SOLUTIONS

MEAITS 2018

UNIT TEST-4

(MAIN & ADVANCED PATTERN)

Test Date: 10-12-2017

Mentors® Eduserv™

Corporate Office: Paruslok, Boring Road Crossing, Patna-01
Kankarbagh Office: A-10, 1st Floor, Patrakar Nagar, Patna-20
Bazar Samiti Office : Rainbow Tower, Sai Complex, Rampur Rd., Bazar Samiti Patna-06
Call : 9569668800 | 7544015993/4/6/7

JEE MAIN

PHYSICS

1. (A)

Let CB = x m

5 = 2i

$$\Rightarrow i = \frac{5}{2} = \frac{15}{3+9x}$$

$$\frac{1}{2} = \frac{3}{3+9x} = \frac{1}{1+3x}$$

$$x = \frac{1}{3}\text{cm} = 33.33 \text{ cm}$$

$$\Rightarrow \text{AC} = 66.7 \text{ cm}$$

2. (B)

From Energy conservation

$$\frac{1}{4}\rho AgH^2 = \frac{1}{2}\rho A.4Hv^2$$

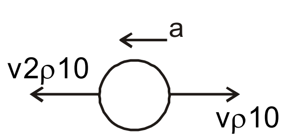
$$\frac{gH}{2} = 4v^2 \Rightarrow v = \sqrt{\frac{gH}{8}}$$

3. (C)

$$32 = \int_0^4 \frac{36t^2}{R}dt = \frac{36}{R} \times \frac{64}{3} = \frac{12 \times 64}{R}$$

$$R = 24\Omega$$

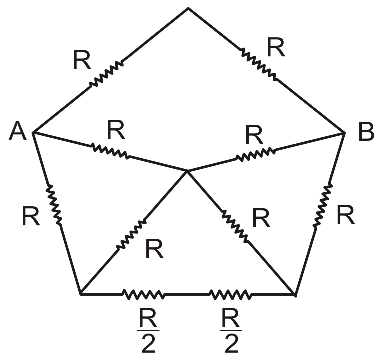
4. (B)

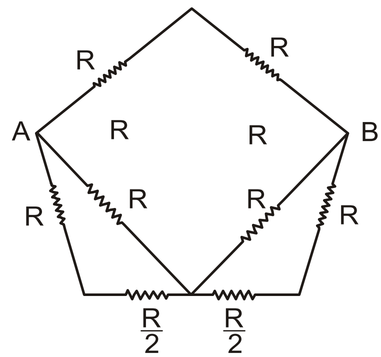


$$a = \frac{V\rho 10}{V\rho} = 10 \text{ m/s}^2$$

5. (A)

Equivalent Network





$\Rightarrow R_{AB} = \frac{8R}{11}$

6. (A)

$v = \sqrt{(20)^2 - 2 \times 10 \times 15}$

$= \sqrt{400 - 300} = 10\,m/s$

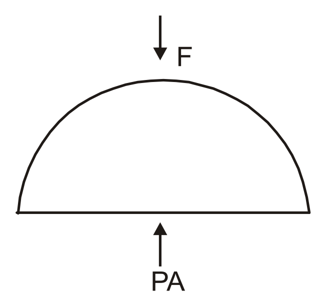
Now, $av = 10^{-3}$

$a = \frac{10^{-3}}{10} = 10^{-4}$

$M \times 10 = 10^3 \times 10^{-4} \times 10^2$

$M = 1Kg$

7. (A)



$PA - F = F_g = \frac{2\pi}{3} r^3 \rho_1 g$

$\Rightarrow (P_0 + \rho_1 gh)\pi r^2 - F = \frac{2\pi}{3} r^3 \rho_1 g$

$\Rightarrow F = P_0 \pi r^2 + \left(h - \frac{2}{3} r\right) \pi r^2 \rho_1 g$

8. (B)

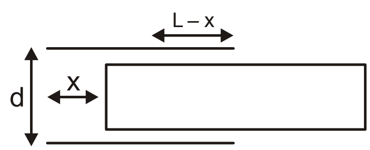
$\sum w_F = \Delta k$

$$\int_a^b \frac{-GMm}{r^{2.1}} dr = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$\frac{-GMm}{1.1}\left(\frac{1}{a^{1.1}} - \frac{1}{b^{1.1}}\right) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$\Rightarrow v^2 = \frac{-2GM}{1.1}\left(\frac{1}{a^{1.1}} - \frac{1}{b^{1.1}}\right) + u^2 \Rightarrow v^2 = \frac{2GM}{1.1}\left(\frac{1}{b^{1.1}} - \frac{1}{a^{1.1}}\right) + u^2$$

9. (A)

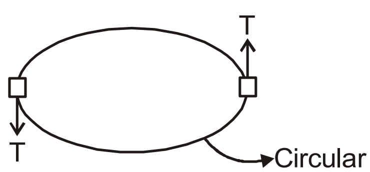


$$C = \frac{\epsilon_0 b}{d}\left[K\ell - (K-1)vt\right]$$

It is linear equation so capacitance decreases with constant rate.

10. (B)

$$\text{Angular impulse} = \int 2TR\,dt$$



By angular impulse theorem

$$2TRt = L_f - L_i$$

$$\Rightarrow 2TRt = mR^2\omega$$

$$F_{net} \text{ towards centre } = ma_N$$

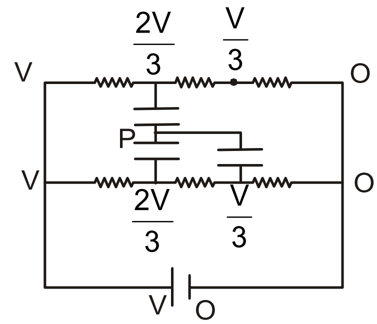
$$\Rightarrow mg = m\omega^2 R$$

$$\Rightarrow \omega = \sqrt{\frac{g}{R}}$$

$$\Rightarrow t = \frac{M\sqrt{Rg}}{2T}$$

11. **(C)**

Let potential of point P = x volt



Now, $\left(x-\frac{2V}{3}\right)C+\left(x-\frac{V}{3}\right)C+\left(x-\frac{2V}{3}\right)C=0$

$\Rightarrow 3x=\frac{4V}{3}+\frac{V}{3}=\frac{5V}{3}$

$\Rightarrow x=\frac{5V}{9}$

Potential difference in capacitor $C_3=\frac{5V}{9}-\frac{V}{3}$

$=\frac{2V}{9}$

$\therefore$ (C)

12. **(B)**

Water rise due to capillary action > H

$\Rightarrow \frac{2T}{\rho rg}>H$

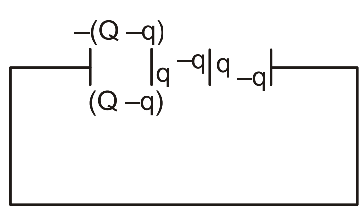
13. **(C)**

Due to connecting wire

$R_{xy}=2R$

14. **(D)**

Let Q charge is given to



$\frac{Q-q}{\epsilon_0 A}x$

$=\frac{q}{\epsilon_0 A}2x$

$q=\frac{Q}{3}$

15. (B)

$$r = R_{eq} \Rightarrow r = \frac{3R \cdot 6R}{9R}$$

$$\Rightarrow r = \frac{6R}{3} = 2R$$

$\therefore$ (B)

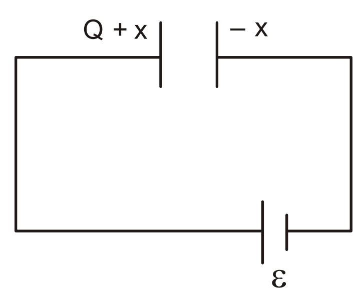
16. (B)

Initial extension of spring $= x_0 = \frac{mg}{k}$ just after collision of B with A the speed of combined mass is $\frac{v}{2}$. For spring to just attain natural length the combined mass must rise up by $x_0 = \frac{mg}{k}$

Now, $\frac{V}{2} = \sqrt{\frac{K}{2m}}\sqrt{\frac{4m^2g^2}{k^2} - \frac{m^2g^2}{k^2}}$

$$V = g\sqrt{\frac{6m}{k}}$$

17. (B)



Election field between the capacitor plated $= \frac{\sigma_1}{2\varepsilon_0} + \frac{-\sigma_2}{2\varepsilon_0}$

$$E = \frac{Q+x}{2A\varepsilon_0} + \frac{x}{2A\varepsilon_0}$$

Potential difference

$$\varepsilon = \frac{d}{2A\varepsilon_0}(Q+2x) \Rightarrow -x = \frac{Q}{2} - C\varepsilon$$

18. (C)

Let elongation in spring are x_1 and x_2.

Now, $x_1 + x_2 = 2x$

$3kx_1 = kx_2$

and $3kx_1 + kx_2 = k_{eq}x$

$$\Rightarrow T = 2\pi\sqrt{\frac{m}{k_{eq}}} = 2\pi\sqrt{\frac{m}{3k}}$$

19. (B)

We have $T = 2\pi\sqrt{\frac{R^3}{GM}}$

$V = A\omega$

$A_1 > A_2$

$\Rightarrow v_1 > v_2$

20. (A)

$$\frac{Q^2}{2 \times 4\pi\varepsilon_0 R} + 8\pi R^2 T = U$$

$$\frac{dU}{dR} = 0$$

$$\Rightarrow R^3 = \frac{Q_2}{8\pi\varepsilon_0 \cdot 16\pi T}$$

21. (A)

For circular orbit, $v = \sqrt{\frac{GM}{2R}}$ .....(i)

For elliptical orbit

$$\frac{1}{2}mv_0^2 - \frac{GMm}{2R} = -\frac{GMm}{4R}$$ ......(ii)

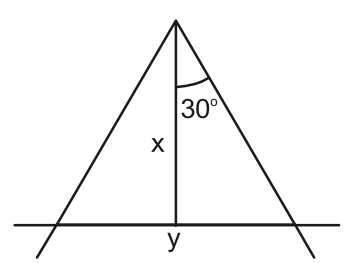
From (i) & (ii)

$v = v_0$

22. (A)

$y = 2x\tan 30^\circ$

$$y = \frac{2x}{\sqrt{3}}$$



Now,

$y\,2T = mg$

And $dy = \frac{2}{\sqrt{3}}\,dx$

$$\Rightarrow 2T dy = -ma$$

$$\Rightarrow 2T\frac{2}{\sqrt{3}}dx = -ma$$

$$\Rightarrow a = \frac{-4T}{\sqrt{3}m}dx$$

$$\Rightarrow \frac{-4T}{\sqrt{3}m} = \omega^2 \Rightarrow \omega = \sqrt{\frac{4T}{\sqrt{3}m}} = \frac{2\pi}{T'}$$

$$\Rightarrow T = \text{time period} = 2\pi\sqrt{\frac{\sqrt{3}m}{4T}} = \pi\sqrt{\frac{\sqrt{3}m}{T}}$$

23. (C)

For the given situation disc will perform translator motion in radius ℓ.

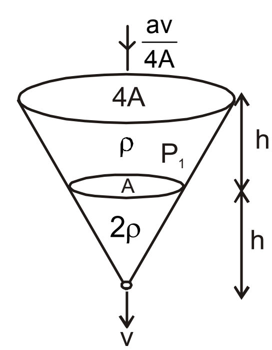
$$\therefore\ 2\pi\sqrt{\frac{\ell}{g}} = T$$

24. (D)

We have

$$P_0 + \frac{1}{2}2\rho v^2 = P_1 + 2\rho gh + \frac{1}{2}2\rho\left(\frac{av}{A}\right)^2 \qquad \text{......(i)}$$

$$P_0 + \frac{1}{2}\rho\left(\frac{av}{4A}\right)^2 + \rho gh = P_1 + 0 + \frac{1}{2}\rho\left(\frac{av}{A}\right)^2 \qquad \text{.......(ii)}$$



$$\Rightarrow v^2\left[1 - \frac{a^2}{32A^2} - \frac{a^2v^2}{A^2} + \frac{a^2v^2}{2A^2}\right] = 3gh$$

From (i) & (ii)

$$\Rightarrow v^2\left[1 - \frac{a^2}{32A^2} - \frac{a^2v^2}{2A^2}\right] = 3gh$$

$$\Rightarrow v = \sqrt{\frac{3gh}{1 - \frac{17a^2}{32A^2}}}$$

25. (B)

$$F = \left(P_0 - P_{average}\right)\ell h$$

$$F = \left(\frac{2T}{d} - \frac{\rho gh}{2}\right)\ell h$$

$$h = \frac{2T}{\rho gd} \Rightarrow F = \frac{2T^2\ell}{\rho gd^2}$$

26. (B)

At some distance from centre inside core

$$F = \frac{G\frac{4}{3}\pi r^3(3\rho)m}{r^2}$$

$$ma = -4\pi G\rho mr$$

$$a = -4\pi G\rho mr$$

$$\Rightarrow \omega = \sqrt{4\pi G\rho} = \frac{2\pi}{T} \quad \Rightarrow T = 2\pi\sqrt{\frac{1}{4\pi G\rho}} = \sqrt{\frac{\pi}{G\rho}}$$

$$t_{AB} = \frac{T}{2} = \frac{1}{2}\sqrt{\frac{\pi}{G\rho}}$$

27. (A)

Let they will be in phase after time t then

$$\frac{t}{3} - \frac{t}{7} = \frac{1}{2} \Rightarrow t = \frac{21}{8}s$$

28. (C)

$$I = \frac{E}{R_1 + R_2}$$

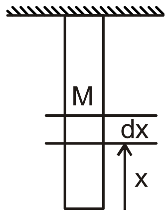
$$E_1 = \frac{1}{2}C_1\left(\frac{E}{R_1} + \frac{R_1}{R_2}\right)^2$$

$$E_2 = \frac{1}{2}C_2\left(\frac{E}{R_1} + \frac{R_2}{R_2}\right)^2$$

$$\Rightarrow \frac{E_1}{E_2} = \frac{C_1}{C_2}\frac{R_1^2}{R_2^2}$$

29. **(B)**

Tension in rod at a distance x from lower end is $\frac{mgx}{\ell}$



Now,

$$Y\frac{dy}{dx}=\frac{T}{A}=\frac{mgx}{\ell A}$$

$$\int_0^{y} Y\,dy=\int_{30}^{70}\frac{mg}{\ell A}x\,dx$$

$$Y\,y=\frac{mg}{\ell A}\left(\frac{70^2-30^2}{2}\right)$$

$$\Rightarrow\ y=\frac{mg}{AY}\times 20$$

$$\therefore\ \text{Total length}=\left(40+\frac{20mg}{AY}\right)\text{cm}$$

30. **(B)**

$$F=P\left(\text{at centroid}\right)\times\text{Area}$$

$$=\left(P_0+\frac{h+h+a+h+a}{3}g\rho\right)\frac{ab}{2}$$

$$F=\left[P_0+\left(h+\frac{2a}{3}\right)g\rho\right]\frac{ab}{2}$$

$$=\left(P_0+\rho gh\right)\frac{ab}{2}+\frac{a^2bg\rho}{3}$$

$$\Rightarrow\ F=\left(P_0+\rho gh\right)\frac{ab}{2}+\frac{a^2b\rho g}{3}$$

CHEMISTRY

31. (C)

For a solution to have twice alkalinity, we have

$[OH^-] = 2\times10^{-7}M$

pOH = 7–log 2= 7–0.3=6.7

pH= 14–6.7 = 7.3

32. (C)

Increase in pressure will affected the amount of B_3A equilibrium. Change in temperature always affects the equilibrium.

33. (D)

$$pH = pK_a + \log\frac{[salt]}{[Acid]}$$

= 4.74 + 0.02

= 4.76

34. (B)

$$A(s) \rightleftharpoons B(g) + 2C(g)$$

$t = O$ –   O   O

$t = teq^m$ –   P   2P

P_{Total} = P + 2P = 3P = 15

P = 5

$K_p = P_B.\,(P_c)^2 = P\,(2P)^2$

$= 5 \times (10)^2 = 500\ atm^3$

35. (C)

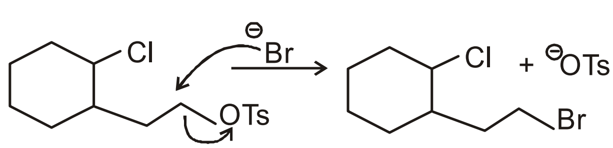
$$\frac{h}{1-h} = \sqrt{K_h} = \sqrt{\frac{K_w}{K_a, K_b}} = \sqrt{\frac{10^{-4}}{\left(3\times10^{-7}\right)^2}}$$

$$\frac{h}{1-h} = \frac{1}{3} \Rightarrow h = 0.25$$

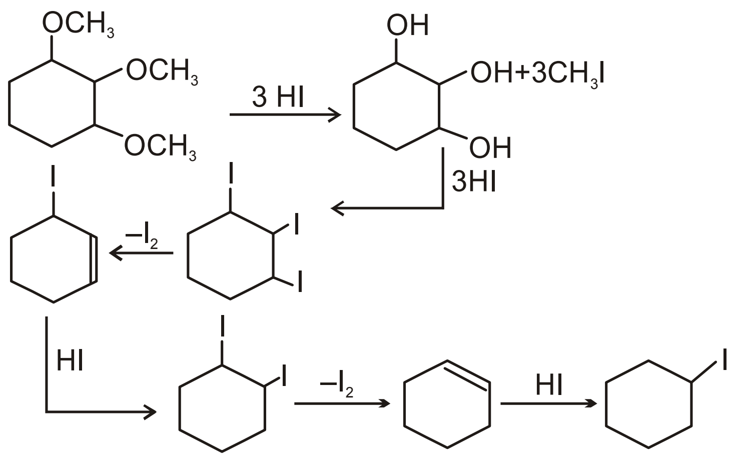
% hydrolysis = 0.25 × 100 = 25%

36. (A)

In DMSO SN^2 is favourable. SN^2 reactions are faster in primary halide then secondary halide



37. (C)



38. (A)

(A) diphenyl ether do not cleaved by HI.

(B) $CH_2=CH-O-CH_2-CH_3 \xrightarrow{HI} CH_2=CH-\overset{H}{\underset{\oplus}{O}}-CH_2-CH_3$

$\downarrow I^{\ominus}$

$CH_2=\underset{}{\overset{OH}{C}}H+CH_3-CH_2-I$

$\downarrow$ tautomorises

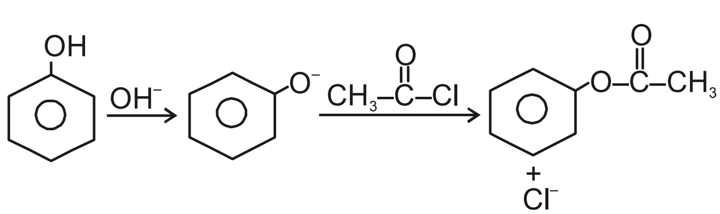
CH_3CHO

final product is CH_3CHO & CH_3-CH_2-I

(C) $Ph-CH=CH_2 \xrightarrow[2.H_2O_2/OH^-]{1.B_2H_6} Ph-CH_2-CH_2-OH$ (major) 97%

\+

$Ph-\underset{OH}{CH}-CH_3$ (minor) (2-4%)

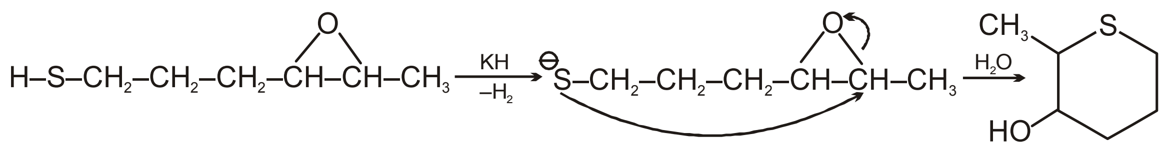
(D) 

39. (D)

$$CHCl_3 + KOH \longrightarrow :\bar{C}Cl_3 \xrightarrow{-Cl^-} :CCl_2 + KCl + H_2O$$

here : CCl_2 as electrophile is present

40. (C)



41. (D)

(A) is based on the difference in wettability of different minerals.

(B) uses sodium ethyl xanthate as collector

(C) uses NaCN as depressed in the mixture of ZnS and PbS When ZnS forms soluble complex and PbS forms froth.

42. (A)

Pig iron consist 2.5% carbon and 0.03% As, S, P, Si as a impurities.

43. (B)

Contains more impurity than the original metal

44. (B)

$$\underset{\text{Basic}}{FeO} + \underset{\text{Acidic}}{SiO_2} \longrightarrow \underset{\text{Slage}}{FeSiO_3} \downarrow$$

45. (A)

$$HgS + HgO \longrightarrow Hg + SO_2 \text{[Self reduction)}$$

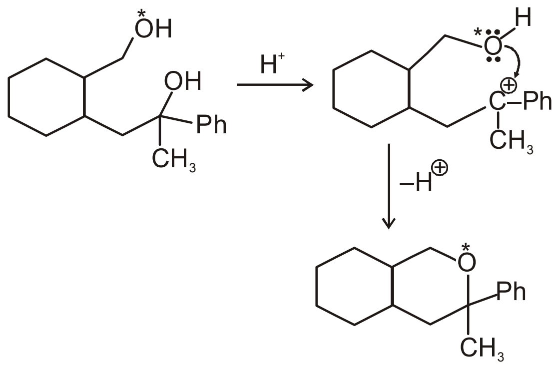
46. (C)

The aromatic halogen compound which contain –R/–M effect showing group at ortho & para position is highly reactive by Aromatic nucleophilic substitution reaction.

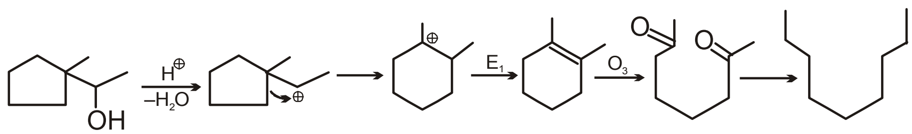
47. (D)

Because $CH_3-C(CH_3)_2-O^-$ is stronger base than $CH_3-\overset{O}{\overset{\|}{C}}-O^-$.

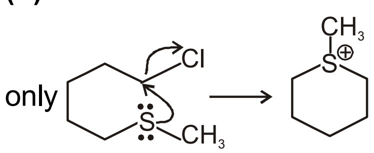
48. (B)



49. (C)



50. (A)



In case of 'b' & 'd' nudcleophilic substitution is not possible at vinylic carbon 'C' also not undergo NGP effect.

51. (B)

In an aq. solution of salt containing amphiprotic anion

$$pH = \frac{1}{2}(PK_2 + PK_3)$$

$$= \frac{1}{2}(-\log K_2 - \log K_3) = \frac{1}{2}\left(-\log 6\times 10^{-8} - \log 1\times 10^{-12}\right) = \frac{1}{2}(8 - \log 6 + 12)$$

$$= \frac{20-0.78}{2} = \frac{19.22}{2} = 9.61$$

52. (B)

$$\text{Mobs.} = \frac{M_{Theo.}}{1+(n-1)\alpha} = \frac{100}{1+(2-1)\alpha}$$

$$75 = \frac{100}{1+\alpha}$$

$$1+\alpha = \frac{100}{75} = \frac{4}{3} = 1.33$$

$$\alpha = 0.33$$

53. (B)

$$\alpha = \frac{D-d}{(n-1)d}$$

$$\frac{D}{d} = 1+(n-1)\alpha$$

$\therefore$ One mole of reactant should produce n mole of product.

54. (C)

In an ag. solution

$[Ba^{2+}] = \left[SO_4^{2-}\right] = 4\times10^{-5}$ M

$K_{sp}(BaSO_4) = (4\times10^{-5})^2 = 1.6\times10^{-9}\ M^2$

$\Rightarrow$ In 0.2 M Na_2SO_4 solution, the concentration of $SO_4^{2-} = 0.2M$

$$\therefore \left[Ba^{2+}\right] = \frac{K_{sp}\left[BaSO_4\right]}{\left[SO_4^{2-}\right]} = \frac{1.6\times10^{-9}}{0.2} = 8\times10^{-9}$$

55. (C)

$$K_c = \left(\frac{mole}{L}\right)^{\Delta n};\Delta n = -2$$

$\therefore$ On increasing pressure reaction will ge shifted in forward direction.

56. (A)

Ellingham diagram represents change of ΔG with temperature:

$\Delta G = \Delta H - T\Delta S$

57. (D)

$$\xrightarrow[\text{Polarization}\uparrow \ \text{Thermal stability}\downarrow]{BaCO_3 > SrCO_3 > CaCO_3 > MgCO_3}$$

58. (A)

$$Mg_2C_3 + 4H_2O \longrightarrow 2Mg(OH)_2 + \underset{\downarrow}{C_3H_4}$$

$$\overset{3-}{C} - \underset{\sigma}{C} \overset{\pi\pi}{\equiv} \overset{1-}{C}$$

$= 2\sigma, 2\pi$ Bonds

59. (B)

$$\xrightarrow[\text{Solubility}\uparrow]{NaHCO_3 < KHCO_3 < RbHCO_3}$$

60. (C)

MATHEMATICS

61. **(B)**

Let number of terms = n, then

1000 = 1 + (n – 1)d

$\therefore n-1=\frac{999}{d},$ as $999=33\times 37$

Total 8 divisors.

But when d = 999 it is rejected so total 7 A.P.S.

62. **(B)**

$(x-iy)=i(x+iy)^2=-2xy+i(x^2-y^2)$

$\therefore -2xy=x$ & $x^2-y^2=-y$

So, $x=0$ or $y=-\frac{1}{2}$ $\Rightarrow$ non zero solutions are

$z=1, \frac{\sqrt{3}-i}{2}, \frac{-\sqrt{3}-i}{2}$

63. **(B)**

$2\alpha+\beta=\frac{15}{2}$

$2\alpha\beta+a^2=18$

$\therefore\ 2\alpha\left(\frac{15}{2}-2\alpha\right)+\alpha^2=18$

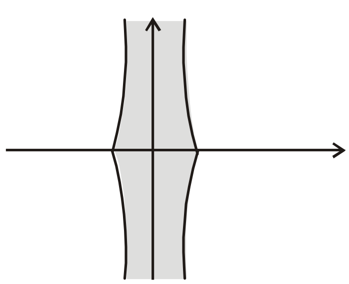
$\alpha^2-5\alpha+6=0$

$\therefore \alpha=2$ or 3. then $\beta=\frac{7}{2}$ & $\frac{3}{2}$ respectively

$\therefore k=28$ & 27

64. **(A)**

By Graph, required area is 4 sq. units.



65. (B)

Given expression is $\frac{S_{2n}-S_n}{S_n}=k$

$\frac{2+(2n-1)d}{2+(n-1)d}=k \qquad \therefore nd(k-2)=k(d-2)-(d-2)=(k-1)(d-2)$

Equality holds when d = 2

66. (D)

equation of ellipse is

$(x-\alpha)^2+(y-\beta)^2=e^2\frac{(\ell x+my+n)^2}{\ell^2+m^2}$

as e is known, there are 4 arbitrary constants.

67. (A)

For $f(x)=ax^2+bx+c$

$f(0)=c$

$f(1)=a+b+c$

So, $f(0)\cdot f(1)<0$

i.e., root exists between 0 & 1 so D > 0

68. (A)

$c=(m^2+4)a^2+(m-2)^2a-4m+2$

$\Rightarrow (a^2+a)m^2-4(a+1)m+4a^2+4a+2-c=0$

must be an identity in m

$\therefore a=-1$ and $c=2$

69. (B)

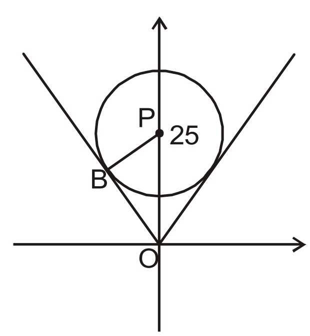
The required Quadratic is

$x^2-(q+q^2+q^3+q^4+q^5+q^6)x+(q+q^2+q^4)\left(\frac{1}{q^4}+\frac{1}{q^2}+\frac{1}{q}\right)=0$

i.e. $x^2+x+2=0$

$\therefore$ D = –7

70. (C)



$\angle POB=\sin^{-1}\frac{15}{25}=\sin^{-1}\frac{3}{5}$

$\therefore \arg(z)_{max}=\frac{\pi}{2}+\sin^{-1}\frac{3}{5}$

$\angle POB=\sin^{-1}\frac{15}{25}=\sin^{-1}\frac{3}{5}$

$\therefore \arg(z)_{max} = \frac{\pi}{2} + \sin^{-1}\frac{3}{5}$

71. **(C)**

$\frac{a^2}{1-r^2} = 10 \cdot \frac{a}{1-r}$ ......(i)

$\frac{a}{1-r} = 2017$ ......(ii)

On solving $r = \frac{2007}{2027}$

72. **(C)**

$\sum\alpha = 0\ ;\ \sum\alpha\beta = -\frac{1}{7},\ \alpha.\beta.\gamma = \frac{2}{7}$

$(\alpha+\beta+\gamma)\left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right) = 3 + \sum\left(\frac{\alpha}{\beta}+\frac{\beta}{\gamma}\right) = 0$

$\therefore \sum\left(\frac{\alpha}{\beta}+\frac{\beta}{\gamma}\right) = -3$

73. **(C)**

Let the elements of set be

$a, a+1, a+2, ..., a+(n-1)$

$a + a + 1 + a + 2 + ... + a + (n-1) = 35$

$\Rightarrow \frac{n}{2}\left[2a + (n-1)\right] = 35 \qquad \Rightarrow n^2 + (2a-1)n - 70 = 0$

$a \in I$ roots of the above equation must be positive integer ≥ 2

$\therefore$ D should be a perfect square

$\Rightarrow (4a-1)^2 + 280 = k^2 \ \Rightarrow k^2 - (4a-1)^2 = 280$

$(k - 2a + 1)(k + 2a - 1) = 280$

Possible positive integral values of a = 17,5,2 i.e. 3 sets.

74. **(B)**

$(i)^{\frac{1}{i}} = \left(e^{i\frac{\pi}{2}}\right)^{\frac{1}{i}} \Rightarrow e^{\frac{\pi}{2}}$

75. **(C)**

Let $z = x + iy$

then, $\left(x + \sqrt{x^2 + y^2}\right) + iy = 2 + 8i$

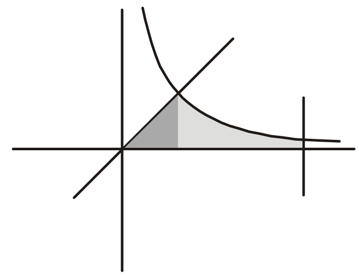
or $x + \sqrt{x^2 + 64} = 2$

$x^2 + 64 = 4 - 4x + x^2$

$\therefore x = -15$

$z = -15 + 8i$

76. **(B)**



Area of tringle $= \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

Total area $= \frac{1}{2} + \int_1^4 \frac{1}{x} dx = \frac{1}{2} + \ell n4$

77. **(C)**

This can be reduced to,

$y = A.\sin(x + B) - D \times \ell n(E \cdot x)$

thus order = 4

78. **(B)**

$\left(y\cos x + 2xe^y\right)dx + \left(\sin x + x^2e^y - 1\right)dy = 0$ is of the form Mdx + Ndy = 0

as $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, so integrating w.r.t x and y respectively.

$y\sin x + x^2e^y - y = c$

79. **(A)**

Re-writing as,

$\frac{1}{y^{2017}} \cdot \frac{dy}{dx} - \frac{1}{y^{2016}} \cdot \frac{1}{x} = 1$ Let $\frac{1}{y^{2016}} = t$

$\frac{dt}{dx} + 2016 \cdot \frac{t}{x} = -2016$ $\Rightarrow \frac{-2016}{y^{2017}}\frac{dy}{dx} = \frac{dt}{dx}$

$I.F = e^{\int \frac{2016}{x}dx} = x^{2016}$

$\therefore$ solution $\Rightarrow \frac{1}{y^{2016}} \cdot x^{2016} = \frac{-2016}{2017}x^{2017} + c$

80. (A)

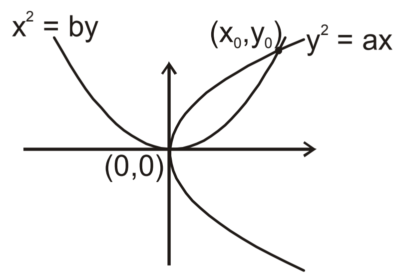
$\frac{dx}{dy} = \frac{e^{2x}}{y^3} + \frac{1}{y}$ Let $u = e^{-2x} \Rightarrow \frac{du}{dy} = -2u\frac{dx}{dy}$

$\therefore -\frac{1}{2u}\frac{du}{dy} = \frac{1}{y^3.u} + \frac{1}{y}$ or $\frac{du}{dy} + \frac{2u}{y} = -\frac{2}{y^3}$

$\because e^{\int 2\frac{dy}{y}} = e^{2\ln y} = y^2$ $\therefore y^2u = -2\ln y + c$

81. (C)

$y^2 = ax$ & $x^2 = by$ is $\frac{16}{3}$ sq.units



Consider a > 0, b > 0

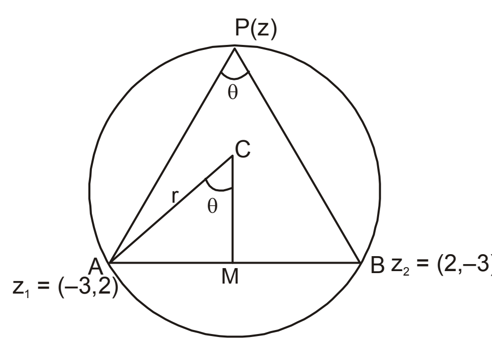
Solving $x^2 = by$

& $y^2 = ax$ we get x = 0, $\left(ab^2\right)^{\frac{1}{3}}$

Now area $= \int_0^{x_0}\left(\sqrt{ax} - \frac{x^2}{b}\right)dx = \frac{16}{3} \Rightarrow ab = 16$

82. (B)

$\arg\left(\frac{z-z_1}{z-z_2}\right) = \cos^{-1}\left(\frac{1}{\sqrt{10}}\right) = \tan^{-1}(3)$



In ΔAMC,

$\sin\theta = \frac{AM}{r} = \frac{AB}{2r}$

$\frac{3}{\sqrt{10}} = \frac{5\sqrt{2}}{2r}$ $\therefore r = \frac{5\sqrt{5}}{3}$

83. (A)

$$x^5 - 40x^4 + ax^3 + bx^2 + cx + d = 0 \quad \frac{a}{r^2},\ \frac{a}{r},\ a,\ ar,\ ar^2$$

$$a\left(\frac{1}{r^2} + \frac{1}{r^2} + 1 + r + r^2\right) = 40 \qquad ....(i)$$

$$\frac{1}{a}\left(r^2 + r^2 + 1 + \frac{1}{r} + \frac{1}{r^2}\right) = 10 \qquad ....(ii)$$

$$\frac{(i)}{(ii)} \Rightarrow a^2 = 4 \Rightarrow a \pm 2$$

$$d = -a^5 = \pm 32$$

84. (D)

$$\text{Area} = \int_0^a \left(\sqrt{a} - \sqrt{x}\right)^2$$

$$= \int_0^a a + x - 2\sqrt{ax}\,dx = \frac{a^2}{6} \text{ sq. units}$$

85. (D)

This is not always a prime.

for p = 11, $2^{11} - 1 = 2047 = 23 \times 89$ is not a prime.

86. (A)

$$k = \frac{x\left(x^6 + 1\right)}{x^2 + 1} \qquad \therefore x^6 + 1 = k\left(\frac{x^2 + 1}{x}\right) \ge 2k$$

So, $2k - x^6 \le 1$

87. (D)

$$\frac{4}{x} + \frac{9}{y} + \frac{16}{z} = \left(\frac{4}{x} + \frac{9}{y} + \frac{16}{z}\right)(x + y + z)$$

≥ 81 (Use A.M. $\ge$ G.M.)

88. (A)

$$\int_0^9 \sqrt{x}dx = \frac{2}{3}x^{\frac{3}{2}}\Big]_0^9 = \frac{54}{3} = 18$$

Now, $\int_0^a \sqrt{x}dx = 9 \;\therefore \frac{2}{3}a^{\frac{3}{2}} = 9$

$$\therefore a = \left(\frac{27}{2}\right)^{\frac{2}{3}} = \frac{9}{2^{\frac{2}{3}}}$$

89. (C)

$f(x) = (x - x_1)(x - x_2)(x - x_4)(x - x_4)$

$|f(i)| = 1 \Rightarrow \sqrt{\left(1+x_1^2\right)\left(1+x_2^2\right)\left(1+x_3^2\right)\left(1+x_4^2\right)} = 1$

$\Rightarrow x_1 = x_2 = x_3 = x_4 = 0$

$\therefore\; a + b + c + d = 0$

90. (C)

making $D \le 0$ of quadratic in x

$\therefore 3y^2 - 2xy + 4 - h^2 \ge 0$

$\Rightarrow 4h^2 - 4 \times 3\left(4 - h^2\right) \le 0$

$\therefore h \in \left[-\sqrt{3}, \sqrt{3}\right]$

Sum of Integer = –1 + 0 + 1 = 0

JEE ADVANCED

PHYSICS

1. **(B)**

Let $V_0 = \sqrt{2gh}$. Then COLM gives

$$mV_0 = (2m + m)v$$

$$\Rightarrow \quad v = \frac{v_0}{3}.$$

Distance of initial position to equilibrium position is $x = \frac{3mg}{k} - \frac{2mg}{k} = mg/k$.

We have $v = \omega\sqrt{A^2 - x^2}$

$$\Rightarrow \quad \frac{v_0}{3} = \omega\sqrt{A^2 - x^2}.\ \text{But}\ \omega = \sqrt{\frac{K}{3m}}.$$

$$\Rightarrow \quad A = \frac{2mg}{K}.\ \left[\text{Use } v_0 = \sqrt{2g \times \frac{4.5mg}{k}}\right]$$

Hence time taken to reach mean position is given as

$$x = A\sin\omega t$$

$$\Rightarrow \quad \frac{mg}{k} = \frac{2mg}{k}\sin\omega t.$$

$$\Rightarrow \quad t_1 = \frac{\pi}{6\omega}$$

And time taken to reach from mean position to maximum, compression is

$$= \frac{T}{4} = \frac{\pi}{2\omega}$$

Hence total time

$$\Delta T = \frac{\pi}{6\omega} + \frac{\pi}{2\omega}$$

= 20π (ms) Ans.

2. **(A)**

Total force = $P_0 \times \pi r^2$ + weight of liquid above the sphere.

3. (A)

$I_{shell} + I_{element} = 0$

$\therefore \ |I_{shell}| = I_{element}$

$$= \frac{G.(\sigma.\Delta A)}{\left(R + \frac{R}{2}\right)^2}$$

4. (C)

Let potential of junction is V. Then current law at junction gives

$$\frac{12 - V}{3} + \frac{6 - V}{2} = \frac{V - 0}{1}$$

$$\Rightarrow \ V = \frac{42}{11} \text{volt}$$

$$\therefore \ i_3 = \frac{V}{R} = \frac{V}{I} = \frac{42}{11} \text{Amp.}$$

5. (D)

$$3\frac{1}{2}m\omega^2 \left(A^2 - x^2\right) = \left(\frac{1}{2}m\omega^2 x^2\right)$$

$$\Rightarrow \ x = \frac{\sqrt{3}}{2} A = A \sin \omega t.$$

$$\therefore \ t = \frac{2}{3} \text{ sec.}$$

6. (D)

Number of droplets is $n = \frac{R^3}{r^3}$

According to question,

Energy released = K.E. of bigger drop

$$\therefore \quad \frac{R^3}{r^3} \times 4\pi r^2.T - 4\pi R^2 T = \frac{1}{2}.\rho.\frac{4}{3}\pi R^3.v^2$$

$$\therefore \quad v = \sqrt{\frac{6T(R - r)}{\rho r R}}$$

7. (B)

$Q_i = C(V_1 + V_2)$ $\quad U_i = \frac{1}{2}C(V_1 + V_2)^2$

$Q_f = CV_1$ $\quad U_f = \frac{1}{2}CV_1^2$

$\therefore \quad \Delta Q = CV_2$

Hence heat generated

$$H = CV_1 \times V_2 - \left[\frac{1}{2}C(V_1 + V_2)^2 - \frac{1}{2}CV_1^2\right]$$

$$= -CV_1V_2 + \left[\frac{1}{2}CV_1^2 + \frac{1}{2}CV_2^2 + CV_1V_2 - \frac{1}{2}CV_1^2\right]$$

$$= \frac{1}{2}CV_2^2$$

Which is independent of V_1

8. (B)

First ring have two symmetrical section as second ring. Hence as far as net upward field in two cases are concerned, we have $2I_2 \cos\phi = I_1$

$$\therefore \quad \frac{I_1}{2I_2} = \cos\phi$$

9. (A)

$$R_1 = \frac{200 \times 200}{60} = \frac{2000}{3}\Omega$$

$$R_2 = \frac{200 \times 200}{100} = 400\Omega$$

$$\therefore \quad R_{net} = \frac{3200}{3}$$

$$\therefore \quad i = \frac{200 \times 3}{3200} = \frac{3}{16}\text{amp.}$$

$$\therefore \quad P = i^2 R_{net}$$

$$= \frac{3}{16} \times \frac{3}{16} \times \frac{3200}{3} = 37.5 \text{ watt.}$$

10. (B)

Net downward force on cube

F_1 = wt. of water + self weight

$$= (P_0 + \rho_w ga)a^2 + \left(a^3 - \frac{a}{2} \times \frac{a}{2} \times a\right)\rho_{cube}.g$$

$$= \frac{7}{2} \times 10^7 N.$$

And net upward force by water

$$F_2 = \left(\frac{a}{2} \times a\right)\left(P_0 + \rho_w.g.\frac{3a}{2}\right)$$

$$= 1.25 \times 10^7 N$$

$\therefore$ Normal reaction

$$N = F_1 - F_2$$

$$= 2.25 \times 10^7 \text{ N}$$

11. (A)

If charge an sphere is Q, then $\frac{Kq}{d+x} + \frac{KQ}{R} = 0$

$$\Rightarrow \quad Q = -\frac{Rq}{d+x}$$

(d + x) will decrease as a result $-$ve charge on sphere will increases. Hence current (flow of positive charge) will be from C to C'

12. (A)

$$Q = -\frac{Rq}{d+x}$$

$$\therefore \quad i = \frac{dQ}{dt} = -Rq\left\{-\frac{1}{(d+x)^2}\frac{dx}{dt}\right\}$$

$$= \frac{Rq}{(d+x)^2}.v$$

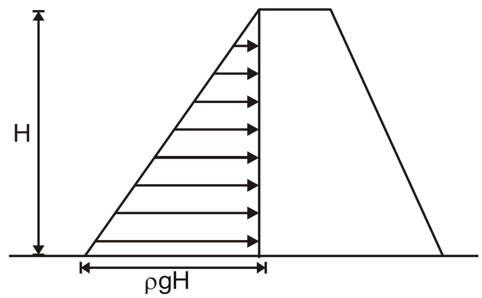
But for maximum current $v = \omega a$

$(d+x)^2 \simeq d^2$ (for small amplitude)

$$\therefore i_{max} = \frac{Rq\omega a}{d^2}$$

13. (C)

Pressure at the base of wall is ρgH. The force per unit length of the water an wall is equal to the area of the pressure diagram.



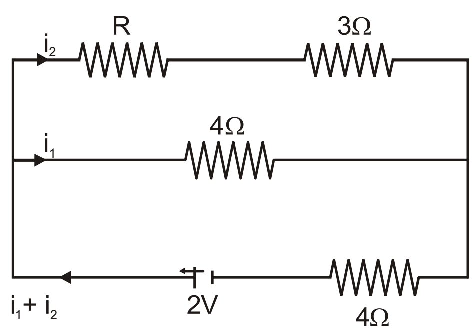
$$\therefore \quad \frac{F}{\ell} = \frac{1}{2}.(\rho gH) \times H.$$

$$\therefore \quad F = \frac{1}{2}\rho gH^2\ell$$

14. (B)

The point of application of the resultant force passes through the centroid of the pressure diagram i.e. at $\frac{H}{3}$ from base.

15. (C)



We have $2 - 4(i_1 + i_2) = 4i_1 = (3 + R)\, i_2$

Getting the value of i_2 from above relation, we have maximum power

in R if $\frac{dp}{dR} = 0$

$R = 5\Omega$

16. (D)

$V_T = \varepsilon - ir$

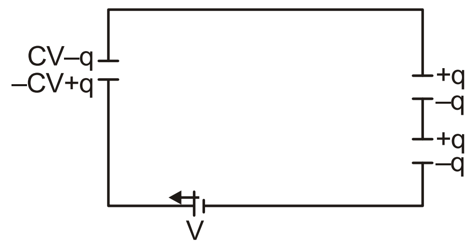
But $i = i_1 + i_2 = \frac{3}{10}$ amp.

$\therefore \quad V_T = 2 - 4 \times \frac{3}{10}$ = 0.8 volt

17. (C)

Let q charge flown through switch.

Then circuit will be as shown in figure for which KVL gives



$$-V + \frac{q - CV}{C} + \frac{q}{C} + \frac{q}{C} = 0$$

$\Rightarrow \frac{3q}{C} = 2V. \qquad \Rightarrow q = \frac{2}{3}CV$

Hence $Q_1 = CV - q = \frac{CV}{3}$ $\qquad P \to (4)$

Final charge on $C_2 = q = \frac{2CV}{3}$ $\qquad R \to (3)$

W.D. by battery V_2 is = $V \times q = \frac{2CV^2}{3}$ $\qquad Q \to (1)$

Change in energy of capacitor system is

$$\Delta U = 2 \times \frac{4CV^2}{18} + \frac{CV^2}{18} - \frac{1}{2}CV^2 = 0$$

Hence heat produced = W.D. by battery = $\frac{2CV^2}{3}$ $\qquad S \to (1)$

18. (D)

$P.E. = mgh = mg\ell(1 - \cos\theta)$

K.E. = loss in P.E.

$= mg\ell(1 - \cos\theta_0) - mg\ell(1 - \cos\theta)$

$= mg\ell(\cos\theta - \cos\theta_0)$

Momentum $mv = \sqrt{2mk}$

For P.E. = K.E.

$$mg\ell(1-\cos\theta) = mg\ell(\cos\theta - \cos\theta_0)$$

$$\therefore \quad \cos\theta = \frac{1+\cos\theta}{2}$$

19. **(C)**

20. **(D)**

If spring is not attached then block will oscillate only for half period.

CHEMISTRY

21. **(B)**

$$2Na(g) \rightleftharpoons Na_2(g)$$

$$P_{Na} = X_{Na}.P_{Total} = \frac{20}{100} \times 10 \text{ bar} = 2 \text{ bar}$$

$$P_{Na_2} = X_{Na_2}.P_{Total} = \frac{80}{100} \times 10 \text{ bar} = 8 \text{bar}$$

$$K_p = \frac{P_{Na_2}}{(P_{Na})^2} = \frac{8}{(2)^2} = 2$$

22. **(A)**

	2NaOH +	$CuSO_4 \longrightarrow$	$Cu(OH)_2$ +	Na_2SO_4
ini.mmoles:	500x	500×0.6		
mmoles: left	$500x - 600$	0	300	300

Initial pOH = – log x

$$\text{Final pOH} = -\log\frac{500x-600}{1000}$$

$$pOH_f - pOH_i = -\log\frac{500x-600}{1000}.\frac{1}{x} = 1 = \log 10$$

$$x = \frac{3}{2}$$

2x = 3

23. (B)

$$H_3PO_4 = \frac{0.98}{98} = 10^{-2}\,\text{Mole}$$

$$\left[H_3PO_4\right] = \frac{10^{-2}}{100} \times 1000 = 0.1\,M$$

pH = 5 ; [H+] = 10–5

$\Rightarrow H_3PO_4$ & $H_2PO_4^-$

$$pH = PK_{a_1} + \log\frac{\left[H_2PO_4^-\right]}{\left[H_3PO_4\right]} \Rightarrow 5 = 3 + \log\frac{\left[H_2PO_4^-\right]}{\left[H_3PO_4\right]}$$

$$\because \left[H_3PO_4] + [H_2PO_4^-\right] = 0.1$$

$$\therefore \left[H_3PO_4] = 10^{-3}M \quad \& \quad [H_2PO_4^-\right] = 10^{-1}M$$

For $H_2PO_4^-$ & HPO_4^{2-}

$$pH = PK_{a_2} + \log\frac{\left[HPO_4^{2-}\right]}{\left[H_2PO_4^-\right]}; 5 = 8 + \log\frac{\left[HPO_4^{2-}\right]}{\left[H_2PO_4^-\right]}$$

$$\because \left[H_2PO^-\right] + \left[HPO_4^{2-}\right] = 0.1$$

$$\left[H_2PO_4^-\right] = 0.1M \;\&\; \left[HPO_4^{2-}\right] = 10^{-4}\,M$$

$\Rightarrow$ For HPO_4^{2-} & PO_4^{3-}

$$pH = PK_{a_3} + \log\frac{\left[PO_4^{3-}\right]}{\left[HPO_4^{2-}\right]}$$

$$5 = 12 + \log\frac{\left[PO_4^{3-}\right]}{\left[HPO_4^{2-}\right]}$$

$$\because \left[HPO_4^{2-}\right] + \left[PO_4^{3-}\right] = 10^{-4}$$

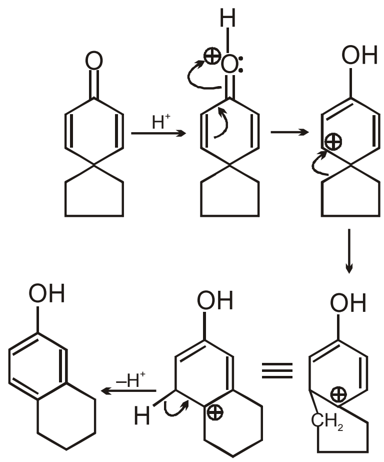
$$\therefore \left[PO_4^{3-}\right] = 10^{-11}M$$

$$\& \left[HPO_4^{2-}\right] = 10^{-4}M$$

24. (A)

Common pressure in the vessels just after opening the valve is 4.5 atm so first reaction moves in forward direction and second reaction moves in reverse direction. After a long time first reaction goes to completion and only second equilibrium exist. So final pressure is 4 atm.

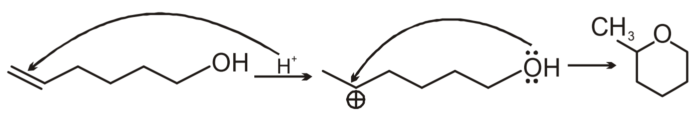
25. (D)



26. (A)

Rate of solvolysis $\propto$ stability of carbocation.

27. (A)



28. (A)

29. (C)

30. (C)

In roasting ore heated in the presence of air.

31. (C)

	$2CH_3COOH \rightleftharpoons$	$(CH_3COOH)_2$
Initial	0.1 M	0
Eqn.	(0.1 – 2x)M	x M

$$K = \frac{[(CH_3COOH)_2]}{[CH_3COOH]^2}$$

or $2 \times 103 = \dfrac{x}{(0.1-2x)^2} \Rightarrow x = 0.05\ \&\ [CH_3COOH] = \dfrac{1}{200}$

$$\therefore \frac{[(CH_3COOH)_2]}{[CH_3COOH]} = \frac{0.05}{\frac{1}{200}} = 10:1$$

32. (C)

	$2CH_3COOH$	$\rightleftharpoons$	$(CH_3COOH)_2$
Initial	0.1 M		0
Eqn.	(0.1 – 2x)M		x M

$$K = \frac{[CH_3COOH]}{[CH_3COOH]^2}$$

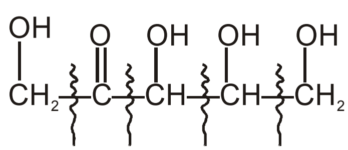
$$\text{or } 3.6 \times 10^{-3} = \frac{x}{(0.1-2x)^2} \approx \frac{x}{(0.1)^2}$$

$$\therefore x = 3.6 \times 10^{-5}$$

$$\frac{[(CH_3COOH)_2]}{[CH_3COOH]} = \frac{x}{0.1-2x} \approx \frac{x}{0.1}$$

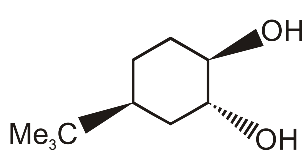
$$= \frac{3.6\times10^{-5}}{0.1} = \frac{9}{25000}$$] Instability constant is the reverse of formation constant.

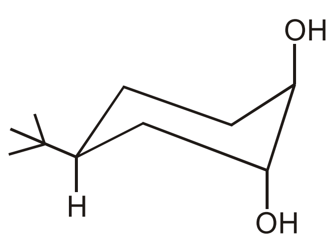
33. (B)



CH_3–OH is not formed, only oxidised product is formed. ($2HCHO + 2HCOOH + CO_2$)

34. (C)





both –OH gr are far distant so
cyclic intermediates are not formed

35. (C)

36. (B)

Solubility product of Mg^{2+} is less than of Ca^{2+} ion

37. (B)

(A — R); (B — R); (C — Q); (D — P)

(A) $\Delta ng > 0$

When   the backward shifting takes place. Total pressure even after shifting remain same.

(B) $\Delta ng < 0$, when V then backward shift will take place but $P_{final} < P_{initial}$

(C) No change but $P_{final} < P_{initial}$ as volume has increased

(D) Forward shifting will take place and $P_{final} < P_{initial}$

(C) No change but $P_{final} < P_{initial}$ as volume has increased

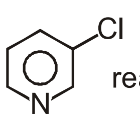
(D) Forward shifting will take place and $P_{final} < P_{initial}$

38. (C)

(A — S); (B — Q, R); (C — P); (D — P)

In case of KOH phenol changes into $Ph-\bar{O}\overset{+}{K}$ due to larger size of K+, product formed is para but in case of NaOH, $Ph\bar{O}\overset{+}{Na}$ is formed and smaller size of Na+ leads to ortho derivative

For Salicyldehyde reagent is $CHCl_3$/NaOH

For 

reagent is $CHCl_3$/NaOH

39. (D)

(A — S); (B — R); (C — P); (D — Q)

Given compound mainly undergo solvolysis by S_N^1 path.

So rate of solvolysis α stability of carbocation

40. (B)

(A-Q; B-S; C-R; D-P)

MATHEMATICS

41. (A)

$f(1) = a + b + c$

$f(-2) = 4a - 2b + c$, Hence $f(1) - f(-2) = 3(b - a)$

So $E = \dfrac{a+b+c}{b-a} = \dfrac{3f(1)}{f(1)-f(-2)} = \dfrac{3}{1-\dfrac{f(-2)}{f(1)}}$

Hence E_{min} occurs when $f(-2) = 0$ $\left(\text{as } f(x) \geq 0, \forall x \in R\right)$

$\therefore$ E_{min} = 3

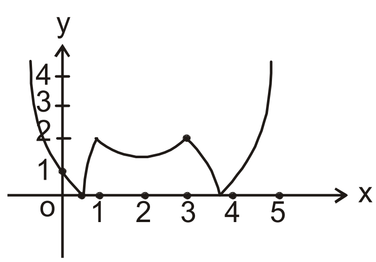
42. (C)

$\left(z^{89} + i^{97}\right)^{94} = z^n \Rightarrow \left((i \cdot \omega^2)^{89} + i^{97}\right)^{94} = \left(i\omega^{178} + i\right)^{94}$

$\Rightarrow i^{94}(-\omega^2)^{94} = -\omega^2 \Rightarrow i^n\omega^{2n} = -\omega^2$ for $n = 10$

$\Rightarrow \left({}^nC_r\right)_{max} = {}^{10}C_5 = 252$

43. (D)



we can see by the graph of y = f(x) and y = m, two roots of different signs possible, only for m > 2

44. (A)

We have sum of roots $= \sin\theta + \cos\theta = -(a-1)$ … (i)

and product of roots $= \sin\theta \cdot \cos\theta = \dfrac{1-2a}{4}$ … (ii)

Squaring equation (i) $\Rightarrow 1 + \dfrac{1-2a}{2} = (1-a)^2 \Rightarrow a = \dfrac{1 \pm \sqrt{3}}{2}$

But $\Rightarrow a = \frac{1+\sqrt{3}}{2}$ rejected as $\sin\theta + \cos\theta$ can not be taken negative

$\Rightarrow a = \frac{1-\sqrt{3}}{2}$ and from (i) $\Rightarrow \theta = 30^o$ and 60^o

45. (D)

$$\frac{2}{c-a} = \frac{1}{a-b} + \frac{1}{b-c}$$

$$\Rightarrow 2(a-b)(b-c) = -(a-c)^2 \Rightarrow (a-c)^2 + 2(a-b)(b-c) = 0$$

$$\Rightarrow (a+c)^2 - 4ac + 2(ab - ac - b^2 + bc) = 0$$

$$\Rightarrow (a+c)^2 - 4b^2 + 2b(a+c) - 2ac - 2b^2 = 0 \quad (\because b^2 = ac)$$

$$\Rightarrow (a+c)^2 + 2b(a+c) - 8b^2 = 0$$

$$\Rightarrow (a+b+c)^2 = 9b^2 \Rightarrow (a+4b+c)(a+c-2b) = 0$$

$$\Rightarrow \frac{a+c}{b} = -4, \text{ as } (a+c \neq 2b)$$

$$\Rightarrow \left|\frac{a+c}{b}\right| = 4$$

46. (C)

Let common difference = d

$$\text{Now } \frac{1}{d}\left[\frac{a_2-a_1}{a_1a_2} + \frac{a_3-a_2}{a_2a_3} + \frac{a_4-a_5}{a_3a_4} + \ldots + \frac{a_{4001}-a_{4000}}{a_{4000}a_{4001}}\right] = 10$$

$$\Rightarrow \frac{1}{d}\left[\frac{1}{a_1} - \frac{1}{a_{4001}}\right] = \frac{(a_{4001}-a_1)}{da_1a_{4001}} = 10$$

$$\Rightarrow \frac{4000d}{da_1a_{4001}} = 10 \Rightarrow a_1a_{4001} = 400 \quad \ldots (I)$$

But $a_2 + a_{4000} = a_1 + a_{4001} = 50$ Hence $(a_1 - a_{4001})^2 = 2500 - 4(400) = 900$

$$\Rightarrow |a_1 - a_{4001}| = 30$$

47. (C)

Diff of the roots of 1^{st} equation $=4|\cos^4\alpha-\sin^4\alpha|=4|\cos 2\alpha|=\sqrt{4k^2-4k}$ … (i)

And from II^{nd} quadratic equation $\Rightarrow 4|\cos 2\alpha|=\sqrt{(25)-4.3}=\sqrt{13}$ … (ii)

From (i) and (ii) we get $\Rightarrow$ $4k^2-4k=13$ $\Rightarrow$ sum of k_1 and $k_2=1$

48. (C)

$$\text{Area}=\int_0^a\left(\sqrt{a}-\sqrt{x}\right)^2dx=\int_0^a\left(a+x-2\sqrt{a}\sqrt{x}\right)dx=\frac{a^2}{6}$$

49. (D)

Let d_1 be the common difference

$s_1=a=T_1$

$S_2=4a+\frac{d}{2}=T_1+T_2$ $\Rightarrow$ $d_1=T_2-T_1=\left(4a+\frac{d}{2}-a\right)-a=2a+\frac{d}{2}$

50. (C)

Roots of given quadratic are ω and ω^2 cube roots of unity.

$\Rightarrow$ $(1+1+4)+(1+1+4)+.....9\text{ times}=54$

Solution for 51. & 52

Let $\int_0^2 f(x)dx=\lambda$

$\Rightarrow f'(x)=f(x)+\lambda$

$\Rightarrow f'(x)=f(x)+\lambda$ $\Rightarrow \int\frac{f'(x)}{f(x)+\lambda}dx=\int 1dx$

$\Rightarrow \ell n\left(f(x)+\lambda\right)=x+c$ $\Rightarrow f(x)=-\lambda+(1+\lambda)e^x$ as f(0) = 1

$\Rightarrow \lambda=\int_0^2 f(x)dx=\int_0^2\left(-\lambda+(1+\lambda)e^x\right)dx$

$=-2\lambda+(1+\lambda)\left(e^2-1\right)$ $\Rightarrow 3\lambda=\lambda(e^2-1)+e^2-1$

$\Rightarrow \lambda=\frac{e^2-1}{4-e^2}$

$f(x)=\frac{1-e^2}{4-e^2}+\left(\frac{3}{4-e^2}\right)e^x$

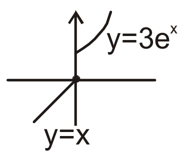
51. (B)

f(x) is decreasing function with range $\left(-\infty, \frac{1-e^2}{4-e^2}\right) \Rightarrow f(x)=0$ for only one value of x

52. (A)

$$g(x)=\begin{cases} 3e^x & ; x \ge 0 \\ x & ; x < 0 \end{cases},$$

graph of y = g(x) will be like as shown.



Solution for 53. & 54

As given conditions we can define

$P'(x)=A(x-1)(x-2)(x-3)$ ... (i)

$$\Rightarrow P(x)=A\left(\frac{x^4}{4}-2x^3+\frac{11}{2}x^2-6x\right)+B$$

As $P(-1)=0 \Rightarrow B=\frac{-55}{4}A$

Now $\int_{-2}^{2} P(x)=\frac{-1348}{15} \Rightarrow A=4$

53. (C)

$P'(-1)=-96$ and $P(-1)=0 \quad \Rightarrow$ Tangent $\equiv y=-96(x+1)$,

54. (B)

Two points of minimum are $A(1,-64)$ and $B(3,-64)$ so distance AB = 2

Solution for 55. & 56

55. (D)

AB chord is $T=S_1 \Rightarrow ky-hx=k^2-h^2$

Clearly slope $=\frac{h}{k}=\beta(\text{given})$

$\Rightarrow$ locus of (h, k) is

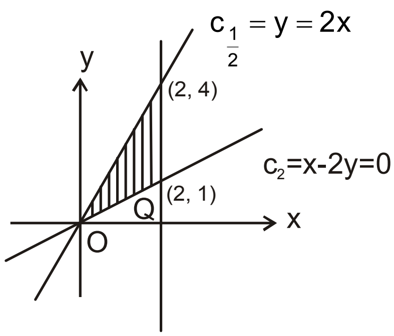
$x=\beta y \Rightarrow c_\beta = x=\beta y$

56. **(C)**

$c_2 = x = 2y$

$c_{\frac{1}{2}} = 2x - y = 0$

Area enclosed by the curve is $A = \frac{1}{2} \times 3 \times 2 = 3$ sq. units



57. **(B)**

(P) $z = x + iy$

$\Rightarrow (x+iy)^2 + \sqrt{x^2+y^2} = 0$

on comparing real and imaginary parts both sides

we get $z = 0, i, -i$

(Q) $2(x^2+y^2) + (x+iy)^2 - 5 + i\sqrt{3} = 0$

$\left.\begin{array}{l} 2x^2 + x^2 - 5 - y^2 + 2y^2 = 0 \\ 2xy = -\sqrt{3} \end{array}\right\}$ on solving we get $x = \pm\frac{1}{\sqrt{6}}, \pm\frac{\sqrt{3}}{\sqrt{2}}$

(R) $x - iy = i(x^2 - y^2 + 2ixy)$ $\left.\begin{array}{ll} x = -2xy & \text{...(i)} \\ -y = x^2 - y^2 & \text{...(ii)} \end{array}\right\}$ Solving we get value of z

(S) From (i) equation, $9|z-12|^2 = 25|z-8|^2$

$\Rightarrow 9\left\{(x-12)^2 + y^2\right\} = 25\left\{(x-8)^2 + y^2\right\} \Rightarrow 9\left\{(36) + y^2\right\} = 25\left\{4 + y^2\right\}$

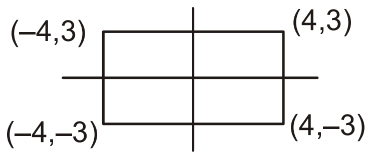
Also from (ii) equation x = 6 $\Rightarrow z = 6 \pm \sqrt{14}\,i$

58. (A)

(P) Let $z = x + iy$, put in the given equation $z\bar{z}(\bar{z}^2 + z^2) = 350$

$\Rightarrow x^2 + y^2 = 25$ and $x^2 - y^2 = 7 \Rightarrow x = \pm 4, y = \pm 3$

Area = $8 \times 6 = 48$



(Q) $\frac{A}{B} + \frac{B}{A} = 1 \Rightarrow A^2 + B^2 = AB$

$\therefore$ triangle with vertices at A, B and origin (O) is equilateral.

(S) $m = |z|_{least} = m = 13 - 1 = 12$

$n = |z|_{greatest} = n = 13 + 1 = 14$

$\Rightarrow m + n = 26$

59. (C)

$S_n = 1! + 2! + 3! + 4! + 5! + 6! + 7I$ where I be an integer

$S_n = 873 + 7I$

$\frac{S_n}{7} = 124.71 + I \Rightarrow \left[\frac{S_n}{7}\right] = 124 + I$

$\therefore 7\left[\frac{S_n}{7}\right] = 868 + 7I$

$\Rightarrow S_n - 7\left[\frac{S_n}{7}\right] = (873 + 7I) - 868 + 7I = 5$

Now $\sin^{-1}(\sin 5) = \sin^{-1}(\sin(5 - 2\pi)) = 5 - 2\pi$

$\cos^{-1}(\cos 5) = \cos^{-1}(\cos(2\pi - 5)) = 2\pi - 5$

$\tan^{-1}(\tan 5) = \tan^{-1}(\tan(5 - 2\pi)) = 5 - 2\pi$

$\cot^{-1}(\cot 5) = \cot^{-1}(\cot(5 - \pi)) = 5 - \pi$

60. (D)

(P) $TS = y - 5 = \frac{-p}{2}\left(x - \frac{p}{2}\right)$ Put y = 0 $\Rightarrow x = \frac{10}{p} + \frac{p}{2}$ hence $p = \pm 2, \pm 10$ for $x \in I$

(Q) Let $f(x) = ax^3 + bx^2 + cx + d$ $\Rightarrow f(0) = d, f'(0) = c, f''(0) = 2b, f'''(0) = 6a$

$$\Rightarrow f(x) = \frac{f'''(0)}{6}x^3 + \frac{f''(0)}{2}x^2 + f'(0)x + f(0)$$

$$\Rightarrow \int f(x)dx = \frac{f'''(0)}{24}x^4 + \frac{f''(0)}{6}x^3 + \frac{f'(0)}{2}x^2 + f(0)x + E$$

$$\Rightarrow 8A + B + C + D = \frac{1}{3} + \frac{1}{6} + \frac{1}{2} + 1 = 2$$

(R) Constant term $= \sum\left(\frac{1}{k+1}\right)\left(\frac{1}{k+2}\right) = 1 - \frac{1}{n+2}$

$$\Rightarrow \text{constant term} = 1 \text{ as } n \to \infty$$

(S) Given that $x_1x_2x_3x_4 = -51$ and $x_1 = 1 - 4i, x_2 = 1 + 4i$

$$\Rightarrow x_2x_3x_4 = \alpha + i\beta = \frac{-51}{1 - 4i} = -3 - 12i$$

$$\Rightarrow \alpha = -3, \beta = -12 \Rightarrow \frac{\beta}{\alpha} = 4$$