

SOLUTIONS

WEEKLY TEST-B

RBA & RBS-1801

(JEE ADVANCED PATTERN)

Test Date: 10-12-2017



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PHYSICS

1. The displacement between first stone and aeroplane after t second $(h_1) = \frac{1}{2}(g+f)t^2$

After time t ,

Velocity of aeroplane = $u + ft$

Velocity of first stone = $u - gt$

Where u is velocity of aeroplane when first stone is dropped.

The relative speed of second stone with respect to first stone = $(u + ft) - (u - gt)$

$$= (g + f)t$$

The relative displacement between first and second stone after time $t'(h_2)$

$$= (g + f)tt'$$

$$h_1 + h_2 = \frac{1}{2}(g + f)t^2 + (g + f)tt' = \frac{1}{2}(g + f)(t + 2t')t$$

\therefore (B)

2. $\frac{1}{2}gt^2 = H$... (i)

$$gt = v_y \quad \dots(ii)$$

$$v_x = v_y$$

$$\text{Range} = u_x t = v_y t = gt^2 = 2H$$

\therefore (B)

3. $\frac{dx}{dt} = 2t - 2 = 0 \Rightarrow t = 1$, So, $x_{t=0} = 1\text{m}$, $x_{t=1\text{s}} = 0$, $x_{t=3} = 4\text{m}$

Total distance = 5m

\therefore (D)

4. $a = v \frac{dv}{dx} = \frac{25}{(x+2)^3}$, $\frac{v^2}{2} = 25 \times \left[-\frac{1}{2(x+2)^2} \right]_0^x$, $v^2 = 25 \left[\frac{1}{4} - \frac{1}{(x+2)^2} \right]$

$$v = \sqrt{25 \left[\frac{1}{4} - \frac{1}{(x+2)^2} \right]}, v_{\max} = \frac{5}{2} = 2.5 \text{ m/s (at } x = \infty)$$

\therefore (A)

5. (A)

$$\frac{\pi}{2} = \frac{1}{2} \alpha t_1^2 \quad \pi = \frac{1}{2} \alpha t_2^2$$

$$t_1 = \sqrt{\frac{\pi}{\alpha}} \quad t_2 = \sqrt{\frac{2\pi}{\alpha}}$$

$$v_1 = r\alpha t_1 \quad v_2 = r\alpha t_2$$

$$a_{\text{avg}} = \frac{|\Delta v|}{\Delta t} = \frac{\sqrt{v_1^2 + v_2^2}}{t_2 - t_1} = \frac{r\alpha\sqrt{3}}{\sqrt{2} - 1}$$

6. Force of friction on chain = weight of hanging chain.

$$\mu(l-x)\rho g = x\rho g \quad \Rightarrow \quad \mu(l-x) = x$$

$$x = \frac{\mu l}{1+\mu} = 0.2l \quad \Rightarrow \quad \frac{x}{l} = 0.2 = 20\%$$

 \therefore (A)7. $m_A = 0.5\text{ kg}$; $m_B = 1\text{ kg}$

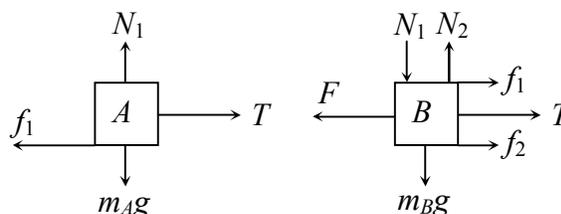
From F.B.D. of block A,

$$T = f_1 = \mu m_A g = 2\text{ N}$$

From F.B.D. of block B,

$$f_2 = \mu N_2 = 6\text{ N}$$

$$F = T + f_1 + f_2 = 2 + 2 + 6 = 10\text{ N}$$

 \therefore (B)

8. Block will return after maximum elongation.

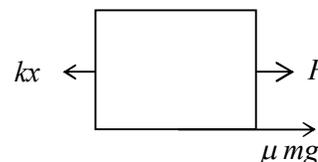
$$\text{i.e. } F \cdot x_{\text{max}} - \frac{1}{2} Kx_{\text{max}}^2 - \mu mgx_{\text{max}} = 0$$

$$x_{\text{max}} = \frac{2(F - \mu mg)}{k} = \frac{8\mu mg}{k}$$

So block will finally come to rest while returning i.e. $v = 0$ & $a = 0$

By work energy theorem while returning

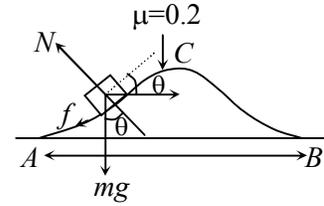
$$-\left(\frac{1}{2} kx^2 - \frac{1}{2} kx_{\text{max}}^2\right) - (F + \mu mg)(x_{\text{max}} - x) = 0 \Rightarrow x = \frac{4\mu mg}{k}$$

 \therefore (B)

9. Work done by friction - $\int \vec{F} \cdot d\vec{s}$

$$-\int_0^x \mu mg \cos \theta \frac{dx}{\cos \theta} = -\mu mgx = -20 \text{ J}$$

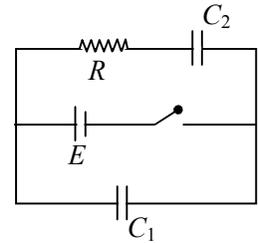
\therefore (C)



10. Rearranging the circuit, we observed that C_1 is joined directly to the cell and acquires its full charge when S is closed. It plays no part in the charging of C_2 through R.

$$\text{So, } q_2 = Q_0(1 - e^{-t/\tau})$$

\therefore (B)



11. For $r < R$

$$\text{From Gauss's law, } \oint \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} q$$

$$\text{i.e. } E4\pi r^2 = \frac{1}{\epsilon_0} \int_0^r \rho_0 \left(1 - \frac{r}{R}\right) 4\pi r^2 dr$$

$$\text{or } E = \frac{\rho_0}{\epsilon} \left[\frac{r}{3} - \frac{r^2}{4R} \right]$$

\therefore (A)

12. For $r > R$

$$E \cdot 4\pi r^2 = \frac{1}{\epsilon_0} \int_0^R \rho_0 \left(1 - \frac{r}{R}\right) 4\pi r^2 dr \Rightarrow E = \frac{\rho_0 R^3}{12\epsilon r^2}$$

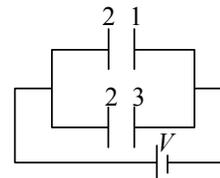
\therefore (B)

13. Both the capacitor are in parallel

Charge on plate (2) initially

$$= \frac{2\epsilon_0 A}{d} V$$

\therefore (B)



14. Charge on plate (2) finally = $\left(\frac{\epsilon_0 A}{3d/2} + \frac{\epsilon_0 A}{d/2}\right)V = \frac{8\epsilon_0 A}{3d}V$

∴ (C)

15. $V_{CM} = \frac{Mv_0 + m(0)}{M + m}$

∴ (D)

16. (A)

17. (A)

At A, $mg \sin \theta + T_A = \frac{mv^2}{r}$, $T_A = m\left(\frac{v^2}{r} - g \sin \theta\right)$

At B, $T_B = \frac{mv_B^2}{r} = \frac{m}{r}(v^2 + 2gr \sin \theta)$

At C, $T_C - mg \sin \theta = \frac{mv_C^2}{r}$, $T_C = mg \sin \theta + \frac{m}{r}(v^2 + 2g \cdot 2r \sin \theta)$

$T_C = m\left(\frac{v^2}{r} + 5g \sin \theta\right)$

∴ I – A, II-B, III-C, IV-D

18. (B)

To shift block A, $Kx = \mu m_1 g$, $x = \frac{16}{100} \text{ m}$

Applying work energy theorem on block B

$W_F + W_{fr} + W_{sp} = 0$, $Fx - \frac{1}{2}Kx^2 - \mu m_2 gx = 0$, $F = 40 \text{ N}$

$W_{fr} \text{ on } B = -\mu m_2 gx = -(0.2)(16)(10)\left(\frac{16}{100}\right) = -5.12$

$W_F = Fx = (40)\left(\frac{16}{100}\right) = 6.4$

To shift block B , $Kx = \mu m_2 g$, $x = \frac{32}{100} m$

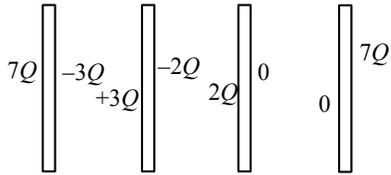
Applying work energy theorem on block A, $W_F + W_{Fr} + W_{sp} = 0$

$$F = \mu m_1 g + \frac{1}{2} Kx = (0.2)(8)(10) + \frac{1}{2}(100)\left(\frac{32}{100}\right) = 32 \text{ N}$$

\therefore (A) – 2; (B) – 1; (C) – 5; (D) – 4

19. (A)

Charge distribution shown in diagram



\therefore (A) – 2, (B) – 4, (C) – 1, (D) – 3

20. (C)

Acceleration of 2kg relative to wedge = 2 m/s^2

Acceleration of 2 kg relative to ground

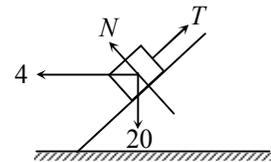
$$= \sqrt{2^2 + 2^2 + 2 \times 2 \times \cos 120^\circ} = 2 \text{ m/s}^2$$

$$20 \sin 60^\circ + 4 \cos 60^\circ - T = 4$$

$$T = 10\sqrt{3} - 2$$

$$N = 20 \cos 60^\circ - 4 \sin 60^\circ = 10 - 2\sqrt{3} \text{ N} \quad \text{Net force} = 2 \times 2 = 4 \text{ N}$$

\therefore (A) – 2 , (B) – 1 , (C) – 3 , (D) – 4



CHEMISTRY

21. (A)

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^{(\gamma-1)} = x^{(1.4-1)} = x^{0.4}$$

$$\frac{Z_{1i}}{Z_{1f}} = \frac{V_{avi} V_2}{V_{avf} V_1} = \sqrt{\frac{T_1}{T_2}} x = x^{1.2}$$

22. (B)

pH = 2, [HCl] = 10^{-2} (M) & pOH = 2

or [NaOH] = 10^{-2} (M)

4 ml of 10^{-2} (M) HCl \equiv 4×10^{-5} moles HCl.

6 ml of 10^{-2} (M) NaOH \equiv 6×10^{-5} moles NaOH

After mixing excess moles of $\text{OH}^- = 2 \times 10^{-5}$

$$[\text{OH}^-] = \frac{2 \times 10^{-5}}{10} \times 10^3 = 2 \times 10^{-3}$$

or pOH = $3 - \log 2 = 3 - 0.3 = 2.7$

or pH = 11.3

23. (B)

24. (D)

$$\Delta H = \Delta E + \Delta nRT$$

$$\text{or } -\Delta nRT = \Delta E - \Delta H = 1200$$

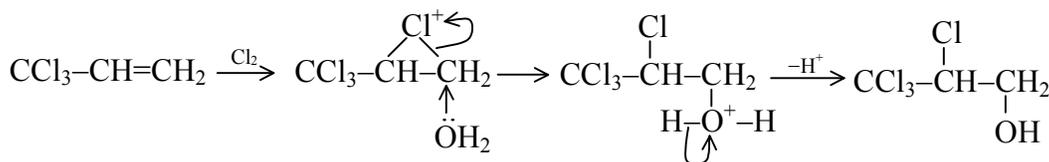
$$\text{or } \Delta n = \frac{-1200}{2 \times 300} = -2$$

$$\frac{K_p}{K_c} = (RT)^{\Delta n} = (0.0821 \times 300)^{-2} = (24.63)^{-2}$$

$$\text{or } = 1.648 \times 10^{-3}$$

$$\text{or } \times 10^4 = 16.48 \approx 16.$$

25. (B)



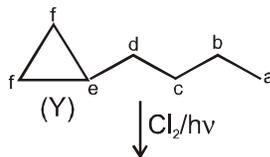
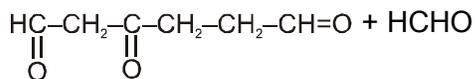
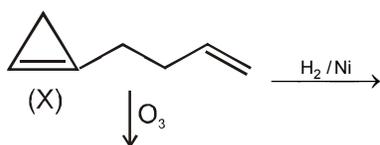
26. (D)

Total products are five and fractional distillation are three.

27. (C)



DU = 3



6-products

28. (C)

29. (A)

30. (D)

In (A) and (B) use (z / e) concept for isoelectronic specie.

In (C) size of neutral atom is greater than its cation.

In (D) Se^{2-} and As^{3-} related with 4th period, while Ba^{2+} and Cs^+ related with 6th period. (These are not isoelectronic species)

31. (A)

$$\Delta_r G^0 = \Delta_r H^0 - T\Delta_r S^0 \quad \text{At } T = 0 \text{ K, } \Delta_r G^0 = \Delta_r H^0$$

$$\therefore \Delta_r H^0 = 100 \text{ kJ/mol}$$

32. (B)

$$\text{When } \Delta_r G^0 = -RT \ln K_p^0 = 0$$

$$\text{or, } K_p^0 = 1$$

$$\therefore T = \frac{100}{4 \times 10^{-2}} = 2500 \text{ K}$$

33. (C)

Two diastereomeric esters and unreacted (+)-2-Butanol are obtained in different fractions.

34. (D)

Both the esters have two dissimilar chiral carbons hence both are optically active.

35. (A)

O-phen & en are symmetrical bidentate ligands

36. (D)

37. (B)

(A — S); (B — P); (C — Q); (D — R)

(A) Since Cu(S) is absent in the equilibrium expression, therefore with the addition of Cu(S) the equilibrium will be unaffected. (A) → S

- (B) On doubling the volume decreases, therefore reaction move to right. (B) \rightarrow P
 (C) $A^- + H_2O \rightleftharpoons HA + OH^-$ in the presence of excess of H^+ the degree of hydrolysis of A^- will be greater. (C) \rightarrow Q
 (D) no of mole in reactant side less (D) \rightarrow R

38. (A)

39. (C)

(A) – (P) ; (B) – (R) ; (C) – (S) ; (D) – (Q)

40. (D)

MATHEMATICS

41. (D)

OADC is a rhombus

$$\therefore M = \frac{3}{4}z_2$$

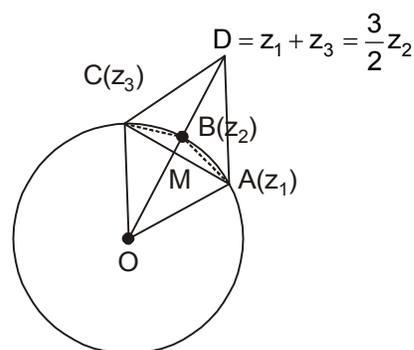
$$\therefore OM = \frac{3}{4}, MB = \frac{1}{4}, OA = OB = 1$$

$$AM = \frac{\sqrt{7}}{4}$$

$$\therefore \text{Ar. of } \triangle OAC = \frac{3\sqrt{7}}{16} \text{ sq. units}$$

$$\text{Also, Ar. of } \triangle ABC = \frac{\sqrt{7}}{16} \text{ sq. units}$$

$$\therefore \text{Ar. of quadrilateral OABC} = \frac{\sqrt{7}}{4} \text{ sq. units}$$



42. (B)

$$a_i = 3^i 5^k$$

$$\therefore \frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n} < \left(\frac{1}{3^0} + \frac{1}{3^1} + \dots \right) \left(\frac{1}{5^0} + \frac{1}{5^1} + \dots \right) = \frac{15}{8}$$

43. (A)

$$I = \int \frac{5x^6 + 12x^5}{(3x^6 + x + 2)^2} dx$$

$$I = \int \frac{\frac{5}{x^6} + \frac{12}{x^7}}{\left(3 + \frac{1}{x^5} + \frac{2}{x^6}\right)^2} dx$$

$$\text{Put } 3 + \frac{1}{x^5} + \frac{2}{x^6} = t$$

$$\left(-\frac{5}{x^6} - \frac{12}{x^7}\right) dx = dt$$

$$I = \int -\frac{dt}{t^2} = \frac{1}{t} + c$$

$$= \frac{1}{3 + \frac{1}{x^5} + \frac{2}{x^6}} + c = \frac{x^6}{3x^6 + x + 2}$$

44. (A)

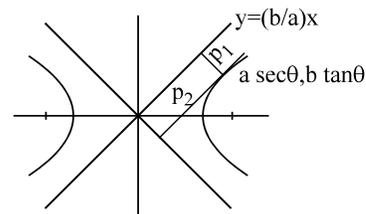
$$\frac{f(0) + f(2)}{2} = f(c); 0 < c < 2$$

45. (B)

$$p_1 p_2 = \frac{a^2 b^2}{a^2 + b^2} = \frac{a^2 \cdot a^2 (e^2 - 1)}{a^2 e^2} = 6$$

$$\frac{2a^2}{3} = 6 \Rightarrow a^2 = 9 \Rightarrow a = 3$$

$$\text{hence } 2a = 6$$

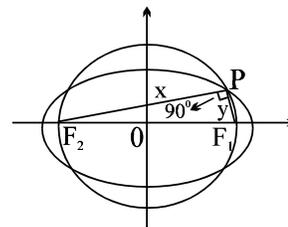


46. (C)

$$x + y = 17; xy = 60, \text{ To find } \sqrt{x^2 + y^2}$$

$$\begin{aligned} \text{now, } x^2 + y^2 &= (x + y)^2 - 2xy \\ &= 289 - 120 = 169 \end{aligned}$$

$$\Rightarrow \sqrt{x^2 + y^2} = 13$$



47. (C)

$$\sin x \cos 2y \leq 1 \text{ and } \cos x \sin 2y \leq 1$$

$$\Rightarrow (a^2 - 1)^2 + 1 \leq 1 \text{ and } a + 1 \leq 1$$

$$\Rightarrow a = \pm 1 \text{ and } a \leq 0$$

Hence $a = -1$

48. (D)

Maximum and minimum possible slopes of PQ is along transverse common tangents of the circles.

Points of intersection of transverse common tangents is (9, 3)

Hence equation of PQ is

$$y - 3 = m(x - 9)$$

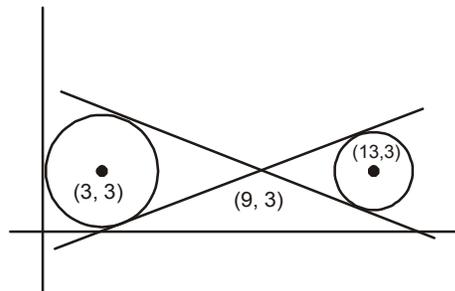
$$\Rightarrow mx - y - 9m + 3 = 0$$

It is tangent to circle C_1

$$\text{Therefore } \frac{|6m|}{\sqrt{m^2 + 1}} = 3 \Rightarrow 3m^2 = 1$$

$$\Rightarrow m = \pm \frac{1}{\sqrt{3}}$$

$$m \in \left[\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right]$$



49. (B)

$$\lim_{x \rightarrow \infty} (\sqrt[3]{(x+a)(x+b)(x+c)} - x)$$

$$= \lim_{x \rightarrow \infty} \frac{(x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc) - x^3}{((x+a)(x+b)(x+c))^{2/3} + x^2 + x((x+a)(x+b)(x+c))^{1/3}} \left(\because x - y = \frac{x^3 - y^3}{x^2 + xy + y^2} \right)$$

$$= \lim_{x \rightarrow \infty} \frac{(a+b+c)x^2 + (ab+bc+ca)x + abc}{((x+a)(x+b)(x+c))^{2/3} + x^2 + x((x+a)(x+b)(x+c))^{1/3}}$$

$$= \frac{a+b+c}{3} \quad (\text{Dividing numerator and denominator by } x^2)$$

50. (C)

$$\text{For } x = k - 1, \quad f(x) = (x - k)^2 \cdot \cos\left(\frac{1}{x - k}\right) - |x - (k - 1)|$$

Since $|x - (k - 1)|$ is not diff. at $x = k - 1$

Hence $f(x)$ is also not diff. at $x = k - 1$

$$\text{For } x = k, \quad f'(k + h) = f'(k - h) = -1$$

51. (B)

$$\begin{aligned} AP^2 + BP^2 + CP^2 &= |z - z_1|^2 + |z - z_2|^2 + |z - z_3|^2 \\ &= 3|z|^2 + |z_1|^2 + |z_2|^2 + |z_3|^2 - z(\bar{z}_1 + \bar{z}_2 + \bar{z}_3) - \bar{z}(z_1 + z_2 + z_3) \end{aligned}$$

$\therefore \triangle ABC$ is equilateral and circumcentre is at origin.

$$\text{Hence, } z_1 + z_2 + z_3 = \bar{z}_1 + \bar{z}_2 + \bar{z}_3 = 0$$

$$\text{and } |z_1| = |z_2| = |z_3| = 2$$

Also, $|z| = 1$ (\because circumradius is 2)

$$\therefore AP^2 + BP^2 + CP^2 = 3 \times 1 + 12 = 15$$

52. (D)

O, F, B, D are concyclic.

$$\text{So, } z_f - z_2 = (z_d - z_2)e^{i\theta} \text{ and } z_d = z_f e^{i(\pi-\theta)}$$

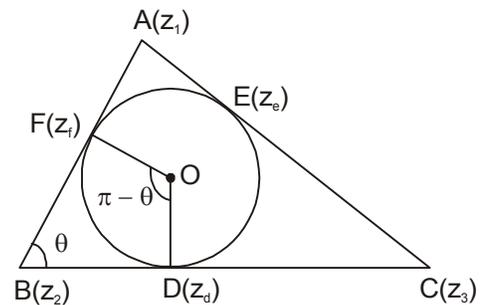
$$\Rightarrow (z_f - z_2)z_d = -z_f(z_d - z_2)$$

$$\Rightarrow \frac{1}{z_d} + \frac{1}{z_f} = \frac{2}{z_2}$$

$$\text{Similarly, } \frac{1}{z_d} + \frac{1}{z_e} = \frac{2}{z_3} \text{ and } \frac{1}{z_e} + \frac{1}{z_f} = \frac{2}{z_1}$$

$$\Rightarrow \frac{1}{z_d} + \frac{1}{z_e} + \frac{1}{z_f} = \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} = 0$$

$$\therefore \operatorname{Re}\left(\frac{1}{z_d} + \frac{1}{z_e} + \frac{1}{z_f}\right) = 0$$



53. (A)

$$f(x) = \left(1 + 2x + \frac{2^2 x^2}{2!} + \frac{2^3 x^3}{3!} + \dots \text{to } \infty\right) + 1$$

$$\text{and } g(x) = \left(1 + 2x + \frac{2^2 x^2}{2!} + \frac{2^3 x^3}{3!} + \dots \text{to } \infty\right) - 1$$

$$\text{Hence } f(x) = e^{2x} + 1 \text{ and } g(x) = e^{2x} - 1$$

$$\int \frac{e^{2x} - 1}{e^{2x} + 1} dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx = \log |e^x + e^{-x}| + C.$$

54. (C)

$$\begin{aligned} \int \frac{e^{2x} + 1}{\sqrt{e^{2x} - 1}} dx &= \int \frac{e^{2x}}{\sqrt{e^{2x} - 1}} dx + \int \frac{1}{\sqrt{e^{2x} - 1}} dx \\ &= \int \frac{e^{2x}}{\sqrt{e^{2x} - 1}} dx + \int \frac{e^x}{e^x \sqrt{e^{2x} - 1}} dx \\ &= \sqrt{e^{2x} - 1} + \sec^{-1}(e^x) + C \end{aligned}$$

55. (D)

Since $f(x)$ is symmetric about $x = 1$ and it is twice differentiable. so $f'(x)$ must have one root at $x = 1$.

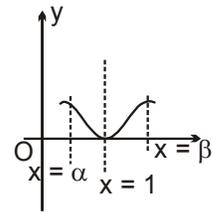
$$f'(x) = a(x-1)(x-\alpha)(x-\beta) = a(x-1)(x^2 - (\alpha+\beta)x + \alpha\beta)$$

Here $\frac{\alpha+\beta}{2} = 1$ so $\alpha+\beta = 2$

$$f'(x) = a(x^3 - 3x^2 + (\alpha\beta+2)x - \alpha\beta)$$

$$f''(2) = 0 \Rightarrow \alpha\beta = -2$$

\therefore Sum of roots of $f'(x) = 0$ is $1 + \alpha + \beta$ i.e., 3.



56. (C)

$$f'(x) = a(x-1)(x^2 - 2x - 2) = a(x-1)((x-1)^2 - 3)$$

$$f(x) = a \left(\frac{(x-1)^4}{4} - \frac{3}{2}(x-1)^2 \right) + C$$

$$\because f(1) = 0 \text{ so } c = 0; f(2) = 1 \text{ so } a = -\frac{4}{5}$$

$$f(x) = -\frac{4}{5} \left[\frac{(x-1)^4}{4} - \frac{3}{2}(x-1)^2 \right]$$

$$f(3) = \frac{8}{5}$$

57. (D)

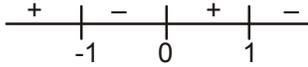
$$(P) f(x) = x^{4/3} - 4x^{1/3}$$

$$f'(x) = \frac{4}{3} \left(\frac{x-1}{x^{2/3}} \right)$$



$$(Q) f(x) = 5x^{2/5} - x^2$$

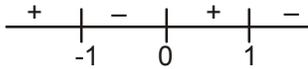
$$f'(x) = 2 \left(\frac{1 - x^{8/5}}{x^{3/5}} \right) = 2 \left(\frac{(1 - x^{1/5})(1 + x^{1/5})(1 + x^{2/5})(1 + x^{4/5})}{x^{3/5}} \right)$$



$$(R) f(0) = \lim_{x \rightarrow 0} \frac{1}{x} \log \left(\frac{e^x - 1}{x} \right) = \lim_{x \rightarrow 0} \frac{1}{\frac{e^x - 1}{x}} \cdot \frac{xe^x - (e^x - 1)}{x^2} = \frac{1}{2}$$

$$(S) f(x) = 3x^{2/3} - x^2$$

$$f'(x) = 2 \left(\frac{1 - x^{4/3}}{x^{1/3}} \right) = 2 \frac{(1 - x^{1/3})(1 + x^{1/3})(1 + x^{2/3})}{x^{1/3}}$$



58. (A)

$$(P) PF_1 + PF_2 = 8$$

\therefore locus of $P(z)$ is an ellipse

$$\therefore |z|_{\max} = 4$$

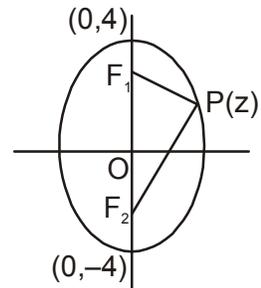
$$(Q) iz^3 + z^2 - z + i = 0$$

$$\Rightarrow (z^2 + i)(iz + 1) = 0 \Rightarrow z^2 = -i \text{ or } z = \frac{-1}{i} = i \quad \therefore |z| = 1$$

$$(R) \text{ Let } z = x + iy, \text{ then } \arg \left(\frac{z - z_1}{z - z_2} \right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left(\frac{y-6}{x-10} \right) - \tan^{-1} \left(\frac{y-6}{x-4} \right) = \frac{\pi}{4} \Rightarrow \tan^{-1} \left[\frac{\frac{y-6}{x-10} - \frac{y-6}{x-4}}{1 + \frac{y-6}{x-10} \times \frac{y-6}{x-4}} \right] = \frac{\pi}{4}$$

$$\Rightarrow |(x-7) + i(y-9)|^2 = (3\sqrt{2})^2 \Rightarrow |z - (7+9i)| = 3\sqrt{2}$$



(S) $|z_1 - 2z_2|^2 = |2 - z_1\bar{z}_2|^2$

or $|z_1|^2 + 2^2|z_2|^2 - 2\text{Re}(z_1(2\bar{z}_2)) = 2^2 + |z_1|^2|\bar{z}_2|^2 - 2\text{Re}(2z_1\bar{z}_2)$

$(1 - |z_2|^2)(|z_1|^2 - 4) = 0 \quad \because |z_2| \neq 1 \quad \therefore (|z_1|^2 - 4) = 0$

or $|z_1| = 2$

59. (C)

(P) Given curve is $(x + 6)(y - 5) = -30$ is a rectangular hyperbola.

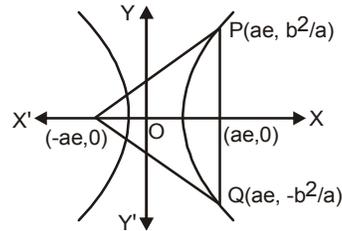
$\Rightarrow e = \sqrt{2}$

(Q) $\tan 30^\circ = \frac{b^2/a}{2ae}$

$\Rightarrow \frac{2}{\sqrt{3}}e = e^2 - 1 \Rightarrow \sqrt{3}e^2 - 2e - \sqrt{3} = 0$

$\Rightarrow e = \frac{2 \pm \sqrt{4+12}}{2\sqrt{3}} = \frac{2 \pm 4}{2\sqrt{3}}$

$\Rightarrow e = \frac{3}{\sqrt{3}} = \sqrt{3}$



(R) Eccentricity of the hyperbola = $\frac{AB}{PA - PB} = \frac{6}{4} = \frac{3}{2}$. If eccentricity of conjugate hyperbola is e'

then $\frac{1}{\left(\frac{3}{2}\right)^2} + \frac{1}{e'^2} = 1 \Rightarrow e' = \frac{3}{\sqrt{5}}$.

(S) Angle between the asymptotes is $\tan^{-1} \left| \frac{2ab}{a^2 - b^2} \right| = \frac{\pi}{3} \Rightarrow \left| \frac{2\frac{a}{b}}{\frac{a^2}{b^2} - 1} \right| = \sqrt{3} \Rightarrow \frac{2\sqrt{e'^2 - 1}}{|e'^2 - 2|} = \sqrt{3}$

(where e' is eccentricity of conjugate hyperbola)

$\Rightarrow e' = 2$

60. (D)

(P) $f(x)$ is not differentiable at $x = 0$ only.

(Q) $g(x)$ is differentiable in R .

(R) from the graph $h(x)$ is not differentiable at four points.

(S) $\phi'(x) = \frac{2 - 5x^2}{3x^{1/3}\sqrt{1-x^2}}$

So $\phi(x)$ is not differentiable at $x = 0, \pm 1$.

