\lambda Rg=\frac{3}{2}\left(\lambda\ell g+\frac{\lambda Rg}{2}\right)$$

$$\Rightarrow \frac{\lambda\ell g}{2}=\frac{1}{4}\lambda Rg \qquad \Rightarrow \ell=\frac{R}{2}$$

X - component of normal reaction N is given by

$$N_x=\int_0^{\pi/2}\lambda R\sin\theta\cos\theta g d\theta+T_B$$

$$=2\lambda Rg$$

13. (A)

14. (D)

If $\tan\theta > \mu$

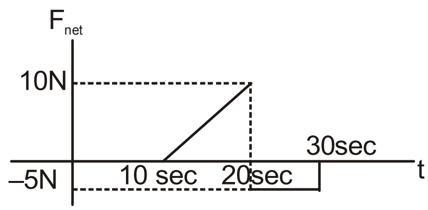
$$a_{min} = g\left(\frac{\tan\theta - \mu}{1+\mu\tan\theta}\right) \quad \& \quad a_{max} = g\left(\frac{\tan\theta + \mu}{1-\mu\tan\theta}\right)$$

15. (D)

16. (B)

Friction force is self adjustable till there is no relative sliping. So net force is zero till the applied fore is less than 10N. But after that (F – 10) N is the net force applied untill the block keeps moving on the surface.

So,



$$\frac{1}{2}\times 10\times 10 = 5\times(t-20)$$

$$\Rightarrow t = 30\,\text{sec}$$

17. (D)

As sources s_3 and s_4 are symmeterically placed intensities at s_3 and s_4 are equal. Path difference

$$\Delta x = s_2 s_3 - s_1 s_3 = \frac{d(z/2)}{D}$$

phase difference $\Delta\phi = \dfrac{\Delta x}{\lambda}2\pi$

Intensity at s_3 and s_4 is equal to $4I\cdot\cos^2\left(\dfrac{\Delta\phi}{2}\right)$

So, Intensity at P is

$$16I\cdot\cos^2\left(\frac{\Delta\phi}{2}\right) = 16I\cdot\cos^2\left(\frac{\pi}{2}\cdot\frac{z}{\beta}\right)$$

18. (C) **19. (A)** **20. (A)**

CHEMISTRY

21. (A,B,C)

$$\frac{W_1}{\text{Mol wt Silver Salt}} = \frac{W_2}{A + \text{Wt Silver}}$$

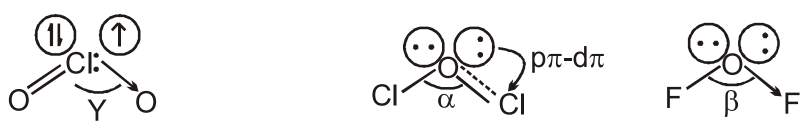
$$\frac{W_1}{W_2} = \frac{\text{Mol wt Silver Salt}}{\text{At wt of Silver}} = \sqrt{3}$$

Molar wt silver salt = $108\sqrt{3}$

Molar wt acid = $108\sqrt{3} - 107 = 108\left(\sqrt{3}-1\right)+1$

$$\%\text{ of Ag} = \frac{W_2}{W_1} \times 100 = \frac{1}{\sqrt{3}} \times 100$$

22. (A,B,C,D)



$\alpha > \beta$
$Y > \alpha$
$Y > \beta$

23. (A,B)

$$\left(P + \frac{a}{V_{m^2}}\right)\left(V_m - b\right) = RT$$

$$PV_m - Pb + \frac{a}{V_m} - \frac{ab}{V_{m^2}} = RT$$

$P \to 0 \;\; V_m \to \infty$

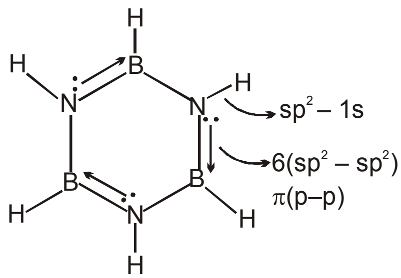
$PV_m = Pb + RT$

Slop = b intercept RT

If $b \to o$ $PV_m = \frac{-a}{V_m} + RT$

$$PV_m = \frac{-a}{ZRT}P + RT$$

24. (A,B,D)



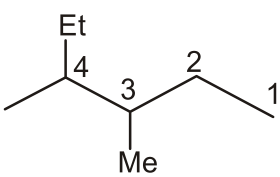
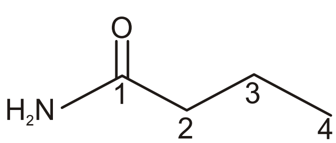
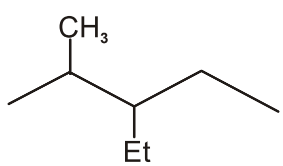
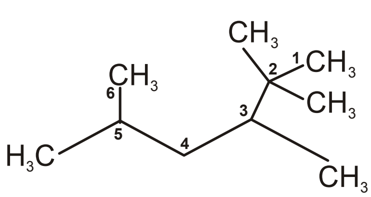
25. (D)

ABC are enantiomers. D is diestereomer of the given compound.

26. (A,B,C,D)

Conceptual

27. (A,B,C,D)

Structure	**Correct IUPAC name**
(A) 	3,4-Dimethylhexane
(B) 	Butanamide
(C) 	3-Ethyl -2-methyl pentane
(D)  ⟶ 2,2,3,5-Tetramethylhexane	

28. (B,C,D)

Let $Fe_2(SO_4)_3$ = a milimole & Fe C_2O_4 = b millimole

$3b = 48\times0.4\times5$ $b = 32$

After reduction Fe^{++} in mixture = b + 2a

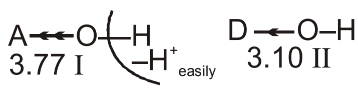
Meq of $KMnO_4$ add = 90×0.4×5=180

Excess $KMnO_4$ for $= 6\times\left[\frac{1000\times 560}{10^6\times 56}\times 1000\right]$ = 60 meq

2a + b = 180 –60=120

$a = \frac{120-32}{2} = 44$

29. **(A)**



So I > II
acidic

30. **(A)**

31. **(A)**

$$P = \frac{nRT}{V-nb} - \frac{an^2}{V^2} = \frac{2\times .0821\times 300}{5-2\times .037} - \frac{4.17\times 2^2}{5^2} = 9.33\,\text{atm}$$

32. **(A)**

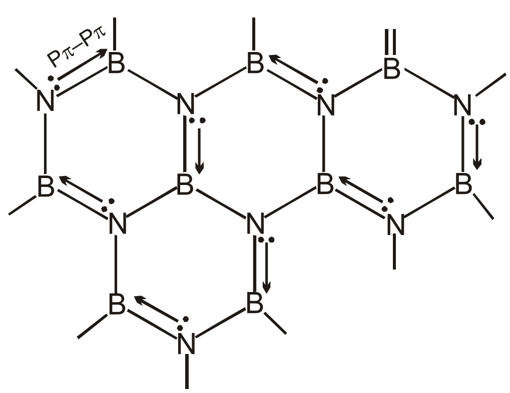
At low pressure $PV_m = RT - \frac{a}{V_m}$

slop = –a = –4.17

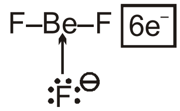
33. **(B)**

34. **(C)**

35. **(C)**



36. (A)



37. (A)

A→R B→P C→S D→Q

(A) $A_2 + B_2 \rightarrow 2AB$ $AB + B_2 \rightarrow AB_3$

2 2 4 4 4 4

(1 mole B_2 left) (R)

(B) $A_2 + B_2 \rightarrow 2AB$ $AB + B_2 \rightarrow AB_3$

1 1 2 2 1

1 0 1

Equal mole of AB & AB_3 (P)

(C) $A_2 + B_2 \rightarrow 2AB$ $AB + B_2 \rightarrow AB_3$

1 1 2 2 2

0 0 2

Only AB_3 is formed. (s)

(D) $A_2 + B_2 \rightarrow 2AB$ $AB + B_2 \rightarrow AB$

1 1 2

Only AB is formed (Q)

38. (B)

A→S B→R C→Q Q→P

(A) P_1 = 2 atm P_2

T_1= – 23ºC= 250 T_1= 27º = 300K

$$P_2 = \frac{P_1}{T_1} \times T_2 = \frac{2}{250} \times 300 = 2.4\,\text{atm}$$

(B) 1.5×2 + 2× 3= P×5 P=1.8 atm

(C) NH_3(b) + HCl (g) → NH_4Cl (s)

2 1

1 0 1

ng = 1 $P = \frac{1 \times .0821 \times 300}{8.21} = 3\,\text{atm}$

(D) P_{gas} = 770 – 20 = 750

When volume is reduced pressure of gas = 750 × 2 = 1500 mm

P_{Total} = 1500 × 20 = 1520 mm = 2 atm

39. (B)

(A-Q; B-P; C-R; D-Q)

$PBr_5(s) \longrightarrow \underset{sp^3}{PBr_4^+} + Br^-$

$N_2O_5(s) \longrightarrow \underset{sp}{NO_2^+} NO_3^-$

$AlCl_3(s) \longrightarrow$ octahedral

$SO_3(s) \longrightarrow [SO_3]_3$ or polymeric linear str.

40. (C)

(a) $\rightarrow$ p
(b) $\rightarrow$ p,r
(c) $\rightarrow$ p,q,r,s
(d) $\rightarrow$ p
Acidity order
(c) > (b) > (d) > (a)

MATHEMATICS

41. (A,B,D)

$f(x) = a(x-2)^2 + 1$

$\because$ $f(1) = 2 \quad \Rightarrow \quad a = 1$

$f(x) = x^2 - 4x + 5$

$g(\ln x) = x^2 - 4x + 5 = (x-2)^2 + 1$

$g(x) = (e^x - 2)^2 + 1$

$x \in (-\infty, \ln 2] \quad \Rightarrow g(x) \in [1,5)$

$g^{-1}(x) = \ln\left(2 - \sqrt{x-1}\right)$

42. (A,C,D)

Given Limit is

$$L = \lim_{x\to\infty} \frac{2 - \left(\frac{\tan^{-1}x}{x}\right)^3}{\frac{8}{\pi}\cot^{-1}|k\,x| + \frac{k^2 \sin\frac{1}{x^3}}{\frac{1}{x^3}} - 3k} = \frac{1}{2}$$

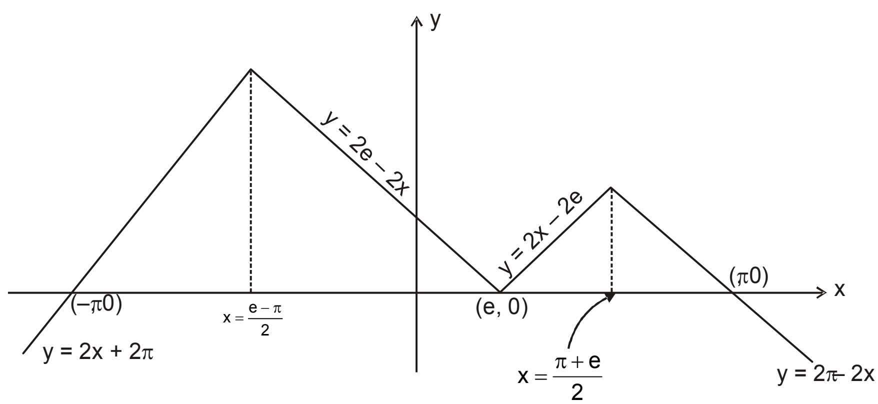
If k = 0, then $L = \frac{2-0}{\frac{8}{\pi}\cdot\frac{\pi}{2}} = \frac{1}{2}$

If k ≠ 0, then $L = \frac{2}{k^2 - 3k} = \frac{1}{2}$

$\Rightarrow$ k = –1, 4

43. (A, C)

The graph of f(x) will be



Range of f(x) is (– ∞, π + e]

44. (C, D)

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} 3 & -4 & 4a \\ 2 & -3 & 4a \\ 5 & -1 & a \end{vmatrix}$$

C_1 = Cofactor of c_1 = 13

C_2 = Cofactor of c_2 = –17

C_3 = Cofactor of c_3 = –1

Area of the triangle = $\frac{D^2}{|2C_1C_2C_3|} = \frac{17a^2}{26} = \frac{34}{13}$ (Given)

$\Rightarrow$ $a^2 = 4$

45. (A, B, D)

The normal to $y^2 = 4(x - 1)$ is $y = m(x - 1) - 2m - m^3$

i.e $y = mx - 3m - m^3$, if it is normal to $y^2 = 4kx$ then

$2km + km^3 = 3m + m^3$ $\Rightarrow m = 0, m^2 = \frac{3-2k}{k-1}$

According to the given condition $\frac{3-2k}{k-1} \le 0$ or k = 1 $\Rightarrow k \le 1$ or $k \ge \frac{3}{2}$

46. (A, C)

$\phi(x) = f(x) + f(2a - x)$

$\phi'(x) = f'(x) - f'(2a - x)$

$\because \quad f''(x) > 0$

$\Rightarrow \quad f'(x)$ is an increasing function

Now, $f'(x) > f'(2a - x)$

$\Rightarrow \quad x > 2a - x$

$\Rightarrow \quad x > a$

$\therefore \quad \phi'(x) > 0$ if $x > a$

$\Rightarrow \quad \phi(x)$ increases in $(a, 2a)$

Similarly $\phi(x)$ decreases in $(0, a)$

47. (A, C)

Equation of radical axis is $x = 0$

If one circle lies completely inside the other, then center of both circles should lie on the same side of radical axis and radical axis should not intersect the circles.

$\Rightarrow \quad (-a_1)(-a_2) > 0$

$\Rightarrow \quad a_1 a_2 > 0$

& $y^2 + c = 0$ should have imaginary roots

$\Rightarrow \quad c > 0$

48. (B, C)

$\cos^{-1} x = 0$ & $\sin^{-1} y = \pm 1$

49. (A)

50. (A)

Let $f(x) = Ke^x - x$

$\Rightarrow f'(x) = Ke^x - 1$

$f'(x) = 0$

$\Rightarrow x = -\ln K$

$f''(-\ln K) = 1 > 0$

$\therefore \quad f(-\ln K) = 1 + \ln K$

For one root $1 + \ln K = 0$

$$\Rightarrow k = \frac{1}{e}$$

For two distinct roots, $1 + \ln K < 0$ $(K > 0)$

$$\Rightarrow K \in \left(0, \frac{1}{e}\right)$$

51. (D)

Since minimum occurs before maximum, $\therefore\ p < 0$

$\therefore\ p = -2$

Let $g(x) = px^3 + qx^2 + rx + s$

$g(x) = -2x^3 + qx^2 + rx + s$

$g'(x) = -6x^2 + 2qx + r \equiv -6(x+2)(x-2)$

$\Rightarrow\ q = 0, r = 24 \ \Rightarrow p + q + r = 22$

52. (C)

both g(–2) & g(2) are positive

$\Rightarrow$ s > 32

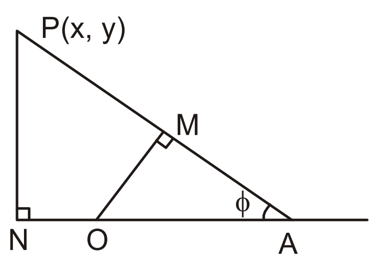
53. (B)

Let $x = a + r\cos\theta,\ y = b + r\sin\theta,$ then the given equation is $|a\sin\theta - b\cos\theta| = c$

$\Rightarrow\ 0 \le c \le \sqrt{a^2 + b^2}$ but c > 0

$\therefore\ 0 < c \le \sqrt{a^2 + b^2}$

54. (D)



The given equation is $\dfrac{|ay - bx|}{\sqrt{a^2 + b^2}} = \dfrac{c}{\sqrt{a^2 + b^2}}\sqrt{(x-a)^2 + (y-b)^2}$ ......(i)

The point A (a, b) lies on ay – bx = 0

If N is the foot of perpendicular from P(x, y) to the line ay – bx = 0, then equation (1) can be written as

PN = K PA, where $K = \dfrac{c}{\sqrt{a^2 + b^2}}$

Clearly $\dfrac{PN}{PA} = \sin\phi = \dfrac{c}{\sqrt{a^2 + b^2}}$

Now, P moves on the line inclined at angle ϕ to the line ay – bx = 0 and passing through A (a, b).

OM = OA sin ϕ = c

55. (D)

56. (B)

$$f(\cot x)=\frac{\cot^2 x+2\cot x-1}{\cot^2 x+1}$$

$$\Rightarrow\ f(t)=\frac{t^2+2t-1}{t^2+1}, t\in R$$

$$\therefore\ g(x)=\frac{\sin^4 2x+32.\sin^2 2x-32}{\sin^4 2x-8\sin^2 2x+32}, x\in R$$

Put $z=\frac{1}{4}\sin^2 2x \qquad \Rightarrow\ z\in\left[0,\frac{1}{4}\right]$

Now, $h(z)=\frac{z^2+8z-2}{z^2-2z+2}; z\in\left[0,\frac{1}{4}\right]$

g(x) has minimum = –1 & maximum = $\frac{1}{25}$

57. (B)

(P) Equation of tangent to $x^2 = 4y$ is $y = mx - m^2$. It passes through P (h, k)

$\Rightarrow\ m^2-hm+k=0\ \Rightarrow\ m_1+m_2=h\ \&\ m_1m_2=k$

Now, $m_1m_2=1\ \Rightarrow k=1\ \Rightarrow$ locus of P is y = 1

(Q) The locus will be the directrix of the parabola

(R) locus of P is $(y-1)^2 = 2x + 3$ whose tangent at the vertex is $2x + 3 = 0$

(S) Tangent to $y^2=12x$ is $y=mx+\frac{3}{m}$ which is tangent to $x^2+y^2=\frac{9}{2}\Rightarrow\ m=\pm1$

$\Rightarrow$ Common tangents are $y=x+3$ or $y=-x-3$

58. (D)

$m=0, M=\frac{\pi}{12}\ \&\ a=3^{\frac{1}{4}}$

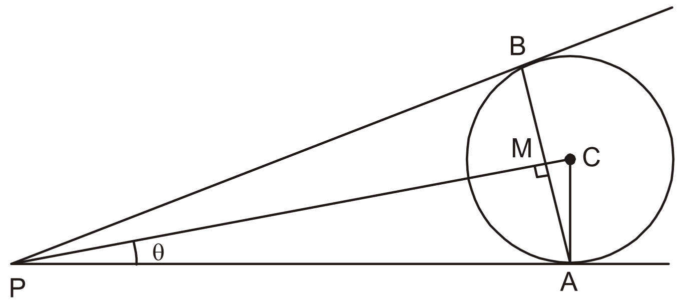
(P) $\tan^{-1}\left(3\left(\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)+\frac{\pi}{12}\right)\right)=-1$

(Q) $\sin^{-1}2\sqrt{x}=3\tan^{-1}\left(\tan\frac{\pi}{12}\right)=\frac{\pi}{4}\ \Rightarrow 8x=1$

(R) $x^2-\tan\left(3\sin^{-1}\left(\sin\frac{\pi}{12}\right)\right)x+3=0 \Rightarrow x^2-x+3=0\Rightarrow \alpha+\beta=1\ \&\ \alpha\beta=3$

(S) $\cos^{-1}x+\cos^{-1}y=2\pi\Rightarrow\ x+y=-2$

59. (D)



(P) $\tan\theta = \frac{r}{PA}$

Given, $\frac{1}{CA^2} + \frac{1}{PA^2} = \frac{1}{16}$

$\Rightarrow \frac{1}{r^2} + \frac{1}{PA^2} = \frac{1}{16}$ $\Rightarrow$ PA $\sin\theta = 4$

$\Rightarrow$ AM $= 4$

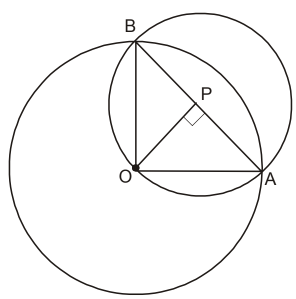
$\therefore$ AB $= 8$

(Q) Any point on the given circle can be taken as

$(x, y) \equiv (-7 + 8\cos\theta, -3 + 8\sin\theta)$

$\therefore\ 3x + 4y = -33 + 8(3\cos\theta + 4\sin\theta) \Rightarrow -73 \le 3x + 4y \le 7$

(R)

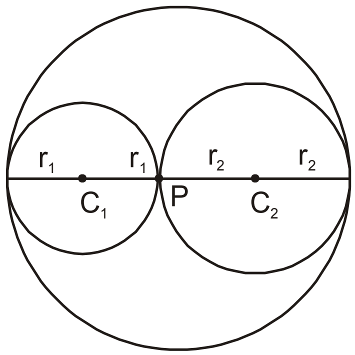


$AP^2 + OP^2 = OA^2 = \left(\sqrt{2}\right)^2$

$\Rightarrow r = 1$

(S) The point P $\left(\sqrt{3}, \sqrt{2}\right)$ lies inside the circle

$\therefore\ r_1 + r_2 = 2$



60. (C)

(P) $\lim_{h\to 0}\left(\frac{f\left(2+3h^4\right)-f\left(2-5h^4\right)}{h^4}\right) = 8f'(2) = 2$

(Q) $\lim_{x\to 0}\frac{\ln\left(1+x+x^2+....+x^n\right)}{x+x^2+.......+x^n}\cdot\frac{x+x^2+....+x^n}{nx} = \frac{1}{5}$

$\Rightarrow\ \frac{1}{n} = \frac{1}{5} \Rightarrow n = 5$

(R) f(x) is discontinuous at x = 3, 6 & 2π

(S) $g(x) = \left|x\left(2x+1\right)\left(2x-1\right)\right|\cos\pi x$ which is non-differentiable at x = 0