JEE (ADVANCED) 2022 PAPER-2

[PAPER WITH SOLUTION]

HELD ON SUNDAY 28TH AUGUST 2022

MATHEMATICS

SECTION1 (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER ranging from 0 TO 9, BOTH INCLUSIVE**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If **ONLY** the correct integer is entered;

  Zero Marks:0 If the question is unanswered;

  Negative Marks : −1 In all other cases.

[:Q.1] Let α and β be real numbers such that $-\frac{\pi}{4} < \beta < 0 < \alpha < \frac{\pi}{4}$. If $\sin(\alpha+\beta) = \frac{1}{3}$ and $\cos(\alpha-\beta) = \frac{3}{2}$, then the greatest integer less than or equal to

$$\left(\frac{\sin\alpha}{\cos\beta} + \frac{\cos\beta}{\sin\alpha} + \frac{\cos\alpha}{\sin\beta} + \frac{\sin\beta}{\cos\alpha}\right)^2$$ is______________

[:ANS] **1**

[:SOLN] $\sin(\alpha+\beta) = \frac{1}{3}$ & $\cos(\alpha-\beta) = \frac{2}{3}$

$$E = \left[\frac{\sin\alpha}{\cos\beta} + \frac{\cos\beta}{\sin\alpha} + \frac{\cos\alpha}{\sin\beta} + \frac{\sin\beta}{\cos\alpha}\right]^2$$

$$= \left[\left(\frac{\sin\alpha}{\cos\beta} + \frac{\cos\alpha}{\sin\beta}\right) + \left(\frac{\cos\beta}{\sin\alpha} + \frac{\sin\beta}{\cos\alpha}\right)\right]^2$$

$$=\left[\frac{\cos(\alpha-\beta)}{\cos\beta\sin\beta}+\frac{\cos(\alpha-\beta)}{\sin\alpha\cos\alpha}\right]$$

$$=\cos^2(\alpha-\beta)\left[\frac{2}{\sin 2\beta}+\frac{2}{\sin 2\alpha}\right]^2$$

$$=4\cos^2(\alpha-\beta)\left[\frac{4\sin(\alpha+\beta).\cos(\alpha-\beta)}{\cos 2(\alpha-\beta)-\cos 2(\alpha+\beta)}\right]^2$$

$$=4\times\frac{4}{9}\left[\frac{4\times\frac{1}{3}\times\frac{2}{3}}{\left(2.\frac{4}{9}-1\right)-\left(1-\frac{2}{9}\right)}\right]^2=\frac{16}{9}$$

[:Q.2] If y(x) is the solution of the differential equation

$xdy-(y^2-4y)\,dx=0$ for $x>0$ $\quad y(1)=2.$

and the slope of the curve y= y(x) is never zero, then the value of $10y(\sqrt{2})$ is _________

[:ANS] **8**

[:SOLN] $xdy-(y^2-4y)\,dx=0$ for $x>0$ $\quad y(1)=2.$

$$\frac{dy}{y^2-4y}=\frac{dx}{x}$$

$$\frac{1}{4}\int\left[\frac{1}{y-4}-\frac{1}{y}\right]dy=\frac{dx}{x}$$

$$\frac{4-y}{y}=kx^4$$

$$y(1)=2\Rightarrow k=1$$

$$y=\frac{4}{1+x^4}$$

$10y(\sqrt{2})=8$

[:Q.3] The greatest integer less than or equal to

$$\int_1^2\log_2(x^3+1)dx+\int_1^{\log_2 9}(2^x-1)^{\frac{1}{3}}dx$$ is _____________

[:ANS] **5**

[:SOLN] $\int_a^b f(x)dx + \int_c^d f^{-1}(x)dx = bd - ac$ if $f(a) = c$ & $f(b) = d$

$$\int_1^2 \log_2(x^3+1)dx + \int_1^{\log_2 9} (2^x-1)^{\frac{1}{3}} dx = (2\log_2 9 - 1) \in (5,6)$$

[:Q.4] The product of all positive real values of x satisfying the equation

$x^{(16(\log_5 x)^3 - 68\log_5 x)} = 5^{-16}$ is ____________

[:ANS] **1**

[:SOLN] $x^{(16(\log_5 x)^3 - 68\log_5 x)} = 5^{-16}$

Taking log on both sides at base 5

$(16(\log_5 x)^3 - 68\log_5 x)\log_5 x = -16$

let $\log_5 x = t \Rightarrow (4t^2-1)(t^2-4) = 0$

[:Q.5] If $\beta = \lim_{x\to 0} \frac{e^{x^3} - (1-x^3)^{\frac{1}{3}} + (1-x^2)^{\frac{1}{2}} - 1)\sin x}{x\sin^2 x}$ then the value of 6β is ___________

[:ANS] **5**

[:SOLN] $$\beta = \lim_{x\to 0} \frac{e^{x^3} - (1-x^3)^{\frac{1}{3}} + \left((1-x^2)^{\frac{1}{2}} - 1\right)\sin x}{x\sin^2 x}$$

$$\beta = \lim_{x\to 0} \frac{e^{x^3} - (1-\frac{1}{3}x^3) + \left(1 - \frac{1}{2}x^2 + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}x^4 + ... - 1\right)\sin x}{x^3}$$

$$= \lim_{x\to 0} \frac{\left(1 + x^3 + \frac{(x^3)^2}{2!} + ...\right) - 1 + \frac{1}{3}x^3 + \left(-\frac{1}{2}x^2 - \frac{1}{8}x^4\right)\sin x}{x^3}$$

$$= \lim_{x\to 0} \frac{x^3\left(1 + \frac{x^3}{2!} + ...\right) + \frac{x^3}{3} + x^3\left(-\frac{1}{2} - \frac{1}{8}x\right)\frac{\sin x}{x}}{x^3}$$

$$= 1 + \frac{1}{3} - \frac{1}{2} = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

$$\beta = \frac{4}{3} \Rightarrow 6\beta = 5 \quad \beta = \frac{5}{6}$$

$$6\beta = 5$$

[:Q.6] Let β be a real number. Consider the matrix

$$A = \begin{pmatrix} \beta & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 1 & -2 \end{pmatrix}$$

If $A^7 - (\beta - 1)A^6 - \beta A^5$ is a singular matrix, then the value of 9β is ____________

[:ANS] **3**

[:SOLN] $A = \begin{pmatrix} \beta & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 1 & -2 \end{pmatrix}$

$A^7 - (\beta - 1)A^6 - \beta A^5$ singular matrix

$\Rightarrow \left|A^7 - (\beta - 1)A^6 - \beta A^5\right| = 0$

$\Rightarrow |A|^5 \left|A^2 - (\beta - 1)A - \beta I\right| = 0$

$\Rightarrow |A|^5 |A + I||A - \beta I| = 0$

$|A| = \begin{vmatrix} \beta & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 1 & -2 \end{vmatrix}$

$|A| = \beta(-2 + 2) - 0 + 1(2 - 3)$

$|A| = -1$

$|A + I| = \begin{vmatrix} \beta + 1 & 0 & 1 \\ 2 & 2 & -2 \\ 3 & 1 & -1 \end{vmatrix}$

$= (\beta + 1)(-2 + 2) - 0 + (2 - 6)$

$= -4$

As $|A| = -1$ & $|A - I| = -4$

$\Rightarrow |A - \beta I| = 0$

$\Rightarrow |A - \beta I| = \begin{vmatrix} 0 & 0 & 1 \\ 2 & -\beta & -2 \\ 3 & 1 & 2 - \beta \end{vmatrix} = 0$

$\Rightarrow 0 - 0 + 1(2 - 3 + 3\beta) = 0$

$-1+3\beta=0$

$\beta=-\frac{1}{3}$

$\Rightarrow 9\beta=3$

[:Q.7] Consider the hyperbola

$\frac{x^2}{100}-\frac{y^2}{64}=1$

with foci S and S_1, where S lies on the positive x–axis. let P be a point on the hyperbola, in the first quadrant. Let $\angle SPS_1=\alpha$, with $\alpha<\frac{\pi}{2}$. The straight line passing through the point S and having the same slope as that of tangent at P to the hyperbola, intersects the straight S_1P at P_1,. Let δ be the distance of P from the straight line SP_1, and $\beta=S_1P$. Then the greatest integer less than or equal to $\frac{\beta\delta}{9}\sin\frac{\alpha}{2}$ is___________

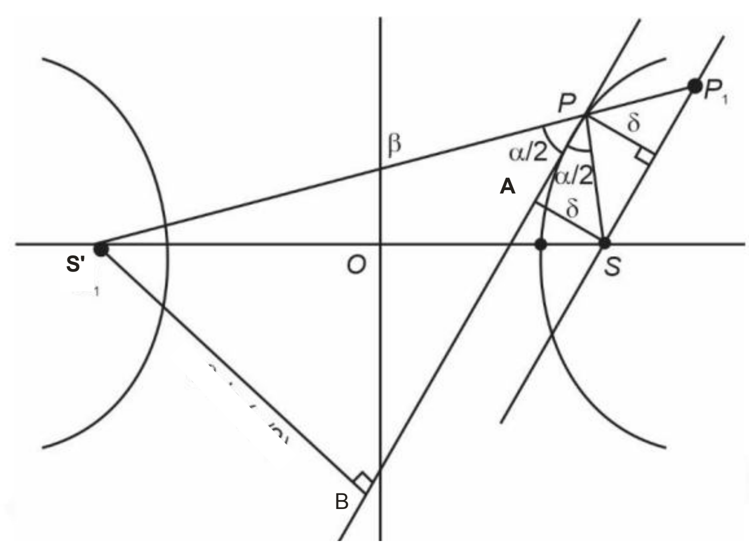
[:ANS] 7

[:SOLN] $\angle SPS_1=\alpha<\frac{\pi}{2}$

As product of perpendicular from two foci on any tangent to hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is b^2

$\Rightarrow SA.S'B=64$

$SA=\delta$ and $S'B=\beta\sin\frac{\alpha}{2}$



$$\Rightarrow SA.S'B = 64 \Rightarrow \Rightarrow \beta\delta ain\frac{\alpha}{2} = 64$$

$$\Rightarrow \frac{\beta\delta ain\frac{\alpha}{2}}{9} = \frac{64}{9} \Rightarrow \left[\frac{\beta\delta Sin\frac{\alpha}{2}}{9}\right] = 7$$

[:Q.8] Consider the functions $f, g : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = x^2 + \frac{5}{12} \quad \text{and} \quad g(x) = \begin{cases} 2\left(1 - \frac{4|x|}{3}\right) & |x| \le \frac{3}{4} \\ 0, & |x| > \frac{3}{4} \end{cases}$$

If α is the area of the region

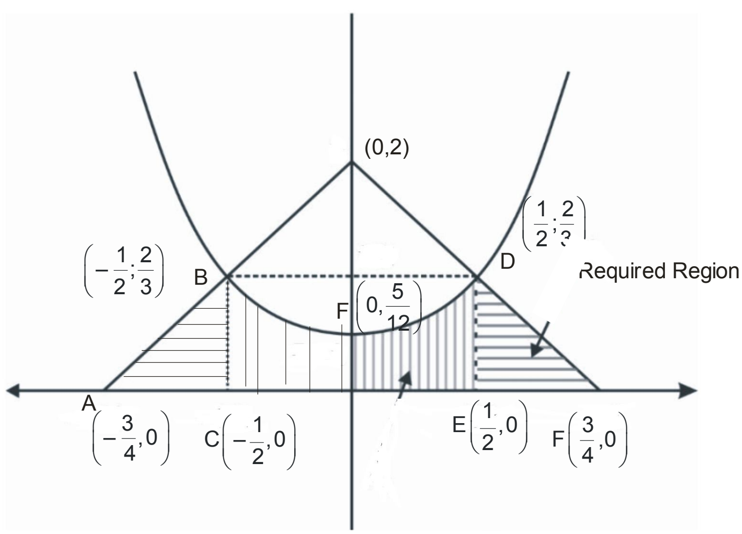
$$\left\{(x, y \in \mathbb{R} \times \mathbb{R} : |x| \le \frac{3}{4}, 0 \le y \le \min\{f(x), g(x)\}\right\}.$$ then the value of 9α is ________

[:ANS] **6**

[:SOLN] $f, g : R \to R$

$$f(x) = x^2 + \frac{5}{12} \text{ and } g(x) = \begin{cases} 2\left(1 - \frac{4|x|}{3}\right) & |x| \le \frac{3}{4} \\ 0 & |x| > \frac{3}{4} \end{cases}$$

$$f(x, y) \in R \times R \ : |x| \le 4, \quad 0 \le y \le \min\{f(x), g(x\}\}$$



$$f(x) = g(x) \Rightarrow \quad x^2 + \frac{5}{12} = 2\left(1 - \frac{4|x|}{3}\right)$$

$$12x^2 + 5 = 24 - 32|x|$$

$$12|x|^2 + 39|x| - 6|x| - 19 = 0$$

$$2|x|(6|x| + 19) - 1(6|x| + 19) = 0$$

$$\Rightarrow |x| = \frac{1}{2} \Rightarrow x = \pm\frac{1}{2}$$

Required Area $\alpha =$ Area of $\triangle ACB +$ Area of $CBFDEC +$ Area of $\triangle DEF$

$$= \frac{1}{2}\times\frac{1}{4}\times\frac{2}{3} + \int_{-1/2}^{1/2}\left(x^2 + \frac{5}{2}\right)dx + \frac{1}{2}\times\frac{1}{4}\times\frac{2}{3}$$

$$= \frac{1}{6} + 2\int_{0}^{1/2}\left(x^2 + \frac{5}{12}\right)dx$$

$$= \frac{1}{6} + 2\left[\frac{x^3}{3} + \frac{5}{12}x\right]_0^{\frac{1}{2}}$$

$$= \frac{1}{6} + 2\left[\frac{1}{24} + \frac{5}{24} - 0\right]$$

$$= \frac{1}{6} + 2\left[\frac{6}{24}\right]$$

$$= \frac{1}{6} + \frac{3}{6} = \frac{4}{6} = \frac{2}{3}$$

$$\alpha = \frac{2}{3} \Rightarrow 9\alpha = 9\times\frac{2}{3} = 6$$

SECTION2 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : **+4** If **ONLY** (all) the correct option(s) is (are) chosen.
  Partial Marks : **+3** If all the four options are correct but **ONLY** three options are chosen.
  Partial Marks : **+2** If three or more options are correct but **ONLY** two options are chosen, both of which are correct options.
  Partial Marks : **+1** If two or more options are correct but **ONLY** one option is chosen and it is a correct option.
  Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks : **–2** In all other cases.

[:Q.9] Let PQRS be a quadrilateral in a plane, where QR=1, $\angle PQR = \angle QRS = 70^0, \angle PQS = 15^0$ and $\angle PRS = 40^0$. If $\angle RPS = \theta^0$, $PQ = \alpha$ and $PS = \beta$, then the interval (s) that contain(s) the value of $4\alpha\beta\sin\theta^0$ is/are

[:A] $(0, \sqrt{2})$

[:B] (1,2)

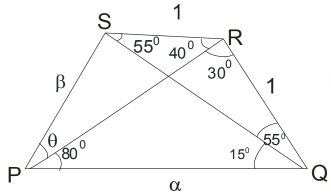
[:C] $(\sqrt{2}, 3)$

[:D] $(2\sqrt{2}, 3\sqrt{2})$

[:ANS] **A,B**

[:SOLN] $\triangle PRQ$

Sine Rule, $\dfrac{RQ}{\text{Sin}80^0} = \dfrac{\alpha}{\sin 30^0} \Rightarrow 2\alpha = \dfrac{1}{\sin 80^0}$.........(i)



RS=RQ=1

also, $\triangle PSR$

$$\frac{RS}{\sin\theta} = \frac{\beta}{\sin 40^0} \Rightarrow \beta\sin\theta = \sin 40^0 \text{.......(ii)}$$

from(i) & (ii)

$$2\alpha\beta\sin\theta = \frac{\sin 40^0}{\sin 80^0} = \frac{1}{2\cos 40^0}$$

$\Rightarrow 4\alpha\beta\sin\theta = \sec 40^0$.........(iii)

$\because\ 30^0 < 40^0 < 45$

$\Rightarrow \sec 30^0 < \sec 40^0 < \sec 45^0$

$\Rightarrow \dfrac{2}{\sqrt{3}} < \sec 40^0 < \sqrt{2}$

from (iii)

$$\frac{2}{\sqrt{3}} < 4\alpha\beta\sin\theta < \sqrt{2}$$

[:Q.10] Let $\alpha = \sum_{k=1}^{\infty} \sin^{2k}\left(\frac{\pi}{6}\right)$

Let $g:[0,1] \to \mathbb{R}$ be the function defined by

$g(x)=2^{\alpha x} + 2^{\alpha(1-x)}$

Then, which of the following statements is/are TRUE ?

[:A] The minimum value of g(x) is $2^{\frac{7}{6}}$

[:B] The maximum value of g(x) is $1+2^{\frac{1}{3}}$

[:C] The function g(x) attains its maximum at more than one point

[:D] The function g(x) attains its minimum at more than one point

[:ANS] **ABC**

[:SOLN] $\alpha = \sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^{2k} = \sum_{k=1}^{\infty}\left(\frac{1}{4}\right)^{k} = \frac{1}{3}$

$\because\ g(x) = 2^{x/3} + 2^{\frac{1}{3}(1-x)}$

$g'(x) = \frac{\ln 2}{3}.\left[2^{x/3} - 2^{\left(\frac{1-x}{3}\right)}\right]$

for max or min g'(x) = 0

$\Rightarrow x = \frac{1}{2}$

$g''\left(\frac{1}{2}\right) > 0$

$\therefore$ minimum value at $x = \frac{1}{2}$ & $g(x)_{min} = 2^{\frac{7}{6}}$

and max. value at x = 0 or x = 1 & $g(x)_{max} = 1+2^{\frac{1}{3}}$

[:Q.11] Let $\overline{Z}$ denote the complex conjugate of a complex number z. If z is non–zero complex number for which both real and imaginary parts of

$\left(\overline{Z}\right)^2 + \frac{1}{z^2}$

are integers, then which of the following is/are possible value(s) of $|z|$?

[:A] $\left(\dfrac{43+3\sqrt{205}}{2}\right)^{\frac{1}{4}}$

[:B] $\left(\dfrac{7+\sqrt{33}}{4}\right)^{\frac{1}{4}}$

[:C] $\left(\dfrac{9+\sqrt{65}}{4}\right)^{\frac{1}{4}}$

[:D] $\left(\dfrac{7+\sqrt{13}}{6}\right)^{\frac{1}{4}}$

[:ANS] A

[:SOLN] $Z = r.e^{i\theta}\ ,\bar{Z} = r.e^{-i\theta}$

then $\bar{Z}^2 + \dfrac{1}{Z^2} = r^2.e^{-i2\theta} + \dfrac{1}{r^2.e^{i2\theta}}$

$= \left(r^2 + \dfrac{1}{r^2}\right).e^{-i2\theta}$

$= \underbrace{\left(r^2 + \dfrac{1}{r^2}\right).\cos 2\theta}_{\text{integer}} - i\underbrace{\left(r^2 + \dfrac{1}{r^2}\right).\sin 2\theta}_{\text{integer}}$

$r^2 + \dfrac{1}{r^2}.\cos 2\theta = I_1$ ....(i)

$\& \, r^2 + \dfrac{1}{r^2}.\sin 2\theta = I_2$ ....(ii)

squaring (i) & (ii) and then add, we get

$r^4 + \dfrac{1}{r^4} = \text{integer}$

Now, check option A,B,C & D, then we get,

r= $\left(\dfrac{43+3\sqrt{205}}{2}\right)^{\frac{1}{4}}$ is correct (as, $r^4 + \dfrac{1}{r^4} = \text{integer}$ is an integer)

[:Q.12] Let G be a circle of radius R > 0. Let $G_1, G_2, \ldots, G_n$ be n circles of equal radius r > 0. Suppose each of the n circles $G_1, G_2, \ldots, G_n$ touches the circle G externally. Also, for i = 1, 2, ……,n–1, the circle G_i touches G_{i+1} externally, and G_n touches G_1 externally. Then, which of the following statement is/are TRUE ?

[:A] If n = 4, then $(\sqrt{2}-1)r < R$

[:B] If n = 5, then r < R

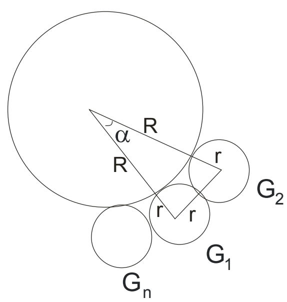
[:C] If n = 8, then $(\sqrt{2}-1)r < R$

[:D] If n = 12, then $\sqrt{2}(\sqrt{3}+1)r > R$

[:ANS] **C,D**

[:SOLN] $\because n\alpha = 360^0$

$$\Rightarrow \alpha = \frac{360^0}{n}$$



cosine Rule

$$\cos\left(\frac{360^0}{n}\right) = \frac{2(R+r)^2 - (2r)^2}{2(R+r)^2}$$

$$\Rightarrow \frac{2r^2}{(R+r)^2} = 1 - \cos\left(\frac{360^0}{n}\right)$$

$$\Rightarrow \frac{r^2}{(R+r)^2} = \sin^2\left(\frac{180^0}{n}\right) \Rightarrow \frac{r}{R+r} = \sin\left(\frac{180^0}{n}\right)$$

$$r = \frac{R}{\operatorname{cosec}\left(\frac{\pi}{n}\right) - 1}$$

for n = 4 $\quad R = (\sqrt{2}-1)r$

for n= 5 $\quad R = (0.7)r \Rightarrow r < r$

for n = 8 $R > (\sqrt{2} - 1)r$

for n=12 $R < (\sqrt{3} + 1)r$

[:Q.13] Let $\hat{i}, \hat{j}$ and $\hat{k}$ be the unit vectors along the three positive coordinate axes. Let

$\vec{a} = 3\hat{i}, \hat{j} - \hat{k},$

$\vec{b} = \hat{i}, +b_2\hat{j} + b_3\hat{k}, \quad b_2, b_3 \in \mathbb{R}$

$\vec{c} = C_1\hat{i} + c_2\hat{j} + c_3\hat{k}, \quad c_1, c_2, c_3 \in \mathbb{R}$

be three vectors such that $b_2b_3 > 0$, $\vec{a}.\vec{b} = 0$ and

$$\begin{pmatrix} 0 & -c_3 & c_2 \\ c_3 & 0 & -c_1 \\ -c_2 & c_1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 3-c_1 \\ 1-c_2 \\ -1-c_3 \end{pmatrix}$$

Then, which the following is/are TRUE ?

[:A] $\vec{a}.\vec{c} = 0$

[:B] $\vec{b}.\vec{c} = 0$

[:C] $|\vec{b}| > \sqrt{10}$

[:D] $|\vec{c}| \le \sqrt{11}$

[:ANS] **B, C, D**

[:SOLN] Since $\bar{a} \,.\, \bar{b} = 0$

$\therefore\ 3 + b_2 - b_3 = 0$

$\therefore\ b_2 - b_3 = -3$

$$\therefore\ |\vec{b}| = \sqrt{1^2 + b_2^{\ 2} + b_3^{\ 2}} = \sqrt{1 + (b_2 - b_3)^2 + 2b_2b_3}$$

$$= \sqrt{1 + 9 + 2b_2b_3}$$

$$= \sqrt{10 + 2b_2b_3} > \sqrt{10}$$

$$\begin{pmatrix} 0 & -C_3 & C_2 \\ C_3 & 0 & -C_1 \\ -C_2 & C_1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 3-C_1 \\ 1-C_2 \\ -1-C_3 \end{pmatrix}$$

$\Rightarrow \vec{c} \times \vec{b} = \vec{a} - \vec{c}$........(1)

$\Rightarrow \left(\vec{c} \times \vec{b}\right).\vec{b} = \vec{a}.\vec{b} - \overrightarrow{c..b}$

$0 = 0 - \vec{b}.\vec{c}$

$\Rightarrow \vec{b}.\vec{c} = 0$

from (i)

$\Rightarrow .(\vec{c} \times \vec{b}).\vec{c} = \vec{a}.\vec{c} - \vec{c}.\vec{c}$

$0 = \vec{a}.\vec{c} - |c|^2$

$\Rightarrow |\vec{c}|^2 = \vec{a}.\vec{c}$

$\Rightarrow |\vec{c}|^2 = |\vec{a}||\vec{c}|\cos\theta \le |\vec{a}||\vec{c}|$

$\Rightarrow |\vec{c}|^2 \le |\vec{a}||\vec{c}|$

$\Rightarrow |\vec{c}| \le |\vec{a}|$

$|\vec{a}| = \sqrt{9+1+1}$

$\Rightarrow |\vec{c}| \le \sqrt{11}$

[:Q.14] For $x \in \mathbb{R}$, let the function y(x) be the solution of the differential equation

$$\frac{dy}{dx} + 12y = \cos\left(\frac{\pi}{12}x\right), \quad y(0) = 0$$

Then, which of the following statements is/are TRUE ?

[:A] y(x) is an increasing function

[:B] y(x) is a decreasing function

[:C] There exists a real number β such that the line $y = \beta$ intersects the curve y = y(x) at infinitely many points

[:D] y(x) is a periodic function

[:ANS] **C**

[:SOLN]

$$\frac{dy}{dx} + 12y = \cos\left(\frac{\pi x}{12}\right)$$

$$I.F. = e^{12x} \Rightarrow y.e^{12x} = \int e^{12x} \cdot \cos\left(\frac{\pi x}{12}\right)dx + C$$

$$\Rightarrow \quad y \cdot e^{12x} = \frac{e^{12x}}{12^2 + \left(\frac{\pi}{12}\right)^2}\left[12\cos\frac{\pi x}{12} + \frac{\pi}{12}\sin\left(\frac{\pi x}{12}\right)\right] + C$$

$$\because \quad y(0) = 0 \Rightarrow C = -\frac{12}{12^2 + \left(\frac{\pi}{12}\right)^2}$$

$$\text{So } y = \frac{1}{\lambda}\left[\underbrace{12\cos\left(\frac{\pi x}{12}\right) + \frac{\pi}{12}\sin\left(\frac{\pi x}{12}\right)}_{f_1(x)} - 12e^{-12x}\right]$$

$$\frac{dy}{dx} = \frac{1}{\lambda}\left[\underbrace{-\pi\sin\left(\frac{\pi x}{12}\right) + \frac{\pi^2}{12^2}\cos\left(\frac{\pi x}{12}\right)}_{f_2(x)} + 12e^{-12x}\right]$$

When x is large then $12e^{-12x}$ tends to zero.

But $f_2(x)$ varies in $\left[-\sqrt{\pi^2 + \left(\frac{\pi}{12}\right)^4}, \sqrt{\pi^2 + \left(\frac{\pi}{12}\right)^4}\right]$

Hence $\frac{dy}{dx}$ is changing its sign.

So y(x) is non monotonic for all real number.

Also when x is very large then again $-12e^{-12x}$ is almost zero but $f_1(x)$ is periodic, so there exist some β for which $y = \beta$ intersect $y = y(x)$ at infinitely many points.

SECTION3 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correctanswer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If **ONLY** the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : −1 In all other cases.

[:Q.15] Consider 4 boxes, where each box contains 3 red balls and 2 blue balls. Assume that all 20 balls are distinct. In how many different ways can 10 balls be chosen from these 4 boxes so that from each box at least one red ball and one blue ball are chosen ?

[:A] 21816

[:B] 85536

[:C] 12096

[:D] 156816

[:ANS] **A**

[:SOLN]

	Bag–1	Bag–2	Bag–3	Bag–4
	3R 2B	3R 2B	3R 2B	3R 2B
Case I	4 Balls	2 Balls	2 Balls	2 Balls
Case II	2 Balls	3 Balls	2 Balls	2 Balls

2 R 1B 2B 1R 2R 1B 2B 1R

$^4C_1$ [3 Red 1 Blue 2 Red 2 Blue] + Case II.

$^4C_1\,[^2C_1+^3C_1](^3C_1\,^2C_1)^3+^4C_2[^3C_2\,^2C_1+^3C_1\,^2C_2]^2[^3C_1\,^2C_1]^2$

$= 4(5)(6)^3+6(3\times2+3)^2(6)^2$

$= 4320+17496$

= 21816 (option A)

[:Q.16] $M=\begin{pmatrix}\frac{5}{2} & \frac{3}{2}\\ -\frac{3}{2} & -\frac{1}{2}\end{pmatrix}$, then which of the following matrices is equal to M^{2022} ?

[:A] $\begin{pmatrix}3034 & 3033\\ -3033 & -3032\end{pmatrix}$

[:B] $\begin{pmatrix}3034 & -3033\\ 3033 & -3032\end{pmatrix}$

[:C] $\begin{pmatrix}3033 & 3033\\ -3032 & -3031\end{pmatrix}$

[:D] $\begin{pmatrix}3033 & 3033\\ 3031 & -3030\end{pmatrix}$

[:ANS] **A**

[:SOLN] $M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 3/2 & 3/2 \\ -3/2 & -3/2 \end{bmatrix} = I + \frac{3}{2}\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$

Let $A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$; $A2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\therefore\ M^{2022} = \left(I + \frac{3A}{2}\right)^{2022} = +2022.\frac{3A}{2} + 0 + ...$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 3033 & 3033 \\ -3033 & -3033 \end{bmatrix} = \begin{bmatrix} 3034 & 3033 \\ -3033 & -3032 \end{bmatrix}$

[:Q.17] Suppose that

Box–I contains 8 red, 3 blue and 5 green balls,

Box–II contains 24 red, 9 blue and 15 green balls

Box–III contains 1 blue, 12 green and 3 yellow balls,

Box–IV contains 10 green, 16 orange and 6 white balls.

A ball is chosen randomly from Box–I; call this ball b. I b is red then a ball is chosen randomly from Box–II, if b is blue then a ball is chosen randomly from BOX–III, and. If b is green then a ball is chosen randomly from Box–IV. The conditional probability of the event 'one' of the chosen balls is white' given that the event at least one of the chosen balls is green' has happened is equal to

[:A] $\frac{15}{256}$

[:B] $\frac{3}{16}$

[:C] $\frac{5}{52}$

[:D] $\frac{1}{8}$

[:ANS] **C**

[:SOLN] $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow P\left(\frac{\text{White}}{\text{At least one green}}\right) = \frac{\frac{1}{16} \times \frac{6}{32}}{\frac{8}{16} \times \frac{15}{48} + \frac{3}{16} \times \frac{12}{16} + \frac{5}{16} \times 1} = \frac{5}{32}$

[:Q.18] For positive integer n, define

$$f(n) = n + \frac{16+5n-3n^2}{4n+3n^2} + \frac{32+n-3n^2}{8n+3n^2} + \frac{4B-3n-3n^2}{12n+3n^2} + ... + \frac{25n-7n^2}{7n^2}$$

Then, the value of $\lim_{n\to\infty} f(n)$ is equal to

[:A] $3 + \frac{4}{3}\log_e 7$

[:B] $4 - \frac{3}{4}\log_e\left(\frac{7}{3}\right)$

[:C] $4 - \frac{4}{3}\log_e\left(\frac{7}{3}\right)$

[:D] $3 + \frac{3}{4}\log_e 7$

[:ANS] **B**

[:SOLN] $f(n) = n + \sum_{r=1}^{n} \frac{16r+(9-4r)n-3n^2}{4rn+3n^2}$

$$= n + \sum_{r=1}^{n} \frac{(16r+9n)-(4rn+3n^2)}{(4rn+3n^2)}$$

$$= n + \sum_{r=1}^{n}\left\{\frac{16r+9n}{4rn+3n^2} - 1\right\} = \sum_{r=1}^{n} \frac{16r+9n}{4rn+3n^2}$$

$$= \lim_{n\to 0} f(n) = \int_0^1 \frac{16n+9}{4n+3}dx = 4n - \frac{3}{4}\log(4x+3)\Big]_0^1$$

$$= 4 - \frac{3}{4}\log\frac{7}{3}$$