JEE (ADVANCED) 2022 PAPER-1

[PAPER WITH SOLUTION]

HELD ON SUNDAY 28TH AUGUST 2022

MATHEMATICS

SECTION-1 (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : + 3 **ONLY** if the correct numerical value is entered;

  Zero Marks : 0 In all other cases.

[:Q.1] Considering only the principal values of the inverse trigonometric functions, the value of

$$\frac{3}{2}\cos^{-1}\sqrt{\frac{2}{2+\pi^2}}+\frac{1}{4}\sin^{-1}\frac{2\sqrt{2}\pi}{2+\pi^2}+\tan^{-1}\frac{\sqrt{2}}{\pi}$$ is ____________

[:ANS] **2.36**

[:SOLN] $$\frac{3}{2}\cos^{-1}\sqrt{\frac{2}{2+\pi^2}}+\frac{1}{4}\sin^{-1}\frac{2\sqrt{2}\pi}{2+\pi^2}+\tan^{-1}\frac{\sqrt{2}}{\pi}$$

$$\frac{3}{2}\tan^{-1}\left(\frac{\pi}{\sqrt{2}}\right)+\frac{1}{4}\tan^{-1}\left(\frac{2\sqrt{2}\pi}{\pi^2-2}\right)+\tan^{-1}\frac{\sqrt{2}}{\pi}$$

$$=\frac{\pi}{2}+\frac{1}{2}\tan^{-1}\left(\frac{\pi}{\sqrt{2}}\right)-\frac{1}{4}\left(2\tan^{-1}\left(\frac{\pi}{\sqrt{2}}\right)-\pi\right)$$

$$=\frac{\pi}{2}+\frac{1}{2}\tan^{-1}\left(\frac{\pi}{\sqrt{2}}\right)-\frac{1}{2}\tan^{-1}\left(\frac{\pi}{\sqrt{2}}\right)+\frac{\pi}{4}$$

$$=\frac{3\pi}{4}$$

[:Q.2] Let α be a positive real number. Let $f:\mathbb{R}\to\mathbb{R}$ and $g:(\alpha,\infty)\to\mathbb{R}$ be the functions defined by

$$f(x)=\sin\left(\frac{\pi x}{12}\right) \quad \text{and} \quad g(x)=\frac{2\log_e(\sqrt{x}-\sqrt{\alpha})}{\log_e(e^{\sqrt{x}}-e^{\sqrt{\alpha}})}.$$

Then the value of $\lim\limits_{x\to\alpha^+} f(g(x))$ is __________.

[:ANS] **0.50**

[:SOLN] $\lim\limits_{x\to\alpha^+} g(x)$

$$= \underset{x\to\alpha^+}{\text{Lt}} \frac{2\ln(\sqrt{x}-\sqrt{\alpha})}{\ln(e^{\sqrt{x}}-e^{\sqrt{\alpha}})}$$

$$\underset{x\to\alpha^+}{\text{Lt}} \frac{2\ln(\sqrt{x}-\sqrt{\alpha})}{\ln(e^{\sqrt{\alpha}}(e^{\sqrt{x}-\sqrt{\alpha}}-1))}$$

$$\underset{x\to\alpha^+}{\text{Lt}} \frac{2\ln(\sqrt{x}-\sqrt{\alpha})}{\ln(e^{\sqrt{\alpha}}(\sqrt{x}-\sqrt{\alpha}))}=2$$

Now, $\lim\limits_{x\to\alpha^+} f(g(x))=f(2)=\sin\left(\frac{\pi}{6}\right)=\frac{1}{2}$

[:Q.3] In a study about a pandemic, data of 900 persons was collected. It was found that

190 persons had symptom of fever,

220 persons had symptom of cough,

220 persons had symptom of breathing problem,

330 persons had symptom of fever or cough or both,

350 persons had symptom of cough or breathing problem or both,

340 persons had symptom of fever or breathing problem or both,

30 persons had all three symptoms (fever, cough and breathing problem).

If a person is chosen randomly from these 900 persons, then the probability that the person has at most one symptom is _____________.

[:ANS] **0.80**

[:SOLN] $n(F)=190, n(C)=220, n(B)=220$

$n(F\cup C)=330, n(C\cup B)=350$

$n(F\cup B)=340, n(F\cap C\cap B)=30$

$n(F\cap C)=190+220-330=80$

$n(C \cap B) = 220 + 220 - 350 = 90$

$n(F \cap B) = 190 + 220 - 340 = 70$

Required Probability = $\dfrac{900-(80+90+70-60)}{900} = 0.8$

[:Q.4] Let z be a complex number with non-zero imaginary part. If $\dfrac{2+3z+4z^2}{2-3z+4z^2}$ is a real number, then the value of $|z|^2$ is_____________.

[:ANS] 0.50

[:SOLN] Let w = $\dfrac{2+3z+4z^2}{2-3z+4z^2}$

for w to be real

$w = \bar{w}$

$\Rightarrow \dfrac{2+3z+4z^2}{2-3z+4z^2} = \dfrac{2+3\bar{z}+4\bar{z}^2}{2-3\bar{z}+4\bar{z}^2}$

$\Rightarrow \dfrac{4+8z^2}{6z} = \dfrac{4+8\bar{z}^2}{6\bar{z}}$

$\Rightarrow z - \bar{z} + 2z\bar{z}(\bar{z} - z) = 0$

$\Rightarrow (z-\bar{z})(1-2z\bar{z}) = 0$

$\Rightarrow z\bar{z} = \dfrac{1}{2} \Rightarrow |z|^2 = \dfrac{1}{2}$

[:Q.5] Let $\bar{z}$ denote the complex conjugate of a complex number z and let $i = \sqrt{-1}$. In the set of complex numbers, the number of distinct roots of the equation $\bar{z} - z^2 = i(\bar{z} + z^2)$ is _______.

[:ANS] 4

[:SOLN] $\bar{z} - z^2 = i(\bar{z} + z^2)$

$\Rightarrow \bar{z}(1-i) = z^2(1+i); z = 0 \rightarrow \text{one solution}$

$\Rightarrow |z|^2 (1-i) = z^3(1+i) \ldots.(i)$

$\Rightarrow |z^3||1+i| = |z|^2 |1-i| \rightarrow |z| = 1$

from (i); $z^3 = \dfrac{1-i}{1+i}$

$\Rightarrow z^3 = -i \rightarrow \text{three solutions}$

$\therefore$ total four solutions.

[:Q.6] Let $l_1, l_2, \ldots, l_{100}$ be consecutive terms of an arithmetic progression with common difference d_1, and let $w_1, w_2, \ldots, w_{100}$ be consecutive terms of another arithmetic progression with common difference d_2, where $d_1 d_2 = 10$. For each i = 1, 2,…, 100, let R_i be a rectangle with length l_i width w_i and area A_i. If $A_{51} - A_{50} = 1000$, then the value of $A_{100} - A_{90}$ is _________.

[:ANS] **18900**

[:SOLN] $A_i = l_i w_i = (l_1 + (i-1)d_1)(w_1 + (i-1)d_2)$

$\Rightarrow A_i = l_1 w_1 + (i-1)(d_1 w_1 + d_2 l_1) + (i-1)^2 d_1 d_2$

$\Rightarrow A_i = l_1 w_1 + (i-1)(d_1 w_1 + d_2 l_1) + 10(i-1)^2$

$A_{51} - A_{50} = 1000$

$\Rightarrow 50(d_1 w_1 + d_2 l_1) + 10 \times 50^2 - 49(d_1 w_1 + d_2 l_1) - 10 \times 49^2 = 1000$

$\Rightarrow d_1 w_1 + d_2 l_1 = 10 \qquad \text{...(i)}$

$A_{100} - A_{90} = 99(d_1 w_1 + d_2 l_1) + 10 \times 99^2 - 89(d_1 w_1 + d_2 l_1) - 10 \times 89^2$

$= 99 \times 10 - 89 \times 10 + 10 \times (99^2 - 89^2)$

$= 100 + 100(99 + 89) = 18900.$

[:Q.7] The number of 4-digit integers in the closed interval [2022, 4482] formed by using the digits 0, 2, 3, 4, 6, 7 is ____________.

[:ANS] **569**

[:SOLN] 20 ——→ 5 × 6 = 30 – 1(2020) = 20

2 ———→ 5 × 6 × 6 = 180

3 ———→ 6 × 6 × 6 = 216

4 ———→4 × 6 × 6 = 144

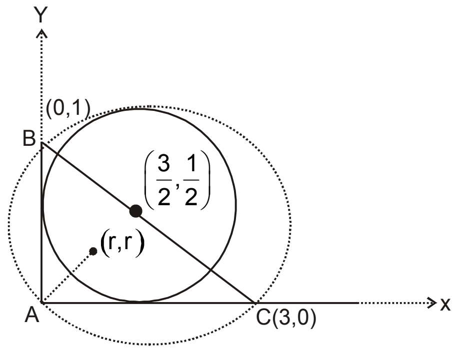
____

569 numbers

[:Q.8] Let ABC be the triangle with AB = 1, AC = 3 and $\angle BAC = \frac{\pi}{2}$. If a circle of radius r > 0 touches the sides AB, AC and also touches internally the circumcircle of the triangle ABC, then the value of r is __________.

[:ANS] **0.84**

[:SOLN]



Larger circle radius

i.e. circum radius = $\frac{\sqrt{10}}{2}$

distance between centres

$$=\sqrt{\left(r-\frac{3}{2}\right)^2+\left(r-\frac{1}{2}\right)^2}$$

Internal touching

$$\Rightarrow \sqrt{2r^2-4r+\frac{10}{4}}=\frac{\sqrt{10}}{2}-r$$

$$\Rightarrow 2r^2-4r+\frac{10}{4}=\frac{10}{4}+r^2-\sqrt{10}r$$

$$\Rightarrow r^2-(4-\sqrt{10})r=0$$

$$\Rightarrow r=4-\sqrt{10}=4-3.162$$

$$\Rightarrow r=0.8377$$

SECTION 2 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: **+4**	If **ONLY** (all) the correct option(s) is (are) chosen.
Partial Marks	: **+3**	If all the four options are correct but **ONLY** three options are chosen.
Partial Marks	: **+2**	If three or more options are correct but **ONLY** two options are chosen, both of which are correct options.
Partial Marks	: **+1**	If two or more options are correct but **ONLY** one option is chosen and it is a correct option.
Zero Marks	: **0**	If none of the options is chosen (i.e. the question is unanswered).
Negative Marks	: **–2**	In all other cases.

[:Q.9] Consider the equation

$$\int_1^e \frac{(\log_e x)^{1/2}}{x(a-(\log_e x)^{3/2})^2}\,dx = 1, \quad a \in (-\infty, 0) \cup (1, \infty)$$

Which of the following statements is/are **TRUE** ?

[:A] No a satisfies the above equation

[:B] An integer a satisfies the above equation

[:C] An irrational number a satisfies the above equation

[:D] More than one a satisfy the above equation

[:ANS] C, D

[:SOLN] $$\int_1^e \frac{(\ln x)^{1/2}}{x(a-(\ln x)^{3/2})^2}\,dx = 1$$

Let, $a-(\ln x)^{3/2} = t$

$$-\frac{3}{2}\frac{(\ln x)^{1/2}}{x}\,dx = dt$$

$$\frac{(\ln x)^{1/2}}{x}\,dx = -\frac{2}{3}dt$$

$$\Rightarrow -\frac{2}{3}\int_a^{a-1}\frac{dt}{t^2} = 1$$

$$\Rightarrow \int_a^{a-1}\frac{dt}{t^2} = -\frac{3}{2} \Rightarrow -\left[\frac{1}{t}\right]_a^{a-1} = -\frac{3}{2}$$

$$\Rightarrow \frac{1}{a}-\frac{1}{a-1} = -\frac{3}{2}$$

$$\Rightarrow 3a^2-3a-2=0 \qquad a=\frac{3\pm\sqrt{33}}{6}$$

(C) and (D)

[:Q.10] Let a_1, a_2, a_3,…. be an arithmetic progression with $a_1 = 7$ and common difference 8. Let T_1, T_2, T_3, …. be such that $T_1 = 3$ and $T_{n+1} - T_n = a_n$ for $n \geq 1$. Then, which of the following is/are TRUE ?

[:A] $T_{20} = 1604$

[:B] $\sum_{k=1}^{20} T_k = 10510$

[:C] $T_{30} = 3454$

[:D] $\sum_{k=1}^{30} T_k = 35610$

[:ANS] B, C

[:SOLN] $a_1, a_2, \ldots\ldots\ldots\ldots a_n$ in A.P. $a_1 = 7$, $d = 8$.

$T_1, T_2, T_3 \ldots\ldots\ldots T_n \ldots\ldots$ (diff. in A.P)

a_1, a_2

$3, 10, 25, \ldots\ldots\ldots\ldots\ldots T_n = an^2 + bn + c$

$T_1 = a + b + c = 3$ …(1)

$T_2 = 4a + 2b + c = 10$ …(2)

$T_3 = 9a + 3b + c = 25$ …(3)

$a = 4$ $b = -5$ $c = 4$

$\because$ $T_n = 4n^2 - 5n + 4$ $\Rightarrow T_{20} = 1600 - 100 + 4$

$T_{30} = 3454$

$S_n = \sum T_n$

$= 4\Sigma n^2 - 5\Sigma n + \Sigma 4$

$$= 4\left\{\frac{n(n+1)(2n+1)}{6}\right\} - 5\frac{n(n+1)}{2} + 4n$$

Now,

$S_{30} = 35615$

and $S_{20} = 10510$

[:Q.11] Let P_1 and P_2 be two planes given by

$P_1 : 10x + 15y + 12z - 60 = 0,$

$P_2 : -2x + 5y + 4z - 20 = 0.$

Which of the following straight lines can be an edge of some tetrahedron whose two faces lie on P_1 and P_2 ?

[:A] $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{5}$

[:B] $\frac{x-6}{-5} = \frac{y}{2} = \frac{z}{3}$

[:C] $\frac{x}{-2} = \frac{y-4}{5} = \frac{z}{4}$

[:D] $\frac{x}{1} = \frac{y-4}{-2} = \frac{z}{3}$

[:ANS] A, B, D

[:SOLN] The edges are skew line with common line of intersection. Edge of tetrahedron be either common intersection line or line intersecting both P_1& P_2 or line lie in P_2/P_1& meeting line.

$\because$ Common intersection line will be

$$\frac{x}{0}=\frac{y}{-4}=\frac{z-5}{5}$$

So by the option

(A), (B) (D) are correct.

[:Q.12] Let S be the reflection of a point Q with respect to the plane given by

$$\vec{r}=-(t+p)\hat{i}+t\hat{j}+(1+p)\hat{k}$$

where t, p are real parameters and $\hat{i},\hat{j},\hat{k}$ are the unit vectors along the three positive coordinate axes. If the position vectors of Q and S are $10\hat{i}+15\hat{j}+20\hat{k}$ and $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ respectively, then which of the following is/are **TRUE** ?

[:A] $3(\alpha+\beta)=-101$

[:B] $3(\alpha+\gamma)=-71$

[:C] $3(\gamma+\alpha)=-86$

[:D] $3(\alpha+\beta+\gamma)=-121$

[:ANS] A, B, C

[:SOLN] $\vec{r}=-(t+P)\hat{i}+t\hat{j}+(1+P)\hat{k}$

mid pt. of S.P. Q lies on the plane.

$$\Rightarrow\therefore\frac{10+\alpha}{2}i+\frac{15+\beta}{2}j+\frac{20+r}{2}k$$

$$=-(t+P)\hat{i}+t\hat{j}+(1+P)\hat{k}$$

on comparing.

$$t+P=-\left(\frac{10+\alpha}{2}\right)\quad ...(1)\qquad t=\frac{15+\beta}{2}\quad ...(2)$$

$$\because 1+P=\frac{20+r}{2}\quad ...(3)$$

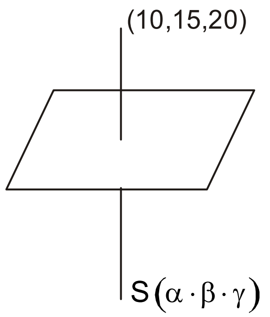
$$\therefore \boxed{P=\frac{18+r}{2}}$$ using t & P in (1)

$$15+\beta+18+r=-10-\alpha$$

$$\boxed{\alpha+\beta+r=-43}$$

Also $X = -t - P \quad \therefore y = t \quad z = 1 + P$

$\boxed{x + y + z = 1}$ Plane in container form.



$$\frac{\alpha - 10}{1} = \frac{\beta - 15}{1} = \frac{r - 15}{1} = -2\left\{\frac{44}{3}\right\} = -\frac{88}{3}$$

So, $\alpha = -\frac{58}{3}, \beta = -\frac{43}{3}, r = -\frac{28}{3}$

Now, $\alpha + \beta = -\frac{101}{3}$ ...(A)

$\beta + \gamma = -\frac{71}{3}$ ...(B)

$\gamma + \alpha = -\frac{86}{3}$ ...(C)

(A) (B) (C) Correct.

[:Q.13] Consider the parabola $y^2 = 4x$. Let S be the focus of the parabola. A pair of tangents drawn to the parabola from the point P = (−2, 1) meet the parabola at P_1 and P_2. Let Q_1 and Q_2 be points on the lines SP_1 and SP_2 respectively such that PQ_1 is perpendicular to SP_1 and PQ_2 is perpendicular to SP_2. Then, which of the following is/are **TRUE** ?

[:A] $SQ_1 = 2$

[:B] $Q_1Q_2 = \frac{3\sqrt{10}}{5}$

[:C] $PQ_1 = 3$

[:D] $SQ_2 = 1$

[:ANS] **B, C, D**

[:SOLN] $y^2 = 4x$

S = (1, 0)

Any point on this parabola is $(t^2, 2t)$

$\therefore$ Equation of tangent at $(t^2, 2t)$ is $yt = x + t^2$

This will pass through p (–2, 1) if t = $-2 + t^2$

$\therefore p_1 = (4, 4)$ and $p_2 = (1, -2)$

Equation of SP_1 is 4x – 3y – 4 = 0 and that of SP_2 is x – 1 = 0

So let $Q_1 = \left(\alpha, \frac{4\alpha - 4}{3}\right)$ and $Q_2 = (1, \beta)$

AΔ PQ, $\perp^r$ to SP_1 and $PQ_2 \perp SP_2$

$$\therefore \left(\frac{\frac{4\alpha-4}{3} - 1}{\alpha + 2}\right)\left(\frac{4}{3}\right) = -1 \text{ and } \left(\frac{\beta - 1}{1 + 2}\right)(\alpha) = -1$$

$\Rightarrow \alpha = \frac{2}{5}$ and $\Delta = 1$

$\therefore Q_1 = \left(\frac{2}{5}, \frac{-4}{5}\right)$ and $Q_2 = (1, 1)$

$\therefore SQ_1 = 1, Q_1Q_2 = \frac{3\sqrt{10}}{5}$, $PQ_1 = 3$, $SQ_2 = 1$

Hence B, C and D are true.

[:Q.14] Let |M| denote the determinant of a square matrix M. Let $g: \left[0, \frac{\pi}{2}\right] \to \mathbb{R}$ be the function defined by where $g(\theta) = \sqrt{f(\theta) - 1} + \sqrt{f\left(\frac{\pi}{2} - \theta\right) - 1}$

$$f(\theta) = \frac{1}{2}\begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix} + \begin{vmatrix} \sin\theta & \cos\left(\theta + \frac{\pi}{4}\right) & \tan\left(\theta - \frac{\pi}{4}\right) \\ \sin\left(\theta - \frac{\pi}{4}\right) & -\cos\frac{\pi}{2} & \log_e\left(\frac{4}{\pi}\right) \\ \cot\left(\theta + \frac{\pi}{4}\right) & \log_e\left(\frac{\pi}{4}\right) & \tan\pi \end{vmatrix}.$$

Let p(x) be a quadratic polynomial whose roots are the maximum and minimum values of the function g(θ), and p(2) = $2 - \sqrt{2}$. Then, which of the following is/are **TRUE** ?

[:A] $p\left(\frac{3 + \sqrt{2}}{4}\right) < 0$

[:B] $p\left(\frac{1 + 3\sqrt{2}}{4}\right) > 0$

[:C] $p\left(\frac{5\sqrt{2}-1}{4}\right)>0$

[:D] $p\left(\frac{5-\sqrt{2}}{4}\right)<0$

[:ANS] A, C

[:SOLN] $f(0)=\frac{1}{2}\left(2+2\sin^2\theta\right)+0=1+\sin^2\theta$

[because second determinant is skew-symmetric of odd order

$\therefore g(\theta)=\sqrt{\sin^2\theta}+\sqrt{\cos^2\theta}$

$= |\sin\theta|+|\cos\theta|$

$= \sqrt{1+|\sin 2\theta|}$

$\therefore$ max and min value of g(θ) are $\sqrt{2}$ and 1 respectively.

$\therefore p(x)=a\left(x-\sqrt{2}\right)(x-1)$

But $p(2)=2-\sqrt{2}$ $\therefore a=1$

$\therefore p(x)=\left(x-\sqrt{2}\right)(x-1)$

$\therefore$ A, C are true

SECTION3 (Maximum Marks : 12)

- This section contains FOUR (04) Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (I), (II), (III) and (IV) and **List-II** has **Five** entries (P), (Q), (R), (S) and (T).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** ofthese four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : −1 In all other cases.

[:Q.15] Consider the following lists:

	List-I		List-II
(I)	$\left\{x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right] : \cos x + \sin x = 1\right\}$	(P)	has two elements
(II)	$\left\{x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3}\tan 3x = 1\right\}$	(Q)	has three elements
(III)	$\left\{x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right] : 2\cos(2x) = \sqrt{3}\right\}$	(R)	has four elements
(IV)	$\left\{x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$	(S)	has five elements
		(T)	has six elements

Codes :

[:A] (I) → (P); (II) → (S); (III) → (P); (IV) → (S)

[:B] (I) → (P); (II) → (P); (III) → (T); (IV) → (R)

[:C] (I) → (Q); (II) → (P); (III) → (T); (IV) → (S)

[:D] (I) → (Q); (II) → (S); (III) → (P); (IV) → (R)

[:ANS] **B**

[:SOLN] cos x + sin x = 1

$$\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x = \frac{1}{\sqrt{2}}$$

$$\cos x \cdot \cos\frac{\pi}{4} + \sin x \cdot \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$$

$$x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$$

$x = 2n\pi + \frac{\pi}{2}$ if $n = 0$ $x = \frac{\pi}{2}$

$x = 2n\pi$ $n = 0$ $x = 0$

so, exactly two solutions $1 \to P$

$$\tan x = \frac{1}{\sqrt{3}} \Rightarrow 3x = n\pi + \frac{\pi}{6}$$

$$x = \frac{n\pi}{3} + \frac{\pi}{18}$$

n = 0

$$x = \frac{\pi}{18} \qquad n = -1$$

$$x = -\frac{\pi}{3} + \frac{\pi}{18} = -\frac{5\pi}{18}$$

so, exactly two solutions 2 $\rightarrow$ P

$$\cos 2x = \frac{\sqrt{3}}{2}$$

$$\cos 2x = \cos\frac{\pi}{4}$$

$$2x = 2\cos\pi \pm \frac{\pi}{6}$$

$$x = n\pi \pm \frac{\pi}{12}$$

$$n = 0 \quad x = \frac{\pi}{12} \qquad n = 1 \quad x = \pi + \frac{\pi}{12}$$

$$x = \pi - \frac{\pi}{12}$$

$$-\pi + \frac{\pi}{12} \qquad -\pi - \frac{\pi}{12}$$

so, exactly six solutions 3 $\rightarrow$ T

$$\cos x - \sin x = -1$$

$$\cos\left(x + \frac{\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right]$$

$$x + \frac{\pi}{4} = 2n\pi \pm \frac{3\pi}{4}$$

$$x = 2n\pi + \frac{\pi}{2}$$

$$x = (2n-1)\pi$$

so, exactly four solutions 4 $\rightarrow$ R

(B) option is correct.

[:Q.16] Two players, P_1 and P_2, play a game against each other. In every round of the game, each player rolls a fair die once, where the six faces of the die have six distinct numbers. Let x and y denote the readings on the die rolled by P_1 and P_2, respectively. If $x>y$, then P_1 scores 5 points and P_2 scores 0 point. If $x = y$, then each player scores 2 points. If $x<y$, then P_1 scores 0 point and P_2 scores 5 points. Let X_i and Y_i be the total scores of P_1 and P_2, respectively, after playing the i^{th} round.

List-I		List-II	
(I)	Probability of $(x_2 \geq Y_2)$ is	(P)	$\frac{3}{8}$
(II)	Probability of $(x_2 > Y_2)$ is	(Q)	$\frac{11}{16}$
(III)	Probability of $(x_3 > Y_3)$ is	(R)	$\frac{5}{16}$
(IV)	Probability of $(X_3 > Y_3)$ is	(S)	$\frac{355}{864}$
		(T)	$\frac{77}{432}$

Codes :

[:A] (I) → (Q); (II) → (R); (III) → (T); (IV) → (S)

[:B] (I) → (Q); (II) → (R); (III) → (T); (IV) → (T)

[:C] (I) → (P); (II) → (R); (III) → (Q); (IV) → (S)

[:D] (I) → (P); (II) → (R); (III) → (Q); (IV) → (T)

[:ANS] **A**

[:SOLN] $P(X_1 > Y_1) + P(X_1 < Y_1] + P(X_1 = Y_1] = 1$

and $P(X_1 > Y_1) = P(X_1 < Y_1) = P$

for i = 2

$P(X_2 = Y_2) = 2P(X > Y), P(X < Y) + P(X = Y)^2$

$$= 2\frac{6C_2}{36} \times \frac{6C_2}{36} + \left(\frac{6C_1}{36}\right)^2$$

$$= \frac{25}{72} + \frac{1}{36} = \frac{77}{72} = \frac{3}{8}$$

$$P(X_2 > Y_2) = \frac{1}{2}\left(1 - \frac{3}{8}\right) = \frac{5}{16}$$

$P(X_2 \geq Y_2) = \frac{5}{16} + \frac{3}{8} = \frac{11}{16}$

$I \rightarrow Q, II \rightarrow R$

for i = 3

$P(X_3 = Y_3) = 6 \times P(X > Y) \times P(X < Y)$

$P(X = Y) + P(X = Y)^3$

$= 6 \times \frac{6C_2}{36} \times \frac{6C_2}{36} \times \frac{6C_1}{36} + \left(\frac{6C_1}{36}\right)^3$

$= \frac{77}{432}$

$III \rightarrow T, IV \rightarrow S$

$P(X_3 > Y_3) = \frac{1}{2}\left(1 - \frac{77}{432}\right) = \frac{355}{864}$

[:Q.17] Let p, q, r be nonzero real numbers that are, respectively, the 10^{th}, 100^{th} and 1000^{th} terms of a harmonic progression. Consider the system of linear equations

$$x + y + z = 1$$

$$10x + 100y + 1000z = 0$$

$$qr\,x + pr\,y + pqz = 0$$

List-I	**List-II**
(I) If $\frac{q}{r} = 10$, then the system of linear equations has	(P) $x = 0, y = \frac{10}{9}, z = -\frac{1}{9}$ as a solution
(II) If $\frac{p}{r} \neq 100$, then the system of linear equations has	(Q) $x = \frac{10}{9}, y = -\frac{1}{9}, z = 0$ as a solution
(III) If $\frac{p}{q} \neq 10$, then the system of linear equations has	(R) Infinitely many solutions
(IV) If $\frac{p}{q} = 10$, then the system of linear equation has	(S) no solution
	(T) at least one solution

Codes :

[:A] (I) → (T); (II) → (R); (III) → (S); (IV) → (T)

[:B] (I) → (Q); (II) → (S); (III) → (S); (IV) → (R)

[:C] (I) → (Q); (II) → (R); (III) → (P); (IV) → (R)

[:D] (I) → (T); (II) → (S); (III) → (P); (IV) → (T)

[:ANS] **B**

[:SOLN] Let d be the common difference

$$\frac{1}{p} = a + 9d$$

$$\frac{1}{q} = a + 99d$$

$$\frac{1}{r} = a + 999d$$

$$x + y + z = 1$$

$$10x + 100y + 1000z = 0$$

$$qrx + pry + pqz = 0$$

Divide by PQR.

$$\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 0$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 10 & 100 & 1000 \\ \frac{1}{p} & \frac{1}{q} & \frac{1}{r} \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 10 & 100 & 1000 \\ a+9d & a+99d & a+999d \end{vmatrix}$$

$$R_2 \to R_2 - R_1$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 9 & 99 & 999 \\ a+9d & a+99d & a+999d \end{vmatrix}$$

$R_3 \to R_3 - (aR_1 + dR_2)$ then

$$\begin{vmatrix} 1 & 1 & 1 \\ 9 & 99 & 999 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$\Rightarrow$ system will have either no solution or Infinite solution.

(1) q = 10 r. $\Rightarrow a = d$

$\Delta = \Delta x = \Delta y = \Delta z = 0$

So, (1) $\rightarrow$Q

(2) $\frac{P}{r} \neq 100$ $\Rightarrow a \neq d$

$\Delta = 0 \quad \Delta_x \neq 0 \;\; \Delta_y \neq 0 \;\; \Delta_z \neq 0$

So, (2) $\rightarrow$S.

(3) $\frac{p}{q} \neq 10 \Rightarrow a \neq d$

So, (3) $\rightarrow$S

(4) $\frac{p}{q} = 10$ then $\Delta = \Delta x = \Delta y = \Delta z = 0$

So, (4) $\rightarrow$R

[:Q.18] Consider the ellipse

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

Let $H(\alpha, 0), 0 < \alpha < 2,$, be a point. A straight line drawn through H parallel to the y-axis crosses the ellipse and its auxiliary circle at points E and F respectively, in the first quadrant. The tangent to the ellipse at the point E intersects the positive x-axis at a point G. Suppose the straight line joining F and the origin makes an angle ϕ with the positive x-axis.

List-I	**List-II**
(I) If $\phi = \frac{\pi}{4}$, then the area of the triangle FGH is	(P) $\frac{(\sqrt{3}-1)^4}{8}$
(II) If $\phi = \frac{\pi}{3}$, then the area of the triangle FGH is	(Q) 1
(III) If $\phi = \frac{\pi}{6}$, then the area of the triangle FGH is	(R) $\frac{3}{4}$
(IV) If $\phi = \frac{\pi}{12}$, then the area of the triangle FGH is	(S) $\frac{1}{2\sqrt{3}}$
	(T) $\frac{3\sqrt{3}}{2}$

Codes :

[:A] (I) → (R); (II) → (S); (III) → (Q); (IV) → (P)

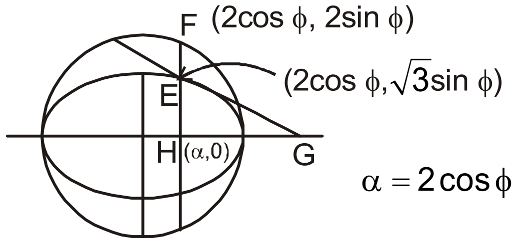
[:B] (I) → (R); (II) → (T); (III) → (S); (IV) → (P)

[:C] (I) → (Q); (II) → (T); (III) → (S); (IV) → (P)

[:D] (I) → (Q); (II) → (S); (III) → (Q); (IV) → (P)

[:ANS] **C**

[:SOLN]



$\alpha = 2\cos\phi$

$$\frac{x^2}{4}+\frac{y^2}{3}=1$$

$$\frac{x^2}{2^2}+\frac{y^2}{(\sqrt{3})^2}=1$$

$a=2 \quad b=\sqrt{3}$

$E=(2\cos\phi, \sqrt{3}\sin\phi)$

tangent at $E \equiv (2\cos\phi, \sqrt{3}\sin\phi)$

$$\frac{x}{2}\times\cos\phi+\frac{y}{\sqrt{3}}\sin\phi=1$$

$y=0$

$$x=\frac{2}{\cos\phi}=2\sec\phi$$

Area of FGH = $(2\sec\phi-\alpha)\times 2\sin\phi\times\frac{1}{2}$

$$=2(\sec\phi-\cos\phi)\times\sin\phi$$

when $\phi=\frac{\pi}{4}$

then Area of FGH = $2\left(\sqrt{2}-\frac{1}{\sqrt{2}}\right)\times\frac{1}{\sqrt{2}}$

$$=2\left(\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\right)=1$$

when $\phi = \frac{\pi}{3}$

Area of FGH = $2\left(2 - \frac{1}{2}\right) \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$

when $\phi = \frac{\pi}{6}$

Area of FGH = $2\left(\frac{2}{\sqrt{3}} - \frac{\sqrt{3}}{2}\right) \times \frac{1}{2} = \frac{4-3}{2\sqrt{3}} = \frac{1}{2\sqrt{3}}$

when $\phi = \frac{\pi}{12}$

Area of FGH. $= 2(\sec 15^\circ - \cos 15^\circ) \times \sin 15^\circ = \frac{\left(\sqrt{3}-1\right)^4}{8}$