SOLUTIONS

WEEKLY TEST-1

GZRS-1902

(JEE MAIN PATTERN)

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PHYSICS

1. At equilibrium, let tension in each spring be *T*. Then

$$2T\cos 60^\circ = Mg$$

$$T = Mg$$

When right spring breaks, the net force on the block is *T*.

$\therefore \quad a = \frac{T}{M} = 10\ m/s^2$

$\therefore$ **(A)**

2. **(B)**

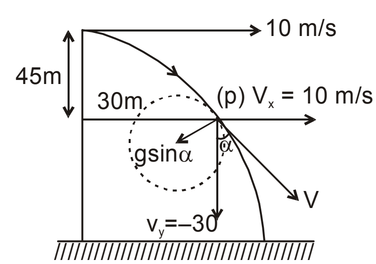
Mathod (I)

After 3 sec.

$V_y = u_y + gt = -30$ m/s

and $V_x = 10$ m/s $\quad \therefore V^2 = V_x^2 + V_y^2$

$\Rightarrow \quad V = 10\sqrt{10}$ m/s

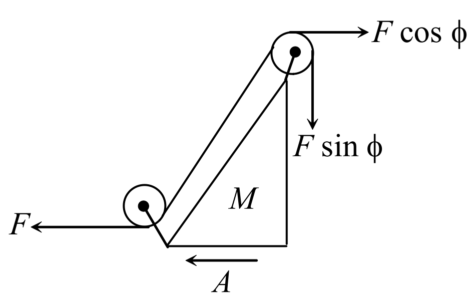


Now, $\tan\alpha = \frac{V_x}{V_y} = \frac{1}{3} \quad \Rightarrow \sin\alpha = \frac{1}{\sqrt{10}}$

Radius of curvature $r = \frac{V_\perp^2}{g\sin\alpha}$

$$r = 100\sqrt{10}\ m$$

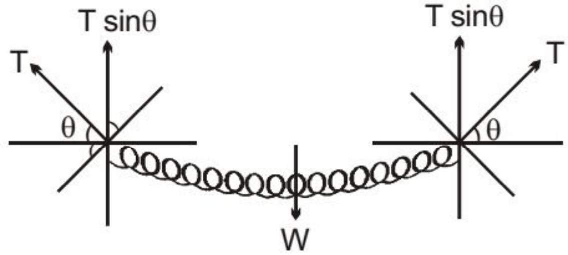
3. $F - F\cos\phi = MA$



$$A = \frac{F - F\cos\phi}{M}$$

$\therefore$ **(B)**

4. (A)



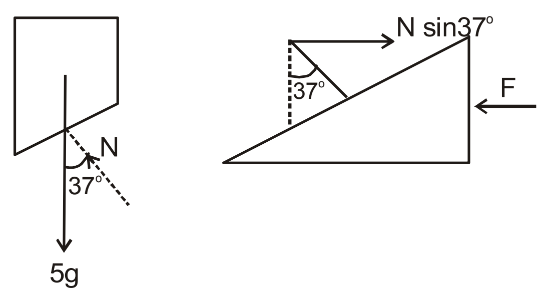
$2T\sin\theta = W$

$T = \frac{W}{2}\operatorname{cosec}\theta$

5. (B)

For equilibrium of 5 kg block

$N\cos 37^{o} = 50$



$\Rightarrow N = 50 \times \frac{5}{4} = 62.5N$

For equilibrium of 10 kg wedge

$N\sin 37^{o} = F$

$\Rightarrow\ 62.5 \times \frac{3}{5} = F$

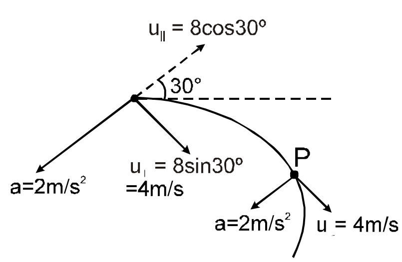
$\Rightarrow\ F = 37.5\ N$

6. (C)

The acceleration vector shall change the component of velocity $u_{||}$ along the acceleration vector.

$r = \frac{v^2}{a_n}$

Radius of curvature r_{min} means v is minimum and a_n is maximum. This is at point P when component of velocity parallel to acceleration vector becomes zero, that is $u_{||} = 0$.



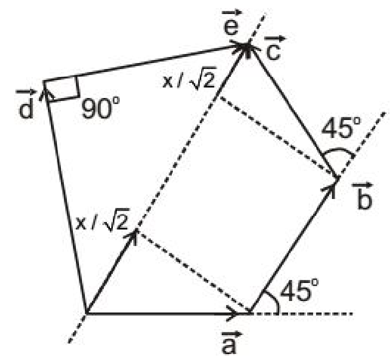
$\therefore\ R = \frac{u_{\perp}^2}{a} = \frac{4^2}{2} = 8$ meter.

7. (B)

For the angle 'θ' normal reaction between A & B becomes zero, they ready to seprate. So, solve 'θ' for $N_{AB} = 0$

8. (B)

The resultant of $\vec{a}, \vec{b}$ and $\vec{c}$ is of magnitude $\frac{x}{\sqrt{2}} + x + \frac{x}{\sqrt{2}}$ whcih is equal to the resultant of $\vec{d}$ and $\vec{e}$.



So,

$$\sqrt{2}x + x = \sqrt{2}y$$

$$\Rightarrow y = \left(1 + \frac{1}{\sqrt{2}} x\right)$$

$$\Rightarrow y = \left(1 + \frac{\sqrt{2}}{2}\right)$$

so,

k = 2

9. (C)

$$R = \frac{u^2}{g}\sin 2\theta = \frac{u^2}{g}$$

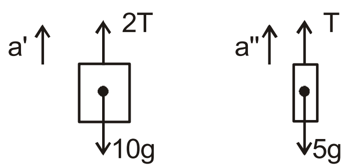
Velocity of take off at *P* or $u = \sqrt{Rg} = \sqrt{90 \times 10} = 30 m/s$

$v = \sqrt{u^2 + 2g\sin\theta S}$ [$v \to$ velocity at point *O*]

$$= \sqrt{(30)^2 + 2 \times 10 \times \frac{1}{\sqrt{2}} \times 80\sqrt{2}} = 50 m/s$$

10. (A)

FBD of 'A' and 'B'



As, froces and mass all are in ration 2 : 1 for blck ‘ A’ and ‘B’. So, their acceleration will be equal From constain realatio

$a' + a'' = a$

$a' + 2a' = a$

$\Rightarrow a' = 1 \text{ m/s}^2$

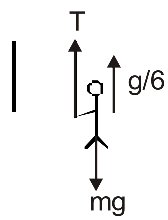
11. (C)

Let the block moves up with acceleration of ‘a’ m/s^2, then as the person is moving up will acceleration $\frac{g}{6}$ relative to string. Acceleration of person in ground frame will be

$\vec{a}_{pg} = \vec{a}_{ps} + \vec{a}_{sg}$

$\Rightarrow a_{pg} = \left(\frac{-g}{6} + a\right)\downarrow$

So, $mg - T = m\left(a - \frac{g}{6}\right)$ .... (i)



and, for block

$T - \frac{mg}{2} = \frac{m}{2}a$ ....(ii)

(i) + (ii)

$\frac{mg}{2} = \frac{3ma}{2} - \frac{mg}{6}$

$\Rightarrow mg\left(\frac{1}{2} + \frac{1}{6}\right) = \frac{3ma}{2} \Rightarrow a = \frac{4g}{9}$

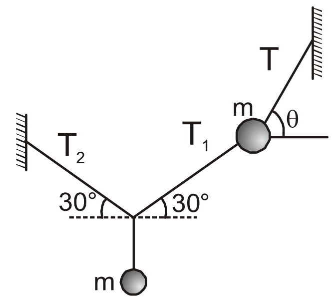
12. (A)

$T = \frac{4m_1 m_2 m_3 g}{4m_1 m_2 + m_2 m_3 + m_1 m_3}$

13. (B)

$v_B \cos 30^0 = v_A \cos 60^0$; $v_B \frac{\sqrt{3}}{2} = \frac{3}{2}$; $v_B = \sqrt{3}$ m/s

14. (C)



$T_1\cos30° = T_2\cos30°$

$\Rightarrow T_1 = T_2$

$(T_1 + T_2)\sin30° = mg$

$T_1 = T_2 = mg$

$T\sin\theta = mg + T_1\sin30°$

$T\sin\theta = mg + \frac{mg}{2}$ ....(i)

$T\cos\theta = T_1\cos30° = mg\times\frac{\sqrt{3}}{2}$ ....(ii)

dividing (i) and (ii)

$$\tan\theta = \frac{3mg/2}{\sqrt{3}mg/2} = \sqrt{3} \Rightarrow \theta = 60°$$

15. (B)

16. (D)

17. (C)

t is the time to reach ground.

$h = \frac{1}{2}at^2$; $\left(1 - \frac{9}{25}\right)h = \frac{1}{2}a(t-1)^2$

$$\left(1 - \frac{9}{25}\right) = \frac{(t-1)^2}{t^2} \;;\; \frac{16}{25} = \frac{(t-1)^2}{t^2}$$

or $\frac{4}{5} = \frac{t-1}{t}$ $\therefore$ t = 5 sec

$h = \frac{1}{2}\times 9.8\times 5^2 = 122.5$ m

$\therefore$ **(C)**

18. (A)

This is the situation similar to elastic collision of ball impinging on floor and bouncing back.

19. (A)

20. (C)

Given equation is $\left(p+\frac{a}{V^2}\right)(V-b)=RT$

We know that $\left[p+\frac{a}{V^2}\right]=\left[ML^{-1}T^{-2}\right]$, P = pressure.

$\left[\frac{a}{V^2}\right]=[ML^{-1}T^{-2}]$

a = $[L^3]^2$ $[ML^{-1}T^{-2}]$ = $[L^6][ML^{-1}T^{-2}]$ = $[ML^5T^{-2}]$ $\left[\because V\equiv\left[L^3\right]\right]$

21. (B)

Value of 1 main scale division = a unit

Now (n+1) vernier scale division = n main scale divisions = na units.

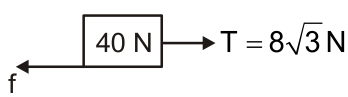
Therefore, value of 1 vernier scale division = $\frac{na}{(n+1)}$ units.

Vernier constant = value of 1 main scale division – value of 1 vernier scale division

$$= a-\frac{na}{n+1}=a\left(1-\frac{n}{n+1}\right)=\frac{a}{(n+1)} \text{ units.}$$

22. (D)

From f.b.d.



$\mu=\frac{8\sqrt{3}}{40}\simeq 0.35$

23. (C)

Use homogenity of dimensión and use

$\mu\rightarrow$ Dimension less quantity

$\lambda\rightarrow$ meter

24. (C)

$v\frac{dv}{dx}$ = 2x + 1

vdv = (2x + 1) dx

$\int_0^v v\,dv=\int_0^x (2x+1)\,dx \quad\Rightarrow\frac{v^2}{2}=x^2+x$

25. (C)

The displacement of the body during the time t as it reaches the point of projection again

$\Rightarrow \; S = 0 \qquad \Rightarrow \; v_0 t - \frac{1}{2} g t^2 = 0 \qquad \Rightarrow t = \frac{2v_0}{g}$

During the same time t, the body moves in absence of gravity through a distance

$D' = v_0 t$, because in absence of gravity g = 0

$$\Rightarrow \; D' = v_0\left(\frac{2v_0}{g}\right) = \frac{2v_0^2}{g} \qquad ...(i)$$

In presence of gravity the total distance covered is

$$= D = 2H = 2\frac{v_0^2}{2g} = \frac{v_0^2}{g} \qquad ...(ii)$$

(i) $\div$ (ii) $\Rightarrow D' = 2D$

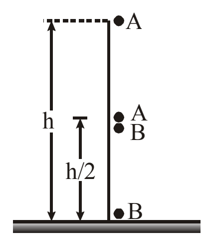
Hence **(C)**

26. (C)

Time of travel of each stone = t

Distance travelled by each stone $= \frac{h}{2}$

For stone $A, \frac{h}{2} = \frac{1}{2} g t^2$ i.e., $t = \sqrt{\frac{h}{g}}$



For stone B, $\frac{h}{2} = ut - \frac{1}{2} g t^2 = u\sqrt{\frac{h}{g}} - \frac{1}{2} g\left(\frac{h}{g}\right)$

$$\Rightarrow \; \frac{h}{2} = u\sqrt{\frac{h}{g}} - \frac{h}{2} \;\Rightarrow\; u\sqrt{\frac{h}{g}} = h$$

$$\therefore \; u = h\sqrt{\frac{g}{h}} = \sqrt{gh}$$

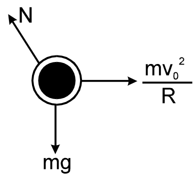
The correct option is **(C)**

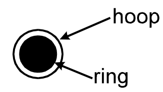
27. (D)

The free body daigram of hoop is

$\therefore$ The normal reaction $N = \sqrt{m^2 g^2 + \frac{m^2 v_0^4}{r^2}}$

$\therefore$ Frictional force $= \mu_k N = \mu_k \sqrt{m^2g^2 + \frac{m^2v_0^4}{r^2}}$





$\therefore$ dangential acceleration $= \frac{\mu_k N}{m} = \mu_k \sqrt{g^2 + \frac{v_0^4}{r^2}}$

28. (B)

$kx = m\omega^2 \ell + m\omega^2 x$

$(k - m\omega^2) x = m\omega^2 \ell$

$x = \frac{m\omega^2 \ell}{k - m\omega^2}$

29. (A)

For a force of 100 N on 10 kg block, relative motion will take place.

$\therefore$ The frictional force between 10 kg block and 40 kg block,

$f = \mu mg = 0.4 \times 10 \times 9.8 \text{N}$

The acceleration of the slab of 40 kg is

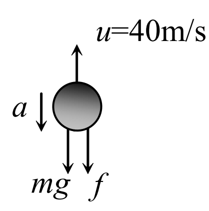
$a = \frac{0.4 \times 10 \times 9.8}{40} = 0.98 \text{ m/s}^2$

30. (D)

Let retardation of body is *a* and air resistance is *f*

$v = u + at$

$0 = 40 - 3a$



$a = \frac{40}{3} \text{ m/s}^2$

$ma = mg + f$

$f = ma - mg = 1.5\left(\frac{40}{3} - 10\right) = 5 \text{ N}$

CHEMISTRY

31. (C)

Acid H_2A

Salt Ag_2A

$$\frac{1}{108\times 2+x}\times 2=\frac{0.108}{108}$$

or, x = 1784

molar mass of H_2A = 1786

32. (C)

P_{gas} =820 – 60 = 760 torr = 1 atm

$$PV=\frac{m}{M}RT$$

or $M=\frac{mRT}{PV}=\frac{10\times 0.082\times 300}{1\times 2}=123$

33. (D)

$${}_{58}Ce=[Xe]4f^{1}5d^{1}6s^{2}$$

$Ce^{+3}=[Xe]4f^{1}$, i.e. only one unpaired electron

$\mu=\sqrt{n(n+2)}=\sqrt{1(1+2)}=\sqrt{3}$ = 1.73 BM

34. (B)

$Fe^{2+}\rightarrow 4s^{\circ}3d^{6}$

↑↓	↑	↑	↑	↑

Spin multiplicity = $2\Sigma s+1=2\left(+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}-\frac{1}{2}\right)+1$ = 4 +1 = 5

35. (D)

$$\frac{\Delta x_e.m_e.\Delta v_{e^-}}{\Delta x_p.m_p.\Delta v_p}=1\Rightarrow\frac{\Delta v_e}{\Delta v_p}=\frac{m_p}{m_e}$$

36. (B)

$IE=+13.6z^{2}=13.6\times 9=122.4ev,$

KE of emitted electron = 122.4–13.6=108.8 ev.

$$\lambda=\sqrt{\frac{150}{V(\text{in volt})}}=\sqrt{\frac{150}{108.8}}=1.17A^{o}$$

37. (D)

$P = 4\pi r^2 \psi^2$

$\frac{dP}{dr} = 0 \quad \Rightarrow z = 4$

38. (D)

39. (B)

200 gm (109%) New oleum

$$\boxed{H_2SO_4 \ + \ SO_3} + \boxed{H_2O} \longrightarrow \boxed{H_2SO_4 \ + \ SO_3}$$

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$

120 gm 80 gm 9 gm 169 gm 40 gm

New oleum

$\because$ 209 gm new oleum can give maximum of 218 gm H_2SO_4.

$\therefore$ 100 gm new oleum can give maximum of $\frac{218}{209} \times 100 = 104.30$

$\therefore$ % labelling of new oleum = 104.3%

40. (C)

$v \propto \frac{z}{n};\ r \propto \frac{n^2}{z};$

frequency of revolution $= \frac{v_n}{2\pi r_n};$

Conulombic force of attraction $= \frac{Ze^2}{(4\pi\varepsilon_0)r^2}$

41. (C)

Let the transition took place from n_2 to n_1.

$n_2 + n_1 = 4$ ... (1)

$n_2 - n_1 = 2$ ... (2)

Using (1) & (2), we get; $n_2 = 3; n_1 = 1$

$$\frac{1}{\lambda} = \bar{v} = (R_H)(3^2)\left[\frac{1}{1^2} - \frac{1}{3^2}\right] = (R_H)(9)\left(\frac{8}{9}\right) = 8R_H$$

42. (C)

$E_{supp} = \phi + K.E.$

$\frac{hC}{\lambda_s} = \phi + K.E$

43. (C)

44. (A)

45. (A)

46. (B)

$\frac{2E_1}{N_O}, \frac{2E_2}{N_O}$

$x(g) \rightarrow x^+_{(g)} + 1e$

$\frac{N_o}{2} \longrightarrow E_1$

$\because \frac{N_o}{2} \rightarrow E_1$

$\therefore 1 \longrightarrow \frac{E_1}{\frac{N_o}{2}} = \frac{2E_1}{N_O}$

$X(g) + 1e - x^-_{(g)} - E_2$

$\frac{N_o}{2} \longrightarrow E_2$

$1 \longrightarrow \frac{E_2}{\frac{N_o}{2}} = \frac{2E_2}{N_0}$

47. (D)

$\xleftarrow[r]{Sc^{+3}, Ti^{+3}, V^{+2}, Cr^{+3}}$

- Non-metals having gaint structure have high M.P. i.e. Si.

48. (A)

The element with atomic number 43 has the configuration :

2,8,18,8 +5,2

$4d^5, 5s^2$

Thus, the element just above 43 belongs to 4th period and has atomic number 25. The configuration is :

2,8, 8,18 + 52

$3d^5, 4s^2$

49. (C)

The atomic number will be 2 + 8 + 8 + 18 + 16 = 52

50. (C)

For isoelectronic species, the size of an ion increases with increase in the negative charge.

51. (C)

If protons are same, they must have same atomic number.

52. (D)

53. (B)

(i) Acidic oxides are generally formed by non-metals, SnO_2 is amphoteric.

(ii) Basic oxides are generally formed by metal.

(iii) ZnO, BeO, SnO_2, Ga_2O_3, PbO, SnO are amphoteric in nature. It is react with strong acid and strong base, and as a result gives weak acid and weak base.

(iv) N_2O, H_2O, NO are neutral oxides. [CaO is basic oxides]

54. (A)

55. (A)

Correct order of B,C & D

$BDE \rightarrow Cl_2 > Br_2 > F_2 > I_2$

Catenation $\rightarrow C >> Si > Ge = Sn >> Pb$

$IP \rightarrow Mn^{+7} > Mn^{+4} > Mn^{+2}$

56. (C)

Due to size of Nitrogen is smaller than another.

57. (D)

Electrons in orbitals bearing a lower 'n' value are more attracted to the nucleus then electrons in orbitals bearing a higher 'n' value. Hence, the removal of electrons from orbitals bearing a higher 'n' value is easier than the removal of electrons from orbitals having a lower 'n' value.

58. (B)

Within a period, the oxidising charactor increases from left to right, therefore among F,O and nitrogen oxidising power decreases in the order F > O > N. However within a group oxidising power decreases from top to bottom. Thus, fluorine is more oxidising aent than CL. Further because 'O' is more eletronegative than Cl, therefore oxygen is more oxidising agent than CL.

Order of oxidising property =F > O > Cl > N

59. (D)

Due to decerasing trend of I.E. reactivity increase along the group but due to standard electrode potential reactivity of halogen in decreasing order along the group.

60. (A)

$\Delta H_{ion.} = -\Delta Heg$

MATHEMATICS

61. (B)

$\sqrt{12-\sqrt{68+48\sqrt{2}}}$

$= \sqrt{12-\sqrt{(6+4\sqrt{2})^2}} = \sqrt{12-6-4\sqrt{2}} = \sqrt{6-4\sqrt{2}} = \sqrt{(2-\sqrt{2})^2} = 2-\sqrt{2}$

62. (B)

$x = \sqrt{3-\sqrt{5}}$ $\qquad$ $y = \sqrt{3+\sqrt{5}}$

$xy = 2$

$x + y = \sqrt{x^2+y^2+2\times 2}$

$= \sqrt{6+4} = \sqrt{10}$

$x - y = \sqrt{6-4} = \sqrt{2}$

Put the value we get the ans.

$(x-y) + 2xy\,(x+y) - xy(x-y)\,(x^2+y^2+xy)$

$-\sqrt{450}+\sqrt{160}$

63. (C)

Given, $|4x + 3| + |3x - 4| = 12$

When $x \le \frac{-3}{4}$

$-(4x + 3) - (3x - 4) = 12$ ...(i)

$-7x = 11 \Rightarrow x = -\frac{11}{7}$ (Accepted)

When $\frac{-3}{4} < x \le \frac{4}{3}$

$4x + 3 - (3x - 4) = 12$; $x = 5$ (Rejected) ...(ii)when $x > \frac{4}{3}$

$4x + 3 + (3x - 4) = 12$; ...(iii)

$7x = 13 \Rightarrow x = \frac{13}{7}$ (Accepted)

From (i) $x = -\frac{11}{7}$ From (ii) $x = 5$ (reject)

From (iii) $x = \frac{13}{7}$

64. (D)

$$= \frac{2^{\log_2(a^4)} - 3^{\log_3(a^2+1)} - 2a}{7^{\log_7(a^2)} - a - 1} = \frac{a^4 - (a^2+1) - 2a}{a^2 - a - 1}$$

$$= \frac{(a^2)^2 - (a+1)^2}{(a^2 - a - 1)} = a^2 + a + 1$$

65. (A)

a, b, c in A.P. $\Rightarrow 2b = a + c$ ...(i)

p, q, r, in H.P. $\Rightarrow q = \frac{2pr}{p+r}$ ...(ii)

ap, bq, cr in G.P. $\Rightarrow b^2q^2 = acpr$ ...(iii)

From (ii) & (iii), we get

$$\Rightarrow \frac{b^2 . 4(pr)^2}{(p+r)^2} = ac\,pr \qquad \Rightarrow \frac{(a+c)^2 pr}{(p+r)^2} = ac \text{(from (i))}$$

$$\Rightarrow \frac{(p+r)^2}{pr} = \frac{(a+c)^2}{ac} \Rightarrow \frac{p^2+r^2}{pr} + 2 = \frac{a^2+c^2}{ac} + 2$$

$$\Rightarrow \frac{p}{r} + \frac{r}{p} = \frac{a}{c} + \frac{c}{a}.$$

66. (A)

$x = 2^{\log_2 8 \log_{11}^{1331}}$

$x = 2^9$ $\qquad$ $y = 2^{\frac{1}{4}}$

$$\text{Log}_B N = \frac{\log 4}{\log 5}\frac{\log 5}{\log 6} \text{------} \frac{\log 36}{\log 37}$$

$$= \frac{\log 4}{\log 37} = \log_{37}{}^4$$

$$Z = \frac{4}{37}$$

$$(x\,y)^z = (2^9.2^{1/4})\frac{4}{37} = \left(2^{\frac{37}{4}}\right)^{\frac{4}{37}} = 2$$

67. (A)

$x_i > 0, i = 1, 2, ..., 50$ & $x_1 + x_2 + x_3 + ... + x_{50} = 50$

or $\sum_{1}^{50} x_i = 50 \Rightarrow \frac{\Sigma x_i}{50} = 1$

$\because$ A.M. $\geq$ H.M.

$$\frac{\left(\sum_{1}^{50} x_i\right)}{50} \geq \frac{50}{\left(\sum_{1}^{50} \frac{1}{x_i}\right)} \Rightarrow 1 \geq \frac{50}{\left(\sum_{1}^{50} \frac{1}{x_i}\right)}$$

$\Rightarrow \sum_{1}^{50} \frac{1}{x_i} \geq 50$, Min value of $\Sigma \frac{1}{x_i} = 50$

68. (B)

Let $x = 5\cos\theta, y = 5\sin\theta$

$0 < 3x + 4y \leq 25 \quad (\because 3x + 4y > 0)$

69. (C)

$$\because \quad \frac{1}{\sqrt{n + \sqrt{n^2 - 1}}} = \frac{1}{\sqrt{\left(\sqrt{\frac{n+1}{2}} + \sqrt{\frac{n-1}{2}}\right)^2}} = \frac{1}{\sqrt{\frac{n+1}{2}} + \sqrt{\frac{n-1}{2}}} = \frac{\sqrt{\frac{n+1}{2}} - \sqrt{\frac{n-1}{2}}}{\frac{n+1}{2} - \frac{n-1}{2}}$$

$$= \sqrt{\frac{n+1}{2}} - \sqrt{\frac{n-1}{2}}$$

Hence $a + b\sqrt{2} = \sum_{n=1}^{49}\left(\sqrt{\frac{n+1}{2}} - \sqrt{\frac{n-1}{2}}\right)$

$$\Rightarrow a + b\sqrt{2} = \left(\sqrt{\frac{2}{2}} - 0\right) + \left(\sqrt{\frac{3}{2}} - \sqrt{\frac{1}{2}}\right) + \left(\sqrt{\frac{4}{2}} - \sqrt{\frac{2}{2}}\right) + \left(\sqrt{\frac{5}{2}} - \sqrt{\frac{3}{2}}\right) + ... + \left(\sqrt{\frac{49+1}{2}} - \sqrt{\frac{49-1}{2}}\right)$$

$= \sqrt{\frac{49+1}{2}} + \sqrt{\frac{48+1}{2}} - \frac{1}{\sqrt{2}} - 0 = 5 + 3\sqrt{2} \quad \Rightarrow \quad a = 5,\ b = 3$ and $a + b = 8$.

70. (C)

Let $f(t) = 9^t + 9^{1-t}$ where $t = \sin^2 x,\ t \in [0,1]$

Use A.M. $\geq$ G.M.

71. (A)

$$10\tan^4\alpha + 15 = 6(\tan^2\alpha + 1)^2 \Rightarrow \tan^2\alpha = \frac{3}{2} \Rightarrow 9\operatorname{cosec}^4\alpha + 8\sec^4\alpha = 75$$

72. (B)

$$t_3 = t_1 + t_2 ; t_7 = 1000 \; ; \; t_1 = 1$$

$$\because \; t_7 = t_1 + t_2 + t_3 + t_4 + t_5 + t_6$$

$$\Rightarrow \; 1000 = 2(t_1 + t_2 + t_3 + t_4 + t_5) = 8(t_1 + t_2 + t_3)$$

$$\Rightarrow \; 1000 = 16(t_1 + t_2) \Rightarrow t_1 + t_2 = \frac{1000}{16} \Rightarrow t_2 = \frac{123}{2}$$

73. (B)

74. (C)

cosecA + cotA = 2

$\Rightarrow$ cosecA – cot A = 1/2

$\Rightarrow$ cosecA = 5/4 & cot = 3/4

$\Rightarrow$ cosA = 3/5

75. (A)

$$\sec\theta - \tan\theta = \lambda \quad \Rightarrow \sec\theta + \tan\theta = \frac{1}{\lambda}$$

$\therefore$ subtracting, $2\tan\theta = \frac{1}{\lambda} - \lambda$ or $2\left(a - \frac{1}{4a}\right) = \frac{1}{\lambda} - \lambda$

or $2a - \frac{1}{2a} = \frac{1}{\lambda} - \lambda \quad \Rightarrow \lambda = \frac{1}{2a}, -2a$

76. (C)

$$(A \cap B) \cup C = \{1,3,5,7,8,9\}$$

$$A' \cap B' = \{10\}$$

$$(A \cup B)' = \{10\}$$

$$(A \cap B) \cap (A \cap C) = \{8\}$$

77. (C)

We have,

$(2x - 3y)^2 + (3y - 4z)^2 + (4z - 2x)^2 = 0 \Rightarrow 2x = 3y = 4z$

$\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in AP $\Rightarrow$ x, y, z are in HP

78. (C)

$$\frac{H_1+2}{H_1-2}+\frac{H_{20}+3}{H_{20}-3} = \frac{\frac{1}{2}+\frac{1}{H_1}}{\frac{1}{2}-\frac{1}{H_1}}+\frac{\frac{1}{3}+\frac{1}{H_{20}}}{\frac{1}{3}-\frac{1}{H_{20}}}$$

$$=\frac{\frac{1}{2}+\frac{1}{2}+d}{\frac{1}{2}-d-\frac{1}{2}}+\frac{\frac{1}{3}+\frac{1}{3}-d}{\frac{1}{3}+d-\frac{1}{3}} = \frac{1+d}{-d}+\frac{\frac{2}{3}-d}{d} = \frac{\frac{2}{3}-1}{d}-2 = 2\times 21-2 = 40$$

79. (A)

$\sin\alpha+\cos\alpha=-\frac{b}{a}$ and $\sin\alpha\cos\alpha=\frac{c}{a}$

$$\Rightarrow 1+2\sin\alpha\cos\alpha=\frac{b^2}{a^2} \Rightarrow 1+\frac{2c}{a}=\frac{b^2}{a^2} \Rightarrow a^2+2ac-b^2=0$$

80. (C)

sec 40°, sec 80°, sec 160° are the roots of $\frac{8}{t^3}-\frac{6}{t}+1=0$

or $t^3-6t^2+8=0$

$\therefore$ Sum of roots = 6.

81. (B)

We have, $\sin\theta+\cos\theta=m$

and $\sec\theta+\text{cosec}\,\theta=n$

$$\Rightarrow \frac{1}{\cos\theta}+\frac{1}{\sin\theta}=n \Rightarrow \frac{\sin\theta+\cos\theta}{\cos\theta\sin\theta}=n$$

$$\Rightarrow \frac{m}{\cos\theta\sin\theta}=n$$

$$\Rightarrow \cos\theta\sin\theta=\frac{m}{n}$$

Squaring (i), we get

$$\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=m^2 \Rightarrow 1+2\cdot\frac{m}{n}=m^2$$

$$\Rightarrow \frac{2m}{n}=m^2-1 \Rightarrow 2m=n(m^2-1).$$

82. (C)

a_1, a_2, a_3, a_4, a_5 are in H.P.

$\Rightarrow a_2 = \dfrac{2a_1 a_3}{a_1 + a_3} \quad \Rightarrow 2a_1a_3 = a_2a_1 + a_3a_2$

$a_4 = \dfrac{2a_3a_5}{a_3 + a_5} \quad \Rightarrow 2a_3a_5 = a_3a_4 + a_5a_4$

$\Rightarrow a_1a_2 + a_2a_3 + a_3a_4 + a_4a_5 = 2a_1a_3 + 2a_3a_5$ ... (i)

$a_3 = \dfrac{2(a_1a_5)}{a_1 + a_5} \Rightarrow a_1a_3 + a_5a_3 = 2a_1a_5$ ... (ii)

using (i) and (ii)

$a_1a_2 + a_2a_3 + a_3a_4 + a_4a_5 = 2(2a_1a_5) = 4a_1a_5$

83. (B)

Case I : When $2x - 3 \geq 0$ i.e., $x \geq \dfrac{3}{2}$

In this case, we have

$|2x - 3| = 2x - 3$

$\therefore |2x - 3| < x - 1 \Rightarrow 2x - 3 < x - 1 \Rightarrow x - 2 < 0 \Rightarrow x < 2$

$\Rightarrow x \in [3/2, 2) \quad [\because x \geq 3/2]$

Case II : When $2x - 3 < 0$ i.e., $x < \dfrac{3}{2}$

In this case, we have

$|2x - 3| = -(2x - 3)$

$\therefore |2x - 3| < x - 1 \Rightarrow -(2x - 3) < x - 1 \Rightarrow 3x - 4 > 0 \Rightarrow x > 4/3$

$\Rightarrow x \in (4/3, 3/2) \quad [\because x < 3/2]$

Thus, the set of the values of x satisfying the given inequation is $(4/3, 3/2) \cup [3/2, 2) = (4/3, 2)$

84. (A)

Solve the inequations

$x^2 - 3x + 2 \leq 0$ and $2x^2 - 3x - 5 \geq 0$

$\Rightarrow 1 \leq x \leq 2$ and $x \leq -1$ or $x \geq \dfrac{5}{2} \quad \therefore x \in \phi$

85. (B)

$$a = \log_{10} 2 = \log_{10} \frac{10}{5} = 1 - \log_{10} 5$$

$$\Rightarrow \log_{10} 5 = 1 - a$$

86. (D)

$$\left|\frac{1-x^2}{x}\right| + |x| = \left|\frac{1-x^2}{x} + x\right| = \left|\frac{1}{x}\right|$$

$$\Rightarrow \frac{1-x^2}{x}. x \geq 0 \Rightarrow x \in [-1, 1] - \{0\}$$

87. (C)

$3^x - 8 = 3^{2-x}$ and $3^x - 8 > 0$

Let $3^x = y (y > 0)$

$$\Rightarrow y - 8 = \frac{9}{y}$$

$$\Rightarrow y^2 - 8y = 9$$

$y = 9, \; y = -1$

$x = 2$

88. (D)

$2^{n+1}(n - 1) + 2 = 2^{n+10} + 2.$

$\therefore \; n = 513.$

Sum of digits = 9.

89. (C)

It is obvious that a, b and c are the roots of the equation $mt^3 + (l - p)\, t - kq = 0$, where (p, q) is the point of concurrency.

Obviously sum of roots = a + b + c = 0

$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$

90. (D)

(1997, 0) lies on y = mx + c

$\Rightarrow 0 = 1997m + c \Rightarrow c = -1997\,m$

$\Rightarrow mc = -1997\,m^2 \leq 0$

which is not possible.