SOLUTIONS

MEAITS 2018

UNIT TEST-3

(JEE MAIN PATTERN)

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PHYSICS

1. (A)

$x^2+y^2=a^2, \quad (x-4a)^2+(y-4a)^2=4a^2$

$\Rightarrow (a\cos\omega t, a\sin\omega t) \Rightarrow (4a+2a\cos\omega t, 4a+2a\sin\omega t)$

Centre of mass $\equiv (h,k)$

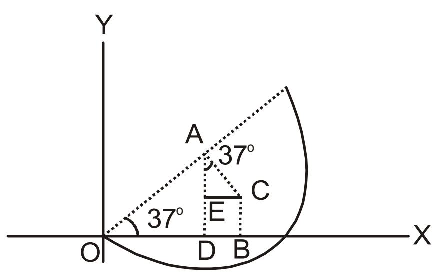
$h=\frac{3a\cos\omega t}{2}+2a \qquad k=\frac{3a\sin\omega t}{2}+2a$

$(h-2a)^2+(k-2a)^2=\frac{9}{4}a^2$

$\Rightarrow \text{rad}=\frac{3}{2}a$

2. (C)

OA = R



$AC=2R/\pi$

$OD=OA\cos 37^\circ$

$=R.\frac{4}{5}$

$DB=CE=AC\sin 37^\circ$

$=\frac{2R}{\pi}.\frac{3}{5}$

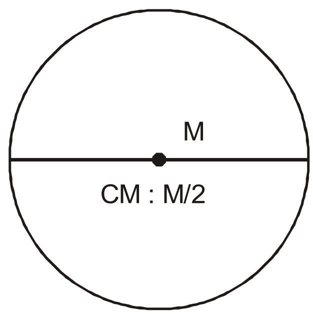
$BC=Y=AD-AE=R\sin 37^\circ-\frac{2R}{\pi}\cos 37^\circ$

$$=\left(\frac{3\pi-8}{5\pi}\right)R$$

$$x = OD + DB = R.\frac{4}{5}+\frac{6R}{5\pi}$$

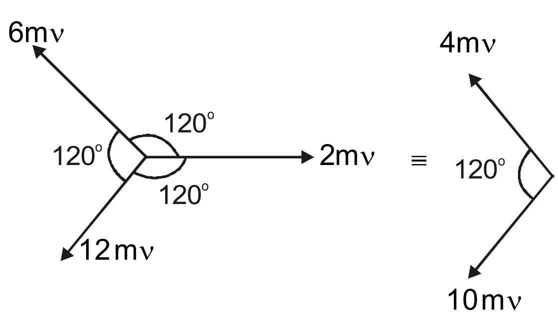
$$=\left(\frac{4\pi+6}{5\pi}\right)R$$

3. (A)



$$y=\frac{M\times 0+\frac{M}{2}\times\frac{3R}{8}}{M+M/2}=\frac{R}{8}$$

4. (D)

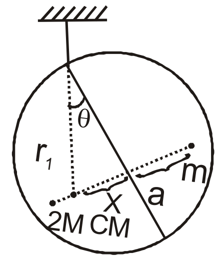


$$\nu_{cm}=\frac{m\nu\sqrt{10^2+4^2+2.10.4\cos 120^\circ}}{9m}$$

$$=\nu\sqrt{\frac{76}{81}}$$

5. (C)

$a = 4R/3\pi$



$r_1 = \frac{M \times 2a}{M + 2M} = \frac{8R}{9\pi}$

$x = a - r_1 = \frac{4R}{9\pi}$

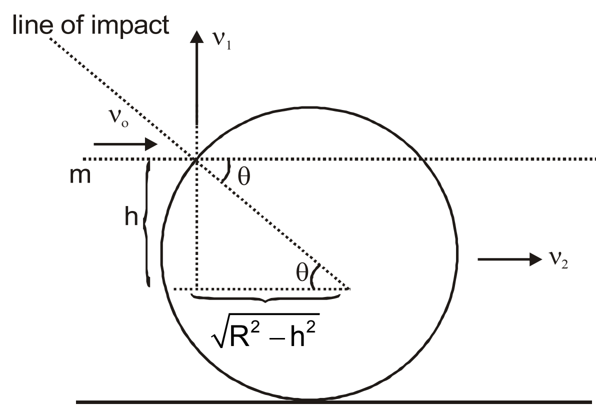
$\tan\theta = \frac{x}{R} = \frac{4}{9\pi} \Rightarrow \theta = \tan^{-1}\left(\frac{4}{9\pi}\right)$

6. (C)

$\nu_1 =$ vel of ball after collision

$\nu_2 =$ vel of sphere after collision

$\nu_o =$ vel of ball before collision.



Along the line of impact :

$\nu_2 \cos\theta + \nu_1 \sin\theta = e.\nu_o \cos\theta$ …. (i)

(e = coefficient of restitution)

Horizontally :

$m\nu_o = 2m \times \nu_2 \Rightarrow \nu_2 = \frac{\nu_o}{2}$ ….(ii)

Perpendicular to the line of impact :

$\nu_o \sin\theta = \nu_1 \cos\theta \Rightarrow \nu_1 = \nu_o \tan\theta$ …(iii)

From (i), (ii) and (iii)

$e = \frac{1}{2} + \tan^2\theta = \frac{1}{2} + \frac{h^2}{R^2 - h^2}$

$= \frac{R^2 + h^2}{2(R^2 - h^2)}$

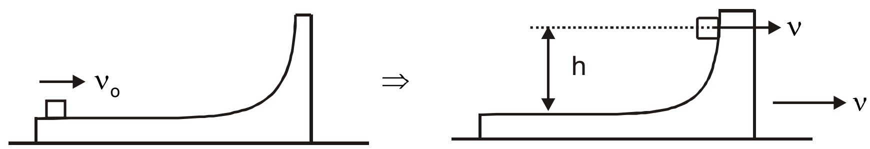
7. (B)

Reading $= 1\text{kg} + 0.5 \times 6\text{kg} + \frac{(0.5 \times dt \times \nu - 0)}{dt}$

$= 1\text{kg} + 3\text{kg} + 0.5 \times \frac{\nu}{10}$

$= 1 + 3 + 0.1 = 4.1\,\text{kg}$

8. (C)



Appling momentum conservation horizontally:

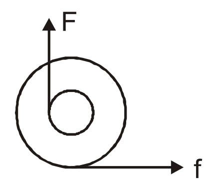
$$m\nu + 2m\nu = m\nu_o \Rightarrow \nu = \frac{\nu_o}{3}$$

$$\frac{1}{2}m{\nu_o}^2 = \frac{1}{2}m\nu^2 + \frac{1}{2}.2m.\nu^2 + mgh$$

$$\Rightarrow h = \frac{{\nu_o}^2 - 3\nu^2}{2g} = \frac{{\nu_o}^2}{3g}$$

9. (B)

For pure rolling :



$$a_{cm} = R\alpha$$

$$\Rightarrow \frac{f}{M} = R.\frac{Fr - fR}{I} \Rightarrow F = f.\left(\frac{I + MR^2}{MrR}\right)$$

10. (D)

$$Mg\left(R - \frac{R}{2}\right) = \frac{1}{2}M\nu^2 + \frac{1}{2}\left(\frac{2}{5}Mr^2\right)\left(\frac{\nu}{r}\right)^2$$

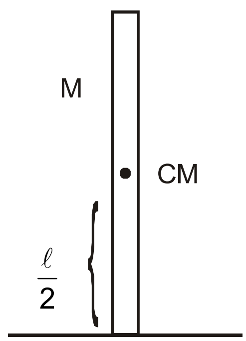
$$\Rightarrow M\nu^2 = \frac{5}{7}.MgR \qquad (\nu = \text{vet at lowest points})$$

$$\frac{1}{2}M\nu^2 = Mg(R - R\cos\theta)$$

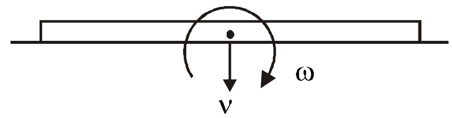
$$\Rightarrow \frac{1}{2} \times \frac{5}{7}MgR = Mg(R - R\cos\theta)$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{9}{14}\right)$$

11. (C)



$\Rightarrow$

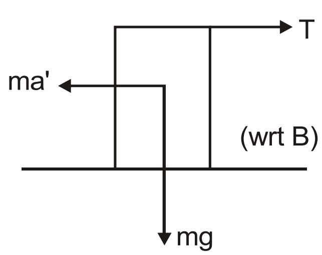


$$\frac{\nu}{\ell/2} = \omega \Rightarrow \nu = \frac{\ell}{2}\omega \qquad \ldots(1)$$

$$\frac{1}{2}m\nu^2 + \frac{1}{2}\left(\frac{m\ell^2}{12}\right)\omega^2 = mg.\frac{\ell}{2} \qquad \ldots(2)$$

From (1) and (2) : $\omega = \sqrt{\frac{3g}{\ell}}$

12. (A)



$$a' = \text{acc of all blocks} = \frac{M'g}{M'+3M} \qquad \ldots.\ (1)$$

$\therefore\ T = 3M \times a' =$ tenstion in the string …. (2)

For the block A, w.r.t. B:

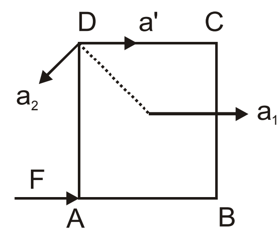
Block A will just topple, if :

$$T.2a = Ma'a + Mg.\frac{a}{2} \qquad \ldots(3)$$

Solving (1), (2), (3), M’ = M

13. (A)

$$a_1 = \frac{F}{M}$$



$\ell =$ side length

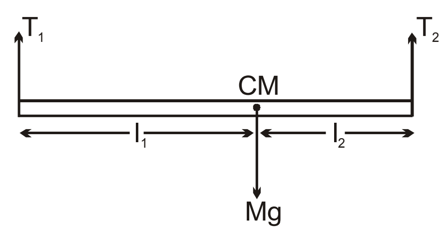
$$\alpha = \frac{F.\ell/2}{M\ell^2/6} = \frac{3F}{M\ell}$$

$$a_2 = R\alpha = \frac{\ell}{\sqrt{2}}.\alpha = \frac{3}{\sqrt{2}}.\frac{F}{M}$$

$$\therefore a = \sqrt{a_1^{\,2} + a_2^{\,2} + 2a_1a_2\cos 135^\circ} = \frac{F}{M}\sqrt{\frac{5}{2}}$$

14. (B)

$$\ell_1 = \frac{\int_0^\ell x\,dm}{\int_0^\ell dm} = \frac{\int_0^\ell x(\ell + x)\,dx\lambda_0}{\int_0^\ell \lambda_0(\ell + x)\,dx}$$



$$= \frac{5}{9}\ell \quad \therefore \ell_2 = \ell - \ell_1 = \frac{4}{9}\ell$$

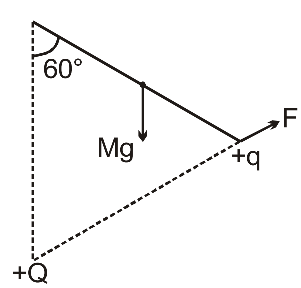
Balancing torque about CM:

$$T_1 \cdot \ell_1 = T_2 \cdot \ell_2 \Rightarrow \frac{T_1}{T_2} = 4/5$$

15. (A)

Balancing torque about hinge:

$$F.\cos 30^\circ\ \ell = Mg.\ \frac{\ell}{2}\sin 60^\circ$$

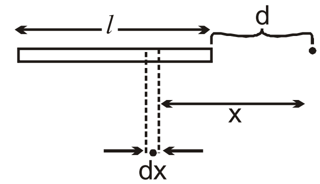


Where $F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qq}{\ell^2}$

$$\Rightarrow Q = \frac{2\pi\varepsilon_0 Mg\ell^2}{q}$$

16. (D)

Electric field at a distance d:



$$E = \int_{d}^{d+\ell} \frac{K\left(\frac{Q}{\ell}\right)dx}{x^2}$$

$$E = \frac{KQ}{d(d+\ell)}$$

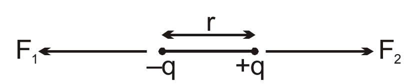
Let p = q.r

r << d and r << l

Net force on the dipole

$F = F_1 - F_2$

$$= \frac{kQq}{d(d+\ell)} - \frac{kQq}{(d+r)(d+r+\ell)}$$



$$= \frac{kQp(2d+\ell)}{d^2(d+\ell)^2}$$

Using $\left(1+\frac{r}{d}\right)^{-1} \simeq 1-\frac{r}{d}$

$$\left(1+\frac{r}{d+\ell}\right)^{-1} \simeq 1-\frac{r}{d+\ell}$$

17. (C)

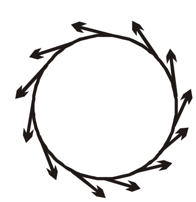
T = F = force on half part

$$= \lambda \cdot \frac{\ell}{2} \cdot E = \lambda \cdot \frac{\ell}{2} \cdot \frac{\sigma}{2\varepsilon_0}$$

$$= \frac{\sigma\ell\lambda}{4\varepsilon_0}$$

18. (A)

$$|\vec{E}| = \frac{E_0}{R}\sqrt{y^2+x^2} = \frac{E_0}{R}.R = E_0$$



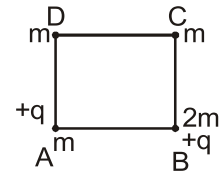
Angular acc of ring

$$\alpha = \frac{\tau}{I} = \frac{QE_0R}{MR^2} = \frac{QE_0}{MR}$$

$$\omega = \omega_0 + \alpha t$$

$$= 0 + \alpha t = \frac{QE_0 t}{MR}$$

19. (B)

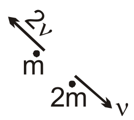


$$U_i = \frac{kq^2}{\ell} \times 4 + \frac{kq^2}{\sqrt{2}\ell} \times 2$$

$$U_f = \frac{kq^2}{\sqrt{2}\,\ell}$$

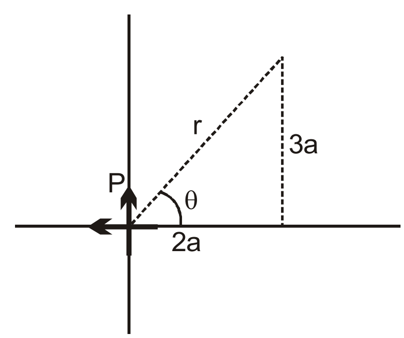
$$\Delta U = K$$

$$\Rightarrow U_i - U_f = \frac{1}{2}2m \cdot \nu^2 + \frac{1}{2}m \cdot (2\nu)^2$$



$$\Rightarrow \nu = \sqrt{\frac{q^2(4\sqrt{2}+1)}{12\sqrt{2}\,m\ell\pi\varepsilon_0}}$$

20. (A)



$$r = \sqrt{3^2a^2 + 2^2a^2}$$

$$= a\sqrt{13}$$

$$\cos\theta = \frac{2a}{a\sqrt{13}} = \frac{2}{\sqrt{13}}$$

$$\sin\theta = \frac{3}{\sqrt{13}}$$

$$v = v_1 + v_2$$

$$= \frac{kp\cos(90-\theta)}{r^2} + \frac{kp\cos(180^\circ - \theta)}{r^2}$$

$$= \frac{kp}{r^2}(\sin\theta - \cos\theta)$$

$$= \frac{kp}{13a^2}\left(\frac{3}{\sqrt{13}} - \frac{2}{\sqrt{13}}\right) = \frac{kp}{13\sqrt{13}a^2}$$

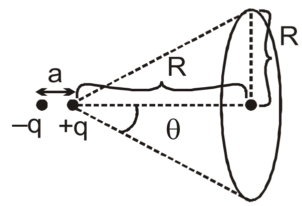
$$= \frac{p}{52\sqrt{13}\pi\varepsilon_0 a^2}$$

21. (D)

Let p = qa

a << R

Flux due to +q is



$$\phi_1 = \frac{q/\varepsilon_0}{4\pi}\cdot 2\pi(1-\cos\theta) = \frac{q}{2\varepsilon_0}\left(1-\frac{x}{\sqrt{R^2+x^2}}\right)$$

$$= \frac{q}{2\varepsilon_0}\left(1-\frac{R}{\sqrt{R^2+R^2}}\right)$$

Flux due to –q is: $\phi_2 = -\frac{q}{2\varepsilon_0}\left(1-\frac{R+a}{\sqrt{R^2+(R+a)^2}}\right)$

So the net flux is:

$$\phi = \phi_1 + \phi_2$$

$$= \frac{q}{2\varepsilon_0}\left[1-\frac{R}{\sqrt{R^2+R^2}}-1+\frac{R+a}{\sqrt{R^2+(R+a)^2}}\right]$$

$$= \frac{a}{2\varepsilon_0}\left[-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\left(1+\frac{a}{R}\right)^{1/2}\right]$$

$$= \frac{q}{2\varepsilon_0}\left[-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{2\sqrt{2}}\cdot\frac{a}{R}\right] \because (a << R)$$

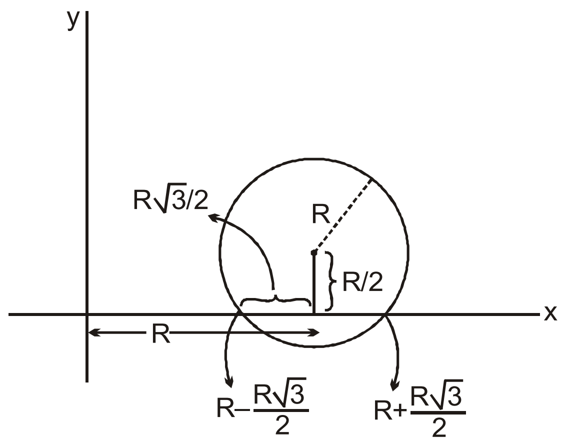
$$= \frac{q}{2\varepsilon_0}\cdot\frac{a}{2\sqrt{2}R} = \frac{p}{4\sqrt{2}\varepsilon_0 R}$$

22. (A)

Charge enclosed within the surface

$$= \int \lambda_0 x\,dx$$

$$= \lambda_0\left[\frac{x^2}{2}\right]_{R-R\sqrt{3}/2}^{R+\frac{R\sqrt{3}}{2}}$$



$= \sqrt{3}\lambda_0 R^2$

$\therefore \phi = \dfrac{\sqrt{3}\lambda_0 R^2}{\varepsilon_0}$

23. (C)

Net enclosed charge = q + q + $4 \times \dfrac{q}{2} = 4q$

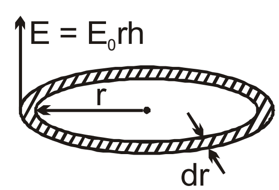
$\therefore$ net flux $= \dfrac{4q}{\varepsilon_0}$

24. (A)

Flux in the bottom surface = 0 (z = 0)

Flux in the cylindrical surface = 0

Flux in the upper circular surface



$= \int_0^R 2\pi r dr \cdot E_0 r h \quad \left(r = \sqrt{x^2 + y^2}, z = h\right)$

$= E_0 \cdot 2\pi h \cdot \dfrac{R^3}{3} = \dfrac{q_{in}}{\varepsilon_0}$

$\Rightarrow q_{in} = \varepsilon_0 \cdot E_0 \cdot 2\pi h \cdot R^3 / 3$

25. (D)

Let $\sigma = kr\, C/m^2$

$$\int_0^R 2\pi r dr\, kr = Q \qquad \Rightarrow k = \frac{3Q}{2\pi R^3}$$

$$q_{in} = \int_0^{R/2} 2\pi r dr \cdot kr = \frac{Q}{8}$$

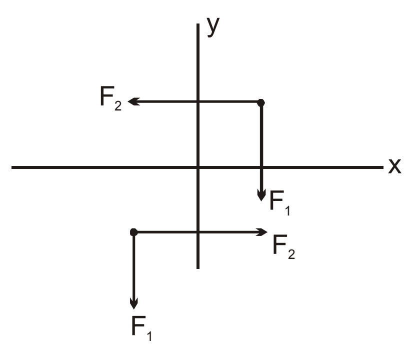
$$\therefore \phi = \frac{Q}{8\varepsilon_0}$$

26. (C)

$$\int_{v_2}^{v_1} dV = V_1 - V_2 = -\int_{2a}^{-a} E_0 x dx = -E_0\left(\frac{x^2}{2}\right)_{2a}^{-a}$$

$$= \frac{3}{2}E_0 a^2$$

27. (A)



$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} = -V_0\hat{i} - 2V_0 y\hat{j}$$

Torque will be only due to $-V_0\ \hat{i}$.

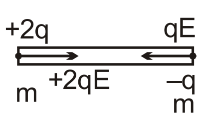
Torque = $V_0\, q \times 2a \cos 45°$

$= V_0\, q\ \sqrt{2}\, a$

28. (C)

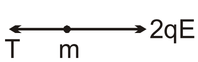
$$a = \frac{2qE - qE}{2m}$$

$= qE/2m$



$2qE - T = ma$

$\Rightarrow T = 2qE - ma$



$= 2qE - m \times \dfrac{qE}{2m} = \dfrac{3}{2}qE$

29. (C)

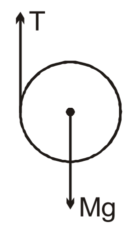
$$\tau = QE \cdot \frac{4R}{3\pi} \times 2$$

$$I = \frac{1}{4}MR^2$$

$$\therefore \alpha = \frac{\tau}{I} = \frac{QE\ 8R/3\pi}{MR^2/4} = \frac{32}{3} \cdot \frac{QE}{\pi MR}$$

30. (A)

$$\frac{Mg - T}{M} = a_{CM}$$



$$\alpha = \frac{TR}{\frac{1}{2}MR^2} = \frac{2T}{MR}$$

$$a_{CM} = R\alpha \Rightarrow \frac{Mg - T}{M} = R \times \frac{2T}{MR} \Rightarrow T = \frac{Mg}{3}$$

CHEMISTRY

31. (B)

$$\underset{\substack{T=300K \\ \therefore n=\frac{10}{300R}}}{(1\,lit,\ 10\,atm)} \xrightarrow{P_{ext}=1atm} \underset{T=\frac{Pv}{nR}=600K}{(4\,lit, 50\,atm)}$$

Now,

$$\Delta H = \Delta U + P_2V_2 - P_1V_1 = q + w + P_2v_2 - P_1v_1$$

$$= q - P_{ext}(v_2 - v_1) + P_2v_2 - P_1v_1$$

$$= 50 \times 300 - 1 \times 3 \times 100 + (20 - 10) \times 100 = 15700\ J = 15.7\ KJ$$

32. (C)

$$\frac{W}{Q_1} = \frac{500 - 300}{500}$$

$$W = \frac{200 \times 1000}{500} = 400\,KJ$$

$$\frac{400}{100} = \frac{1000 - Q_2}{1000}$$

$Q_2 = 600$

33. (B)

$$T \alpha \frac{1}{\sqrt{v}} \Rightarrow Tv^{\frac{1}{2}} = \text{constant}$$

and for adiabatic process

$Tv^{\gamma - 1} = \text{constant}$

Hence

$$\gamma - 1 = \frac{1}{2} \Rightarrow \gamma = 3/2$$

34. (B)

Entropy is state function so ΔS is same along both $A \longrightarrow B$ and $A \longrightarrow C \longrightarrow B$.

35. (D)

$2C_{graphite} + O_2(g) \longrightarrow 2CO(g) : \Delta_r H^\circ = -110.5 \times 2 KJmol^{-1}$

$CO_2(g) \longrightarrow C_{graphite} + O_2(g) : \Delta_r H^\circ = 393.5 \text{ KJ mol}^{-1}$

$C_{diamond} \longrightarrow C_{graphite} : \Delta_r H^\circ = -2 \text{ KJ mol}^{-1}$

Adding : $CO_2(g) + C_{diamond} \longrightarrow 2CO(g) : \Delta_r H^\circ = 170.5 \text{ KJ mol}^{-1}$

36. (D)

Peroxide effect is not noticed in case of $CH_3–CH_2–CH=CH_2$+HCl due to high dissociation energy of

HCl its one of the chain propagating step in which dissociation of HCl takes place is not the thermodynamically favourable. (ΔH=+ve)

37. (A)

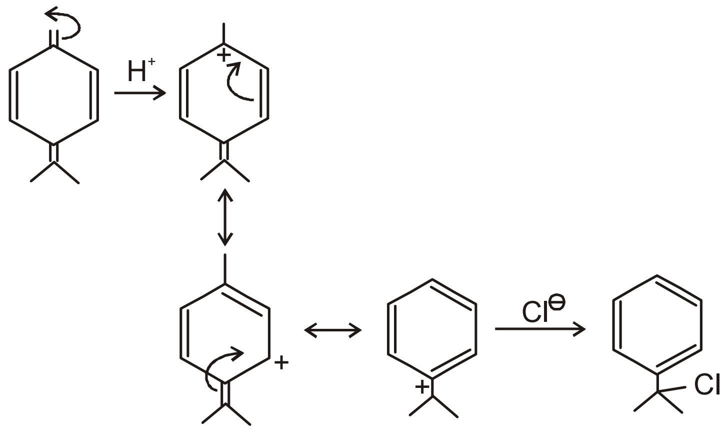
Due to common ion effect dissociation of HF is suppressed as a result F^- ion is not generated.

$HF \rightleftharpoons \boxed{H^\oplus} + F^\ominus$ $HCl = \boxed{H^\oplus} + Cl^\ominus$, H^+ ion is commonion so $Cl^\ominus$, only attack on carbocation formed after electrophilic attack.

$$CH_3 - CH = CH_2 \xrightarrow[Rds]{H^\oplus} H_3C–\overset{\oplus}{C}H–CH_3 \xrightarrow{Cl^\ominus} CH_3 - \underset{\displaystyle Cl}{\underset{|}{C}H} - CH_3$$

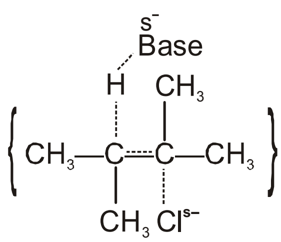
2-Chloropropane

38. (B)



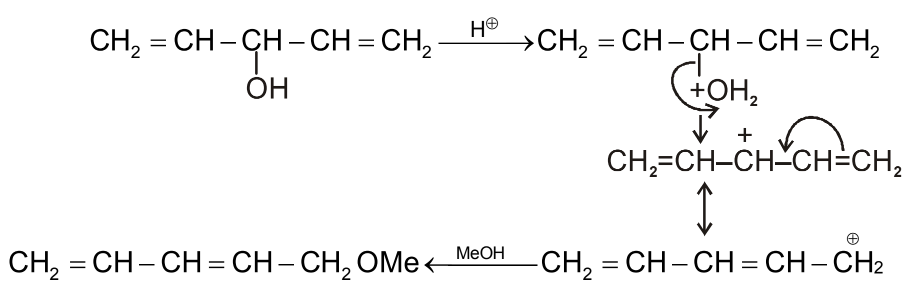
39. (C)

Rate of E_2 reaction increases with increase of α, β alkyl branching because during formation of Transition state of E_2 reaction partial double bond develops in between α and β carbon which is established due to more no. of α c–H bonds of alkyl group and can be formed in low activation energy state.



Transition state of E_2

40. (B)



41. (C)

$Ni^{2+} \rightarrow 3d^8$; Cl^- is WLF.

$\therefore$ n = 0

42. (D)

$Cl \rightarrow$ is weak field ligand CN^{-} is strong field ligand. So, in $[NiCl_4]^{2-}$ is sp^3 & $[Ni(CN)_4]^{2-}$ is dsp^2

43. (B)

$$EF6E = -(0.4 \times 6\Delta_0 + 3p)$$

$$= -\frac{12.5\Delta_0}{5} + 3p$$

44. (C)

For $Fe^{2+}\left[O_2 \rightarrow \text{will be} \xrightarrow{\text{super oxo}} (O_2^{-1})\right]$

45. (A)

$$Cl^{-} < F^{-} < NCS^{-} < NH_3 < CN^{-}$$

46. (A)

$$H_3C-CH=CH-CH_3 \xrightarrow{\dot{B}r} \dot{C}H_2-CH=CH-CH_3 \longleftrightarrow CH_2=CH-\dot{C}H-CH_3$$

$$\dot{C}H_2-CH=CH-CH_3 \xrightarrow{\dot{B}r} \underset{Br}{\underset{|}{C}}H_2-CH=CH-CH_3$$

$\therefore$ No. of G.I. = 2

$$CH_2=CH-\dot{C}H-CH_3 \xrightarrow{\dot{B}r} CH_2=CH-\underset{Br}{\underset{|}{C^*}}H-CH_3$$

C^* = 1 No. of isomers = 2^1 = 2

C^* represents chiral centre.

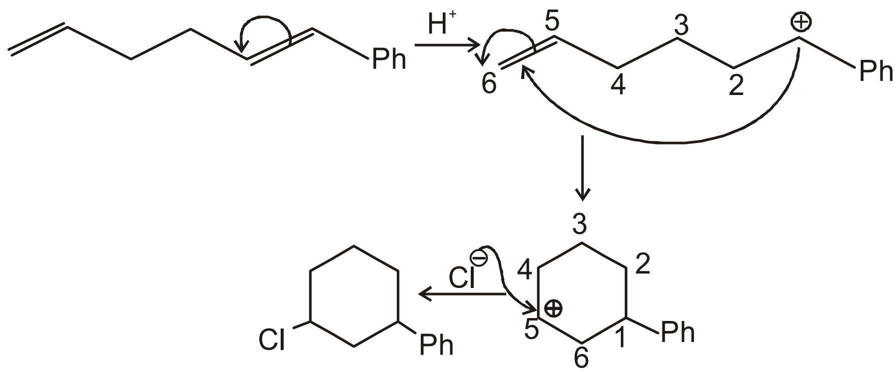
Chiral centre is created so two enantiomer is formed.

Total possible products = 4 (including stereoisomers)

47. (A)

Anti-Markonikoff's product is formed. In case of $F_3C-CH=CH_2$ due to attachment of CF_3 electron withdrawing group CF_3 with $CH=CH_2$

48. (C)



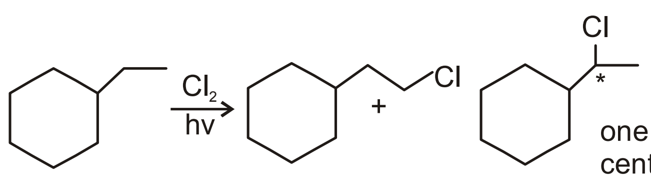
49. (C)

$$CH_3-CH_2-\underset{}{\overset{CH_3}{\overset{|}{C}}}=CH-CH_2-CH_3 \quad CH_3-CH=\overset{CH_3}{\overset{|}{C}}-CH_2-CH_2-CH_3$$

Cis & Trans + Cis & Trans

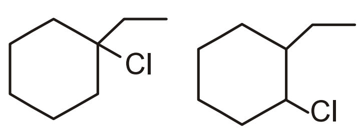
$$+\ CH_3-CH_2-\overset{CH_2}{\overset{||}{C}}-CH_2-CH_2-CH_3$$

50. (A)

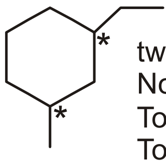


(two isomer)

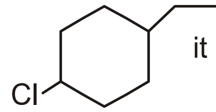
one chiral
centre is created
$2^1=2$ (Isomers)



two chiral centres
C*=2
No Pos total stereo isomers
$2^2=4$



two chiral centres
No Pos
Total stereo isomers=2^2=4
Total monochloro Products = 14



it shows two GI

Including Stereoisomers = 14

51. (B)

$Hg(\ell) \rightleftharpoons Hg(g)$

$\Delta_r S^\circ = 174.4 - 77.4 = 97\ JK^{-1}\ mol^{-1}$

$\because \Delta G^\circ = \Delta H^\circ - T\Delta S^\circ = 0$

$\therefore T = \frac{\Delta H^\circ}{\Delta S^\circ} = \frac{60.8 \times 1000}{97} = 626.8\ K$

52. (B)

For 1 mole combustion of benzene

$\Delta n_g = -1.5$; $\Delta H = \Delta U + \Delta n_g RT$

or, –3271 = $\Delta U - \frac{1.5 \times 8.314 \times 300}{1000}$

$\therefore \Delta U = -3267.25$ KJ

For 1.5 mole combustion of benzene

$\Delta U = -3267.25 \times 1.5 = -4900.88$ KJ

53. (D)

For H_2A and strong base, $\Delta H = 13$ K Cal

enthalpy of dissociation (ionisation) of H_2A = 0.5 K Cal/eq.= 0.5 × 2 = 0.1 K Cal/mole

For $B(OH)_2$ and strong acid, $\Delta H = 10$ KCal

Enthalpy of ionisation of $B(OH)_2$ = 3.5 K Cal/eq = 3.5 × 2 = 7 K Cal/mole

Now,

$\underbrace{B(OH)_2}_{1mole} + \underbrace{H_2A}_{1mole} \longrightarrow BA + 2H_2O : \Delta H = -(13.5 \times 2 - (1 + 7)) : \Delta H = -(27 - 8) = -19$ K Cal

54. (A)

W = – 15 (3 – 6)

= 45 atm L = ΔU

$$\gamma = \frac{Cp_m}{Cv_m} = \frac{(20.91 + 8.314)}{20.91} = 1.4$$

$$\Delta H = \gamma \Delta U$$
$$= 1.4 \times 45 = 63\,L\,atm$$
$$= 6.4 KJ$$

55. (B)

Adiabatic free expansion of ideal gas is an isothermal process.

56. (C)

$$\underset{(O.A.)}{Ce(IV)} \rightarrow Ce(III)$$

Work $\rightarrow$ O.A

$$\underset{(R.A.)}{Eu(II)} \rightarrow Eu(III)$$

work as reducing agent

57. (B)

$Ti^{3+} \rightarrow$ Have one d-electron which is responsible for its colour.

58. (C)

$$2MnO_2 + 4KOH + O_2 \rightarrow 2K_2MnO_4 + 2H_2O$$

$$3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$$

59. (C)

$$\Delta H_{hyd} \propto \frac{1}{\text{Size of ion}}$$

$$Mn^{2+} > Zn^{2+} > Ca^{2+}$$

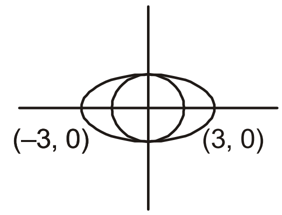
60. (D)

$$Cr_2O_7 + H_2O_2 + H^+ \longrightarrow \underset{(Blue)}{CrO_5}$$

MATHEMATICS

61. (D)

for two ellipse to intersect in 4 points. a must be greater than 1



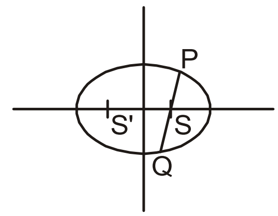
$\Rightarrow$ $r^2 - 11r + 29 > 1$

$\Rightarrow$ $(r - 4)(r - 7) > 0$

$\Rightarrow$ $r \in (-\infty, 4) \cup (7, \infty)$

so r does not lie between [4, 7]

62. (B)



$$\frac{1}{SP} + \frac{1}{SQ} = \frac{2a}{b^2}$$

$$\frac{1}{4} + \frac{1}{SQ} = \frac{6}{4}$$

$$\frac{1}{SQ} = \frac{5}{4}$$

$$SQ = \frac{4}{5}$$

As $SQ + S'Q = 6$

$$S'Q = 6 - \frac{4}{5} = \frac{26}{5}$$

63. (B)

Put $\theta = 0°$, we get a rectangle formed by tangents at the extremities of major and minor axis.

64. (C)

(p, q) lies on the director circle. i.e. $x^2 + y^2 = 25$

so $p = 5\cos\alpha$, $q = 5\sin\alpha$

$K = 15\cos\alpha + 20\sin\alpha$

$K = 5(3\cos\alpha + 4\sin\alpha)$

$\Rightarrow$ $-25 \le K \le 25$

65. (B)

$$y = \frac{1}{2}x \pm \sqrt{\frac{a^2}{4} + b^2}$$

$$2y = x \pm 2\frac{\sqrt{a^2 + 4b^2}}{2}$$

it passes through (–4, 0)

$$0 = -4 \pm \sqrt{a^2 + 4b^2}$$

$$a^2 + 4b^2 = 16$$

using AM $\geq$ GM

$$\frac{a^2 + 4b^2}{2} \geq \sqrt{a^2.4b^2}$$

$$8 \geq 2ab$$

$$ab \leq 4$$

66. (B)

for $m = \frac{3}{2}, a^2m^2 + b^2 = 100$

$\Rightarrow \; 9a^2 + 4b^2 = 400$ ...(i)

for $m = -\frac{1}{6}, a^2 + 36b^2 = 400$ ..(ii)

from (i) & (ii) $a^2 = 40$, $b^2 = 10$

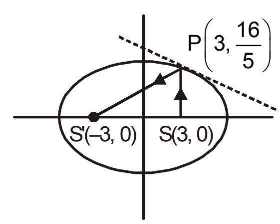
$$e^2 = 1 - \frac{b^2}{a^2}$$

$$e^2 = 1 - \frac{10}{40} = \frac{3}{4}$$

$$e = \frac{\sqrt{3}}{2}$$

67. (B)

The ray from S (3, 0) to $P\left(3, \frac{16}{5}\right)$ is reflected at



P and passes through $S'(-3, 0)$ so slope of

$$S'P = \frac{8}{15}$$

68. (C)

Point on the ellipse is $\left(\sqrt{6}\cos\theta, \sqrt{3}\sin\theta\right)$

Equation of tangent $\frac{x\cos\theta}{\sqrt{6}}+\frac{y\sin\theta}{\sqrt{3}}=1$

It is parallel to x + y = 10

$\frac{\cos\theta}{\sqrt{6}}=\frac{\sin\theta}{\sqrt{3}}$

so required point is (2, 1)

69. (B)

Let the asymptotes be $x + y + \lambda_1 = 0$ and $x + 2y + \lambda_2 = 0$ it passes through (1, 2). Hence $\lambda_1 = -3$, $\lambda_2 = -5$

The equation of hyperbola is $(x+y-3)(x+2y-5)+\lambda=0$

it passes through (3, 5)

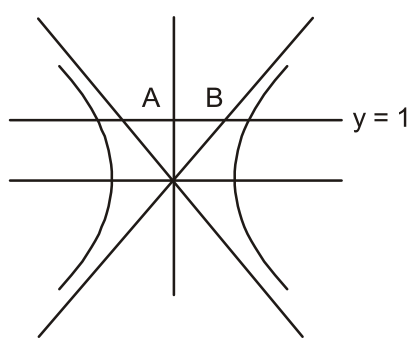
$\Rightarrow\ \lambda = -40$

so equation of hyperbola is

$(x+y-3)(x+2y-5)=40$

70. (A)

for two distinct tangents on different branches the point should lie on the line y = 1 and between A and B (where A and B are points on the asymptotes)



Equation of asymptotes are $5x=\pm 4y,$ solving with y = 1 $x=\pm\frac{4}{5}$

$\Rightarrow\ |\alpha|<\frac{4}{5}$

71. (A)

$(16-\lambda)(9-\lambda)<0$

$\Rightarrow\ 9<\lambda<16$

72. (B)

Equation of tangent to ellipse is

$$y = mx \pm \sqrt{8m^2 + a^2}$$

D = 0, for $4 = x\left(mx \pm \sqrt{8m^2 + a^2}\right)$

$\Rightarrow \quad mx^2 \pm x\sqrt{8m^2 + a^2} - 4 = 0$

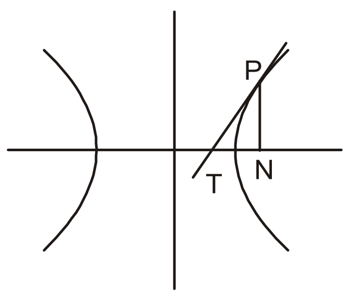
D = 0 $\Rightarrow 8m^2 + a^2 + 6m = 0$

D $\geq$ 0 $\Rightarrow (16)^2 - 4\times 8a^2 \geq 0$

$\Rightarrow \quad a^2 \leq 8$

$|a| \leq 2\sqrt{2}$

73. (B)



OT . ON = a^2

74. (A)

By T = S_1, The equation of chord whose mid point is (h,k) is

3hk – 2yk + 2 (x + h) – 3(y + k) = $3h^2 - 2k^2 + 4h - 6k$

its slope is $\frac{3h+2}{2k+3} = 2$

3h + 2 = 4k + 6

$\Rightarrow \quad \boxed{3x - 4y = 4}$

75. (C)

$$100\left[\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{5}\right)^2\right] = 9(3x + 4y - 7)^2$$

$S = \left(\frac{1}{2}, \frac{1}{5}\right)$, directrix : 3x + 4y – 7 = 0

Latus-rectum : $3x + 4y + \lambda = 0$

passing through $\left(\frac{1}{2}, \frac{1}{5}\right) \Rightarrow \lambda = \frac{-23}{10}$

76. (A)

$f(x) = \tan 8x$

$$\int \tan 8x\, dx = \frac{1}{8}\log(\sec 8x) + C$$

77. (A)

$$\int \sec^{2018} x \operatorname{cosec}^2 x dx - 2018 \int \sec^{2018} x dx$$

$$= -\sec^{2018} x \cot x + \int 2018 \sec^{2017} x \sec x \ \tan x \cot x dx - 2018 \int \sec^{2018} x\, dx$$

$$= -\frac{\cot x}{\cos^{2018}(x)} + C$$

$$\frac{f(x)}{g(x)} = \frac{\cot x}{\cos x} = \frac{1}{\sin x} = \{x\}$$

$\Rightarrow$ no solution.

78. (D)

$$\int \frac{dx}{(x-7)^{\frac{6}{7}}(x+8)^{\frac{8}{7}}} = \int \frac{dx}{\left(\frac{x-7}{x+8}\right)^{\frac{6}{7}} (x+8)^2}$$

Let $\frac{x-7}{x+8} = t$

$$\frac{15}{(x+8)^2} dx = dt$$

$$\frac{1}{15}\int \frac{dt}{t^{6/7}} = \frac{7}{15}\sqrt[7]{\frac{x-7}{x+8}} + C$$

$\Rightarrow k = \frac{7}{15}$

79. (D)

put $\cos x = t \quad \Rightarrow \quad -\sin x\, dx = dt$

$$\Rightarrow \quad -2\int \frac{t+\frac{1}{t}}{t^6+6t^2+4}\,dt$$

$$= \quad -2\int \frac{\frac{1}{t^5}+\frac{1}{t^7}}{1+\frac{6}{t^4}+\frac{4}{t^6}}\,dt = \frac{1}{12}\log\left(1+\frac{6}{t^4}+\frac{4}{t^6}\right)+C$$

$$= \quad \frac{1}{12}\log\left(1+\frac{6}{\cos^4 x}+\frac{4}{\cos^6 x}\right)+C$$

80. (B)

$$\int \frac{e^x\left(1-x^2+1\right)}{(1-x)\sqrt{1-x^2}}\,dx$$

$$= \quad \int e^x\left(\frac{1+x}{\sqrt{1-x^2}}+\frac{1}{(1-x)\sqrt{1-x^2}}\right)dx$$

$$= \quad \int e^x\left(\sqrt{\frac{1+x}{1-x}}+\frac{1}{(1-x)\sqrt{1-x^2}}\right)dx$$

$$= \quad e^x\sqrt{\frac{1+x}{1-x}}+C$$

81. (C)

Let $S = 1+3x+5x^2+7x^3+......$

$\underline{-}xS = \quad \underline{-}x\,\underline{+}\,3x^2\,\underline{+}\,5x^3+......$

$$S(1-x) = 1+2x+2x^2+2x^3+....$$

$$S(1-x) = 1+2\left(\frac{x}{1-x}\right)$$

$$S = \frac{1}{1-x}+2\frac{x}{(1-x)^2}$$

$$\int S\,dx = -\log(1-x)+\log(1-x)^2-\frac{2}{(1-x)}+C$$

82. (B)

$$\int_{-3}^{5} f(|x|)\,dx = \int_{-3}^{3} f(|x|)\,dx + \int_{3}^{5} f(|x|)\,dx$$

$$= 2\int_{0}^{3} f(x)\,dx + \int_{3}^{5} f(x)\,dx$$

$$= 2\left[0 + \frac{1}{2} + \frac{2^2}{2}\right] + \left[\frac{3^2}{2} + \frac{4^2}{2}\right]$$

$$= 5 + \frac{25}{2} = \frac{35}{2}$$

83. (B)

$$\int \frac{2x-4}{(x^2-4x+3)(x^2-4x-5)+16}\,dx$$

Let $x^2 - 4x = t$

$(2x - 4)\,dx = dt$

$$\int \frac{dt}{(t+3)(t-5)+16} = \int \frac{dt}{(t-1)^2}$$

$$= -\frac{1}{t-1} + C$$

$$= -\frac{1}{x^2-4x-1} + C$$

$\Rightarrow\ f(x) = x^2 - 4x - 1$

$\Rightarrow$ a + b + c = 1 – 4 – 1 = – 4

84. (A)

$$I = \int_{2}^{4} (x-1)(x-2)(x-3)(x-4)(x-5)\,dx$$

$$= \int_{2}^{4} (5-x)(4-x)(3-x)(2-x)(1-x)\,dx$$

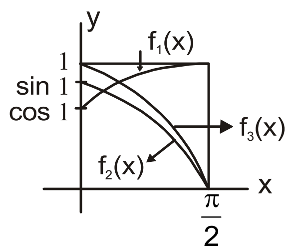
$I = -I$

$2I = 0$

$I = 0$

85. (A)

$$I_1 = \int_0^{\pi/2} \cos(\cos x)\, dx,\ I_3 = \int_0^{\pi/2} \cos x\, dx,$$



Let $f_1(x) = \cos(\cos x), f_2(x) = \sin(\cos x), f_3(x) = \cos x$

it is clear that area under $f_1(x)$ is largest and area under $f_2(x)$ is least

86. (A)

put x tan α = t sin α

dx = cos αdt

$$I = \cos\alpha \int_{\tan\alpha}^{1} f(t \sin\alpha)\, dt$$

$$= -\cos\alpha \int_{1}^{\tan\alpha} f(x \sin\alpha)\, dx$$

87. (B)

$$\int_{-2}^{-1} (ax^2 - 17)\, dx + \int_{1}^{2} (bx + c)\, dx + 17$$

$$= \int_{-2}^{-1} (ax^2 - bx + c)\, dx = 0$$

Hence, $ax^2 - bx + c = 0$ has at least one root in (–2, –1)

88. (B)

Let $f(x) = \dfrac{\sec x + \text{cosec } x - \sec x \text{ cosec } x}{2} = \dfrac{1}{1 + \sin x + \cos x}$

$$\Rightarrow\ f(x) \in \left[\sqrt{2} - 1, 1 - \frac{1}{\sqrt{3}}\right]$$

then $\left[\dfrac{1}{1+\sin x+\cos x}\right]=0$

89. (C)

$$\begin{aligned} f(x) \quad + f(x+6) &= f(x+3)+f(x+9) \\ f(x+3)+f(x+9) &= f(x+6)+f(x+12) \\ \hline f(x) &= f(x+12) \end{aligned}$$

Let $g(x)=\int_{x}^{x+12} f(t)\,dt$

$$g'(x)=f(x+12)-f(x)=0$$

$\Rightarrow$ g(x) is a constant function

90. (B)

$$I=\int_{-\frac{\pi}{4}}^{n\pi-\frac{\pi}{4}} \sqrt{2}\left|\sin\left(x+\frac{\pi}{4}\right)\right| dx$$

$$= n\int_{0}^{\pi} \sqrt{2}\left|\sin\left(x+\frac{\pi}{4}\right)\right| dx$$

$$= \sqrt{2}n\left[\int_{0}^{\frac{3\pi}{4}} \sin\left(x+\frac{\pi}{4}\right).dx-\int_{\frac{3\pi}{4}}^{\pi} \sin\left(x+\frac{\pi}{4}\right)dx\right]$$

$$= 2\sqrt{2}n$$