

SOLUTIONS

PROGRESS TEST-3

GZ-1926, GZK-1909

GZBS-1904-1905

JEE MAIN PATTERN

Test Date: 25-11-2017



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PHYSICS

1. (D)

2. (A)

$$\text{Component of } \vec{A} \text{ along } \vec{B} = \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|} \cdot \hat{B}$$

3. (A)

4. (C)

5. (C)

6. (B)

7. (C)

8. (C)

9. (C)

10. (B)

11. (A)

12. (D)

Let retardation of body is a and air resistance is f

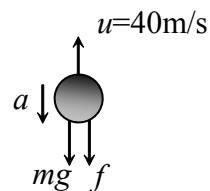
$$v = u + at$$

$$0 = 40 - 3a$$

$$a = \frac{40}{3} \text{ m/s}^2$$

$$ma = mg + f$$

$$f = ma - mg = 1.5 \left(\frac{40}{3} - 10 \right) = 5 \text{ N}$$



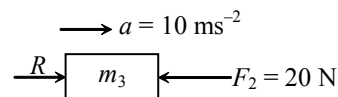
13. (A)

14. (D)

$$a = \frac{F_1 - F_2}{m_1 + m_2 + m_3} = 10 \text{ ms}^{-2}$$

$$R - F_2 = m_3 a$$

$$R = 30 \text{ N}$$



15. (B)

16. (B)

17. (B)

18. (A)

From equilibrium of lower block,

$$T_2 \sin 53^\circ = 60$$

$$\Rightarrow T_2 = 75 \text{ N}$$

$$T_2 \cos 53^\circ = M_2 g$$

$$\Rightarrow 75 \times \frac{3}{5} = m_2 g \Rightarrow m_2 = 4.5 \text{ kg}$$

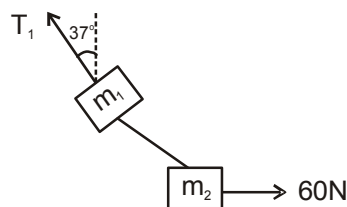
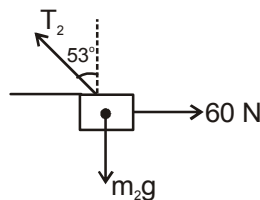
From equilibrium of both the blocks together.

$$T_1 \sin 37^\circ = 60$$

$$\Rightarrow T_1 = 100 \text{ N}$$

$$T_1 \cos 37^\circ = (m_1 + m_2)g$$

$$\Rightarrow 80 = (m_1 + m_2)g \Rightarrow m_1 = 3.5 \text{ kg}$$

19. Average acceleration (A) = $\frac{\text{Change in velocity}}{\text{Time taken}}$ \therefore Change in velocity = Area of acceleration – time graph

$$\therefore \text{Average acceleration} = \frac{\text{Area OABE}}{20 \text{ s}} = \frac{600}{20} = 30 \text{ m/s}^2$$

 \therefore (C)20. Change in velocity $\Delta v = 8 - (-8) = 16 \text{ m/s}$

$$\text{Time taken} \quad \Delta t = \frac{\pi r}{v} = \frac{\pi \times 6}{8} = \frac{3\pi}{4}$$

$$\therefore \text{Average acceleration} = \frac{\Delta v}{\Delta t} = \frac{16 \times 4}{3\pi} = \frac{64}{3\pi}$$

 \therefore (C)21. Velocity = 0 at $t = 5.5 \text{ s}$. $S_{6\text{th}}$

$$= 2 (\text{distance travelled in } (5.5)\text{s} - \text{distance travelled in } 5\text{s}) = 0.5 \text{ m}$$

 \therefore (B)

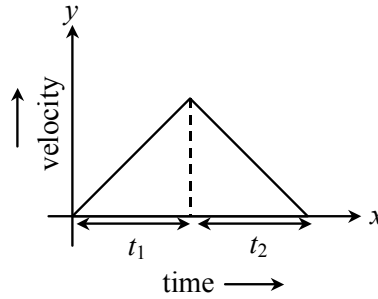
22. $a_1 t_1 = a_2 t_2 \quad \dots (i)$

$\frac{1}{2}(t_1 + t_2)a_1 t_1 = 4 \quad \dots (ii)$

$t_1 + t_2 = 4 \quad \dots (iii)$

$\frac{1}{a_1} + \frac{1}{a_2} = 2$

\therefore (B)

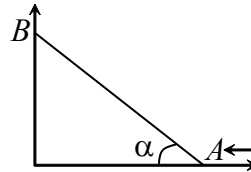


23. (B)

24. $x^2 + y^2 = l^2 \Rightarrow 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \Rightarrow -xv_A + yv_B = 0$

$\therefore v_B = \frac{v_A}{\tan \alpha} = 10\sqrt{3} = 17.3 \text{ m/s}$

\therefore (D)



25. $a = \frac{dv}{dt} = 6t + 5; \quad v = \int_0^t (6t + 5) dt = 3t^2 + 5t; \quad \frac{ds}{dt} = 3t^2 + 5t$

$s = t^3 + \frac{5t^2}{2}; \quad (s)_{t=1s} = 3.5 \text{ m}$

\therefore (B)

26. (A)

27. $8 = 0 + a \left(2 - \frac{1}{2} \right) \quad \dots (1)$

$S_5 = 0 + a \left(5 - \frac{1}{2} \right) \quad \dots (2)$

Dividing equation (2) by (1)

We get, $S_5 = 24 \text{ m}$.

\therefore (B)

28. $x^2 = 1 + t^2$

$$2x \frac{dx}{dt} = 2t \Rightarrow \frac{dx}{dt} = \frac{t}{x} \therefore \frac{d^2x}{dt^2} = \frac{x - t \frac{dx}{dt}}{x^2} = \frac{1}{x} - \frac{t^2}{x^3}$$

\therefore (C)

29. (D)

30. (D)

CHEMISTRY

31. (D)

$$v = K \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \cdot K = (3.29 \times 10^{15} \text{ Hz}) z^2$$

(A) $\frac{1}{2^2} - \frac{1}{5^2} = \frac{21}{100}$

(B) $\frac{1}{2^2} - \frac{1}{3^2} = \frac{5}{36}$

(C) $\frac{1}{2^2} - \frac{1}{4^2} = \frac{3}{16}$

(D) $\frac{1}{1^2} - \frac{1}{3^2} = \frac{8}{9}$ (highest)

32. (D)

$$\frac{1}{\lambda} = R z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

$$\frac{1}{\lambda} = R \times 1^2 \left[\frac{1}{1^2} - \frac{1}{2^2} \right]$$

$$\frac{1}{\lambda} = R \times \frac{3^2}{3^2} \left[\frac{1}{1^2} - \frac{1}{2^2} \right]$$

$$\frac{1}{\lambda} = R 3^2 \left[\frac{1}{3^2} - \frac{1}{6^2} \right]$$

Required Transition $6 \rightarrow 3$ for Li^{++} .

33. (C)

$$\lambda = \frac{h}{mv} \quad \lambda = \frac{h}{\sqrt{2mKE}}$$

$$\lambda_{\text{req}} = \frac{h}{\sqrt{2m9(KE)}} = \frac{1}{3} \frac{h}{\sqrt{2m(KE)}} = \frac{\lambda}{3}$$

34. (B)

$$\Delta x m \Delta V \geq \frac{h}{4\pi}$$

$$\Delta V = \frac{h}{4\pi \times m \times \Delta x} = \frac{6.63 \times 10^{-34}}{4 \times 3.14 \times 1.1 \times 10^{-27} \times 3 \times 10^{-12}} = 1.6 \times 10^4 \text{ m/s}$$

35. (D)

$$\Delta P = \frac{h}{4\pi \Delta x}$$

$\Delta x \longrightarrow 0$ than $\Delta P \longrightarrow \infty$

36. (B)

$$\Delta x = \lambda$$

$$\lambda \Delta P = \frac{h}{4\pi}$$

$$\frac{h}{mv} \times m \Delta V = \frac{h}{4\pi}$$

$$\frac{\Delta V}{V} = \frac{1}{4\pi}$$

$$\% \frac{\Delta V}{V} = \frac{1}{4\pi} \times 100 = \frac{25}{\pi} \cong 8$$

37. (A)

s-orbital is non-directional it does not depend on θ, ϕ angular function $n = 2, \ell = 1, m = 0$ indicate p_z orbital which is directional have angular dependence.

38. (D)

$$-1.6 \times 10^{-19} \text{ and } -4.0 \times 10^{-19}$$

has highest common factor $\therefore .8 \times 10^{-19}$ coulomb

So electronic charge is $-0.8 \times 10^{-19} \text{ C}$

39. (B)

$$KE_1 = hv_1 - hv_0$$

$$KE_2 = hv_2 - hv_0$$

$$\frac{1}{K} = \frac{v_1 - v_0}{v_2 - v_0}$$

$$v_2 - v_0 = kv_1 - kv_0$$

$$(k-1)v_0 = kv_1 - v_2$$

$$v_0 = \frac{kv_1 - v_2}{k-1}$$

40. (B)

$$\frac{hc}{\lambda} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$$

$$\frac{1}{\lambda} = \frac{\lambda_1 + \lambda_2}{\lambda_1 \lambda_2}$$

$$\lambda = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

41. (B)

Let $n_{P_4O_{10}} = x$

$$n_{P_2O_6} = y$$

By POAC on P

$$4 \times \frac{31}{124} = 4x + 4y \quad \dots (i)$$

By POAC on O

$$2 \times 1 = 10 \times x + 6y \quad \dots (ii)$$

On solving (i) & (ii)

$$x = 1/8 \quad y = 1/8$$

$$\therefore \text{Mass of } P_4O_{10} = \frac{1}{8} \times 284 \text{ g} = 35.5 \text{ g}$$

$$\therefore \text{Mass of } P_2O_6 = \frac{1}{8} \times 220 \text{ g} = 27.5 \text{ g}$$

42. (A)

$$\text{Moles of AgCl} = \frac{14.35}{143.5} = 0.1 = \text{Moles of Ag}$$

$$\therefore \text{Mass of Ag} = 0.1 \times 108 \text{ g} = 10.8 \text{ g}$$

$$\% \text{Ag} = \frac{10.8}{21.6} \times 100 = 50\%$$

43. (A)

$$0.5 = \frac{0.4 \times V \times 2 + 50 \times 0.3}{50 + V}$$

$$V = 33.33 \text{ ml}$$

44. (C)

$$120 + 1000 = 1120 \text{ g solution}$$

$$\text{density} = 1.15 \text{ g/ml}$$

$$\text{volume} = \frac{1120}{1.15} = 973.91 \text{ ml}$$

$$M = \frac{2 \times 1000}{973.91} = 2.05 \text{ M}$$

45. (A)

$$\frac{750 \times 0.5 + 250 \times 2}{750 + 250} = 0.875 \text{ M}$$

46. (D)

$$\text{m.mole before addition} = 50 \times 0.06$$

$$\text{m.mole after addition} = 50 \times 0.042$$

$$\text{milimole } \text{CH}_3\text{COOH absorbed} = 50(0.06 - 0.042)$$

$$\text{mass} = \frac{50}{1000}(0.06 - 0.042) \times 60$$

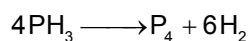
$$\text{mass absorbed per gram} = \frac{50[0.06 - 0.042] \times 60}{1000 \times 3} = 0.018 = 18 \text{ mg}$$

47. (D)

$$\% \text{Br} = \frac{80}{188} \times \frac{W_{\text{AgBr}}}{W_{\text{sample}}} \times 100$$

$$= \frac{80}{188} \times \frac{141}{250} \times 100 = 24$$

48. (C)



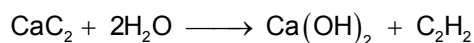
$$100\text{ml} \quad \frac{1}{4} \times 100 \quad \frac{6}{4} \times 100$$

$$25\text{ml} \quad 150\text{ml}$$

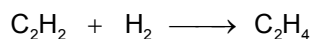
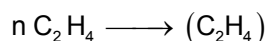
Vol of gases (Product) = 175

Change in Volume = 175 – 100 = + 75 ml.

49. (A)

64 kg 1 K mole

$$\frac{64}{64} \text{K mole}$$

1 K mole 1 K mole

$$1 \text{ K mole} \quad \frac{1}{n} \text{K mole} \times 28n$$

$$\text{mass} = \frac{1}{n} \times 28n \text{ Kg} = 28\text{Kg}$$

50. (A)

$$\text{Molecular wt.} = \frac{4 \times 24 \times 100}{0.096} = 100000$$

51. (D)

$$1 \text{ L H}_2\text{O} = 1000 \text{ g H}_2\text{O}$$

$$\text{No. of mole of H}_2\text{O} = \frac{1000}{18} = 55.55$$

$$\text{No. of H}_2\text{O molecule} = 55.55 N_A$$

52. (B)

$$1 \times n_{\text{CO}_2} = 6 \times n_{\text{K}_4[\text{Fe}(\text{CN})_6]}$$

$$\therefore n_{\text{K}_4[\text{Fe}(\text{CN})_6]} = \frac{1}{6}$$

53. (D)

$$\frac{100 \times 2}{M_{\text{acid}} + 107 \times 2} = \frac{27}{108}$$

$$M_{\text{acid}} + 214 = 800$$

$$M_{\text{acid}} = 586 \text{ g}$$

54. (A)

Let the mixture of O^{17} and O^{18} has at wt = M

$$\frac{90 \times 16 + M \times 10}{100} = 16.12 \quad M = 17.2$$

$$((\%O^{17} \times 17) + (100 - \%O^{17} \times 18)) / 100 = 17.2$$

$$\%O^{17} \text{ is } 80$$

In scale of 10 ans is 8%

55. (B)

$$\lambda = \frac{h}{m_e x} = \frac{h}{m_p V} = \frac{h}{1840 m_e V} [m_p = 1840 m_e]$$

$$\text{Hence, } V = \frac{x}{1840}$$

56. (D)

57. (C)

$$\frac{hc}{\lambda} = 1 + \phi \quad \dots\dots\dots (1)$$

$$3 \times \frac{hc}{\lambda} = 4 + \phi \quad \dots\dots\dots (2)$$

from eq. (1) and (2) $\phi = 0.5 \text{ eV}$

58. (C)

$$\lambda_i = \frac{hc}{5\phi} ; \lambda_e = \frac{h}{\sqrt{2 \times 4\phi m}}$$

$$\frac{\lambda_i}{\lambda_e} = \frac{C}{5} \sqrt{\frac{8m}{\phi}}$$

59. (B)

60. (D)

MATHEMATICS

61. (D)

$$f(x) = \cos^2 \theta + \sec^2 \theta$$

$$= \cos^2 \theta + \frac{1}{\cos^2 \theta}$$

$$\because \cos \theta \neq 0$$

$$f(x) \geq 2$$

62. (C)

We have $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$

$$= (\tan 9^\circ + \tan 81^\circ) - (\tan 27^\circ + \tan 63^\circ)$$

$$= \frac{1}{\sin 9^\circ \cos 9^\circ} - \frac{1}{\sin 27^\circ \cos 27^\circ}$$

$$= \frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ}$$

$$= 2 \left[\frac{\sin 54^\circ - \sin 18^\circ}{\sin 54^\circ \sin 18^\circ} \right]$$

$$= 2 \left[\frac{\cos 36^\circ \sin 18^\circ}{\sin 18^\circ \cos 36^\circ} \right] = 4$$

63. (D)

$$= \frac{2^{\log_2(a^4)} - 3^{\log_3(a^2+1)} - 2a}{7^{\log_7(a^2)} - a - 1} = \frac{a^4 - (a^2 + 1) - 2a}{a^2 - a - 1}$$

$$= \frac{(a^2)^2 - (a+1)^2}{(a^2 - a - 1)} = a^2 + a + 1$$

64. (D)

$$\cos 2A + \cos 2B + \cos 2C$$

$$= 2 \cos(A + B) \cos(A - B) + \cos 2C$$

$$= 2 \cos\left(\frac{3\pi}{2} - C\right) \cos(A - B) + \cos 2C$$

$$= -2 \sin C \cos(A - B) + 1 - 2 \sin^2 C$$

$$\begin{aligned}
 &= 1 - 2 \sin C (\cos(A - B) + \sin C) \\
 &= 1 - 2 \sin C \{ \cos(A - B) + \sin[3\pi/2 - (A + B)] \} \\
 &= 1 - 2 \sin C [\cos(A - B) - \cos(A + B)] \\
 &= 1 - 4 \sin A \sin B \sin C
 \end{aligned}$$

65. (C)

$$\text{We have } \cos^2 A + \cos^2 B - (1 - \cos^2 C) = 0$$

$$\Rightarrow \cos^2 A + \cos^2 B - \sin^2 C = 0$$

$$\Rightarrow \cos^2 A + \cos(B + C) \cos(B - C) = 0$$

$$\Rightarrow 2 \cos A \cos B \cos C = 0$$

$$\Rightarrow \text{Either } A \text{ or } B \text{ or } C \text{ is } 90^\circ$$

66. (B)

$$(x - 2y)\sqrt{2} = (x - 2y) + (x - y - 1)\sqrt{6}$$

$$\Rightarrow (x - 2y)(\sqrt{2} - 1) = (x - y - 1)\sqrt{6}$$

$$\Rightarrow x - 2y = 0 \text{ \& } x - y - 1 = 0 \Rightarrow y = 1, x = 2$$

67. (B)

$$\text{Let ratio be } \lambda : 1 \Rightarrow \frac{6\lambda - 3}{\lambda + 1} = 0, \lambda = \frac{1}{2}$$

68. (C)

$$\sqrt{x^2} = |x| \quad \because x < 0 \Rightarrow |x| = -x$$

$$\Rightarrow \sqrt{\log_{10}(-x)} = \log_{10} \sqrt{(-x)^2}$$

$$\Rightarrow \sqrt{\log_{10}(-x)} = \log_{10}(-x)$$

$$\Rightarrow \log_{10}(-x) = (\log_{10}(-x))^2$$

$$\Rightarrow \log_{10}(-x) (\log_{10}(-x) - 1) = 0$$

$$\Rightarrow \log_{10}(-x) = 0 \text{ or } \log_{10}(-x) = 1$$

$$\therefore (-x) = 1 \text{ or } (-x) = 10 \Rightarrow x = -1 \text{ or } x = -10$$

number of real solution is exactly 2.

69. (A)

Let P is (h, k)

$$\text{then } |h - 2| + |k - 3| = 1$$

$$\Rightarrow |x - 2| + |y - 3| = 1$$

which is a square having centre at (2, 3).

70. (B)

Given that, $\sin 2\theta = k$

$$\begin{aligned} & \frac{\tan^3 \theta}{(1 + \tan^2 \theta)} + \frac{\cot^3 \theta}{(1 + \cot^2 \theta)} \\ &= \frac{\sin^3 \theta}{\cos^3 \theta} \cos^2 \theta + \frac{\cos^3 \theta}{\sin^3 \theta} \sin^2 \theta \\ &= \frac{2(\sin^4 \theta + \cos^4 \theta)}{2(\sin \theta \cos \theta)} \\ &= \frac{2[(\sin^2 \theta + \cos^2 \theta)^2 - 2\sin^2 \theta \cos^2 \theta]}{\sin 2\theta} \\ &= \frac{2\left[1 - \frac{1}{2} \sin^2 2\theta\right]}{\sin 2\theta} = \frac{2 - \sin^2 2\theta}{\sin 2\theta} = \frac{2 - k^2}{k} \end{aligned}$$

71. (B)

Given that,

$$\begin{aligned} f(\theta) &= \sin^2 \theta + \sin^2 \left(\theta + \frac{2\pi}{3}\right) + \sin^2 \left(\theta + \frac{4\pi}{3}\right) \\ &= 1 + \sin^2 \theta - \left[\cos^2 \left(\theta + \frac{2\pi}{3}\right) - \sin^2 \theta \left(\theta + \frac{\pi}{3}\right)\right] \\ &= 1 + \sin^2 \theta - \cos(2\theta + \pi) \cos \frac{\pi}{3} \\ &= 1 + \sin^2 \theta + \frac{\cos 2\theta}{2} \end{aligned}$$

$$= 1 + \sin^2 \theta + \frac{1}{2} - \sin^2 \theta = \frac{3}{2}$$

$$\text{Hence, } f\left(\frac{\pi}{15}\right) = \frac{3}{2}$$

72. (C)

Sum of integers divisible by 2

$$S_2 = \frac{50}{2} [2 + 100] = 25 \times 102 = 2550$$

Sum of integers divisible by 5

$$S_5 = \frac{20}{2} [5 + 100] = 10 \times 105 = 1050$$

Sum of integers divisible by 2 & 5

$$S_{2,5} = \frac{10}{2} [10 + 100] = 5 \times 110 = 550$$

required sum = 2550 + 1050 – 550 = 3050

73. (B)

3. $A_1, A_2, \dots, A_n, 54$ are in AP with common difference d

$$\& \frac{A_8}{A_{n-2}} = \frac{3}{5}$$

$$d = \frac{54 - 3}{n + 1} = \frac{51}{n + 1}$$

$$A_8 = 3 + 8d \ \& \ A_{n-2} = 3 + (n - 2)d$$

$$\frac{A_8}{A_{n-2}} = \frac{3 + 8d}{3 + (n - 2)d} = \frac{3}{5}$$

$$\Rightarrow \frac{3 + 8\left(\frac{51}{n+1}\right)}{3 + (n-2)\left(\frac{51}{n+1}\right)} = \frac{3}{5}$$

$$\Rightarrow \frac{n+1+136}{n+1+(n-2)17} = \frac{3}{5}$$

$$\Rightarrow 5(n + 137) = 3(18n - 33) \Rightarrow 784 = 49n$$

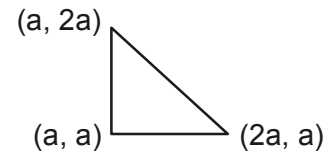
$$\Rightarrow n = \frac{784}{49} = \frac{112}{7} \Rightarrow n = 16$$

74. (D)

$$\frac{1}{2}a^2 = 72$$

$$a = \pm 12$$

$$\text{Centroid} = (16, 16) \text{ or } (-16, -16)$$



75. (B)

$$A = \frac{a+b+c}{3}, G = (abc)^{1/3}, H = \frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}$$

$$H = \frac{3(abc)}{ab+bc+ca}$$

$$x^3 - (a+b+c)x^2 + (ab+bc+ca)x - abc = 0$$

$$\Rightarrow x^3 - 3Ax^2 + (3G^3/H)x - G^3 = 0$$

76. (B)

$$\frac{5-1}{8-2} = \frac{7-5}{x-8} \Rightarrow x = 11$$

77. (C)

$$a^x = b^y = c^z = d^t = k \text{ (let)}$$

$$x \log a = \log k, \quad y \log b = \log k$$

$$z \log c = \log k, \quad t \log d = \log k$$

$$\therefore a, b, c, d \text{ in G.P.}$$

$$\Rightarrow \log a, \log b, \log c, \log d \text{ in A.P.}$$

$$\Rightarrow \frac{\log k}{x}, \frac{\log k}{y}, \frac{\log k}{z}, \frac{\log k}{t} \text{ in A.P.}$$

$$\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z}, \frac{1}{t} \text{ in A.P.} \Rightarrow x, y, z, t \text{ in H.P.}$$

78. (B)

$$\log(-2x) = 2 \log(x+1)$$

$$-2x > 0 \quad \Rightarrow \quad x < 0 \quad \text{.....(i)}$$

$$x+1 > 0 \quad \Rightarrow \quad x > -1 \quad \text{.....(ii)}$$

$$\text{from (i) \& (ii), we get } x \in (-1, 0)$$

$$\therefore -2x = (x+1)^2 \Rightarrow x^2 + 4x + 1 = 0 \Rightarrow x = \frac{-4 \pm 2\sqrt{3}}{2}$$

$$\text{so only one solution lies in } (-1, 0)$$

79. (C) $\log_{\frac{1}{3}}(x^2 + x + 1) > -1$

$$\Rightarrow x^2 + x + 1 < 3$$

$$\Rightarrow x^2 + x - 2 < 0$$

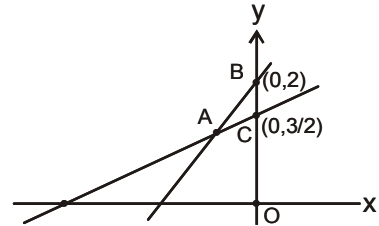
$$\Rightarrow (x + 2)(x - 1) < 0$$

$$\Rightarrow x \in (-2, 1)$$

80. (C)

Co-ordinate of point A $\left(-\frac{1}{7}, \frac{10}{7}\right)$

$$\text{Ar}(ABC) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{7} = \frac{1}{28}$$



81. (C)

Given that $T_m = AR^{m-1} = \frac{1}{n^2}$ and $T_n = AR^{n-1} = \frac{1}{m^2}$

$$\Rightarrow A^2 R^{m+n-2} = \frac{1}{m^2 n^2}$$

$$\Rightarrow AR^{\frac{m+n}{2}-1} = \frac{1}{mn}$$

$$\Rightarrow T_{\frac{m+n}{2}} = \frac{1}{mn}$$

82. (D)

Let d be the common difference

since $a_7 = 9$

$$\therefore a_1 + 6d = 9$$

Let $D = a_1 a_2 a_7$

$$D = (9 - 6d)(9 - 5d)(9)$$

$$= 270 \left[\left(d - \frac{33}{20} \right)^2 - \frac{9}{400} \right]$$

For least value of d .

$$d - \frac{33}{20} = 0 \Rightarrow d = \frac{33}{20}$$

83. (B)

$$A.M \geq G.M$$

$$\frac{a+b+c}{3} \geq (abc)^{\frac{1}{3}}; \text{ for } (a, b, c > 0)$$

$$\Rightarrow a+b+c \geq 3(abc)^{\frac{1}{3}}$$

but given $ab^2c^3, a^2b^3c^4, a^3b^4c^5$ are in A.P

$$\text{Hence } 2abc = 1 + a^2b^2c^2 \Rightarrow (abc - 1)^2 = 0 \Rightarrow abc = 1$$

hence minimum value of

$$a+b+c = 3(abc)^{\frac{1}{3}} = 3.(1)^{\frac{1}{3}} = 3$$

84. (A)

The given expression is

$$1 + 2\sin 3x \sin 2x + \frac{1 - \cos 4x}{2} + \frac{1 - \cos 6x}{2}$$

$$\Rightarrow 1 + 2\sin 3x \sin 2x + \sin^2 2x + \sin^2 3x$$

$$\Rightarrow 1 + (\sin 2x + \sin 3x)^2$$

Thus least value is 1

85. (A)

$$\cos^2 x = \sin x \tan x \Rightarrow \cos^3 x = \sin^2 x$$

$$\Rightarrow \cot^3 x = \cos \operatorname{csc} x \Rightarrow \cot^6 x = \cos^2 \operatorname{csc}^2 x \Rightarrow \cot^6 x - \cot^2 x = 1$$

86. (A)

$$-5 \leq 3 \sin x - 4 \cos x \leq 5$$

$$\log_{20} 10 \leq \log_{20} (3 \sin x - 4 \cos x + 15) \leq \log_{20} 20$$

87. (C)

$$a^{12} r^{66} = 8^{2014} = 2^{6042}$$

$$\therefore a^2 r^{11} = 2^{1007}$$

$$\text{Let } a = 2^\alpha, r = 2^\beta$$

$$\text{then } 2\alpha + 11\beta = 1007$$

$$\Rightarrow (\alpha, \beta) = (498, 1), (487, 3), (476, 5), \dots, (3, 91) \text{ i.e. 46 pairs.}$$

88. (B)

By ratio and proportion

$$\frac{a}{b} = \frac{c}{d} = \frac{xa + yc}{xb + yd}$$

$$\frac{\sin(\alpha + \beta + \gamma)(\sin \alpha + \sin \beta + \sin \gamma) + \cos(\alpha + \beta + \gamma)(\cos \alpha + \cos \beta + \cos \gamma)}{\sin^2(\alpha + \beta + \gamma) + \cos^2(\alpha + \beta + \gamma)}$$

$$\frac{\cos(\alpha + \beta) + \cos(\beta + \gamma) + \cos(\gamma + \alpha)}{1} = 2$$

89. (D)

$$|x^2 - 9| + |x^2 - 4| = 5$$

$$|x^2 - 9| + |x^2 - 4| = |(x^2 - 9) - (x^2 - 4)|$$

$$\Rightarrow (x^2 - 9)(x^2 - 4) \leq 0 \{ \because |a| + |b| = |a - b| \Leftrightarrow a \cdot b \leq 0 \}$$

$$\Rightarrow x \in [-3, -2] \cup [2, 3]$$

90. (A)

$$(\tan \theta + \cot \theta)(\tan^2 \theta + \cot^2 \theta - 1) = 52$$

$$\text{Let } \tan \theta + \cot \theta = t$$

$$t^3 - 3t - 52 = 0 \Rightarrow t = 4$$

$$\tan^2 \theta + \cot^2 \theta = (\tan \theta + \cot \theta)^2 - 2 = 14$$