

JEE (ADVANCED) 2022 PAPER-2

[PAPER WITH SOLUTION]

HELD ON SUNDAY 28TH AUGUST 2022

PHYSICS

SECTION 1 (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from **0 TO 9, BOTH INCLUSIVE**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +3 If **ONLY** the correct integer is entered;
Zero Marks: 0 If the question is unanswered;
Negative Marks : -1 In all other cases.

[:Q.1] A particle of mass 1 kg is subjected to a force which depends on the position as $\vec{F} = -k(x\hat{i} + y\hat{j}) \text{ kg ms}^{-2}$ with $k = 1 \text{ kg s}^{-2}$. At time $t = 0$, the particle's position $\vec{r} = \left(\frac{1}{\sqrt{2}}\hat{i} + \sqrt{2}\hat{j}\right) \text{ m}$ its velocity $\vec{v} = \left(-\sqrt{2}\hat{i} + \sqrt{2}\hat{j} + \frac{2}{\pi}\hat{k}\right) \text{ ms}^{-1}$. Let v_x and v_y denote the x and the y components of the particle's velocity respectively. Ignore gravity, When $z = 0.5 \text{ m}$, the value of $(xv_y - yv_x)$ is _____ m^2s^{-1} .

[:ANS] 3

[:SOLN] $F_x \propto x$ and $F_y \propto y \Rightarrow$ Particle executes SHM along x and y direction.

$$F_x = -kx ; k = 1 \text{ kg/s}^2$$

$$\Rightarrow a_x = \frac{d^2x}{dt^2} = -x; m = 1 \text{ kg} \ \& \ \omega = 1 \text{ rad/s}$$

$$\therefore x = A_x \sin(\omega t + \phi_x); \quad \text{Given : At } t = 0; x = \frac{1}{\sqrt{2}} \text{ m}; v_x = -\sqrt{2} \text{ m/s}$$

$$\text{Also } \Rightarrow \frac{1}{\sqrt{2}} = A_x \sin(\phi_x) \quad \dots(i)$$

$$\Rightarrow v_x = (A_x)\omega \cos(\omega t + \phi_x)$$

$$\text{at } t = 0 \Rightarrow -\sqrt{2} = A_x \cos(\phi_x) \quad \dots(ii)$$

$$(i) \div (ii) \Rightarrow \tan \phi_x = -\frac{1}{2} \Rightarrow \sin \phi_x = \frac{1}{\sqrt{5}} \text{ and } \cos \phi_x = \frac{-2}{\sqrt{5}}$$

$$(i)^2 + (ii)^2 \Rightarrow A_x = \sqrt{\frac{5}{2}} \text{ m}$$

Again

$$F_y = -ky ; k = 1\text{kg/s}^2$$

$$\Rightarrow a_y = \frac{d^2y}{dt^2} = -y; m = 1\text{kg and } \omega = 1\text{rad/s}$$

$$\therefore y = A_y \sin(\omega t + \phi_y) \text{ Given : At } t = 0, y = \sqrt{2}\text{m; } v_y = \sqrt{2} \text{ m/s}$$

$$\Rightarrow \sqrt{2} = A_y \sin(\phi_y) \quad \dots(iii)$$

$$\text{Also } v_y = A_y \omega \cos(\omega t + \phi_y)$$

$$v_y = A_y \omega \cos(\omega t + \phi_y)$$

$$(iii) \div (iv)$$

$$\Rightarrow \tan \phi_y = 1 \Rightarrow \sin \phi_y = \frac{1}{\sqrt{2}} \text{ and } \cos \phi_y = \frac{1}{\sqrt{2}}$$

$$(iii)^2 + (iv)^2$$

$$\Rightarrow A_y = 2$$

$$\text{Now } \Rightarrow xv_y - yv_x = \sqrt{\frac{5}{2}} \sin(\omega t + \phi_x) [2(1) \cos(\omega t + \phi_y)] - 2 \sin(\omega t + \phi_y) \left[\left(\sqrt{\frac{5}{2}} (1) \cos(\omega t + \phi_x) \right) \right]$$

$$\Rightarrow xv_y - yv_x = \sqrt{\frac{5}{2}} (2) [\sin(\omega t + \phi_x) \cos(\omega t + \phi_y)] - \cos(\omega t + \phi_x) \sin(\omega t + \phi_y)$$

$$\Rightarrow xv_y - yv_x = \sqrt{10} [\sin(\phi_x - \phi_y)]$$

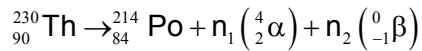
$$\Rightarrow xv_y - yv_x = \sqrt{10} [\sin \phi_x \cos \phi_y - \cos \phi_x \sin \phi_y]$$

$$\Rightarrow xv_y - yv_x = \sqrt{10} \left[\frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{2}} - \left(-\frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{2}} \right) \right] \quad \therefore (xv_y - yv_x) = 3$$

[:Q.2] In a radioactive decay chain reaction, ${}_{90}^{230}\text{Th}$ nucleus decays into ${}_{84}^{214}\text{Po}$ nucleus. The ratio of the number of α to number of β^- particles emitted in this process is _____.

[:ANS] 2

[:SOLN] Let number of α and β^- particles emitted be ' n_1 ' and ' n_2 ' respectively.



$$230 = 214 + 4n_1$$

$$\therefore n_1 = 4$$

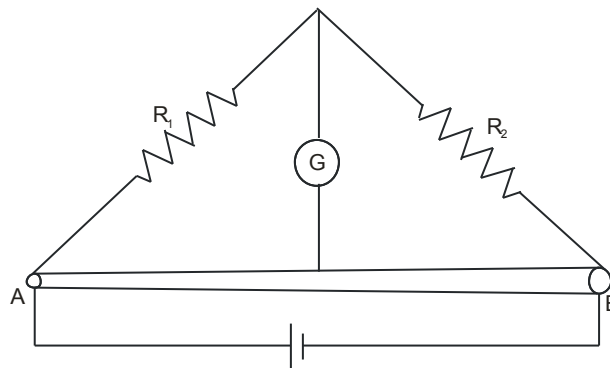
$$90 = 84 + n_1 (2) - n_2 (1)$$

$$\therefore n_2 = 2$$

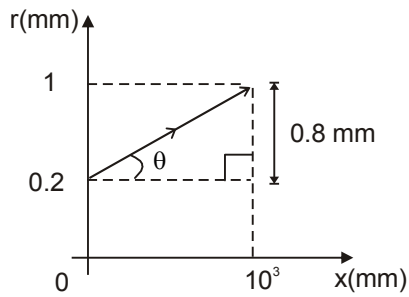
$$\Rightarrow \frac{n_1}{n_2} = \frac{4}{2}$$

$$\therefore \frac{n_1}{n_2} = 2$$

[:Q.3] Two resistances $R_1 = X\Omega$ and $R_2 = 1\Omega$ are connected to a wire AB of uniform resistivity, as shown in the figure. The radius of the wire varies linearly along its axis from 0.2 mm at A to 1 mm at B. A galvanometer (G) connected to the center of the wire, 50 cm from each end along its axis, shows zero deflection when A and B are connected to a battery. The value of X is _____.



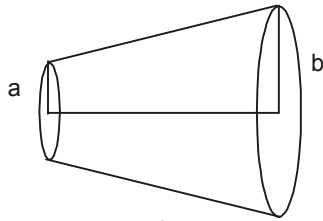
[:ANS] 5



[:SOLN]

$$r = (0.8 \times 10^{-3})x + 0.2; \quad r = \text{radius} \quad x = \text{linear distance from end A}$$

$$\text{At } x = 50 \text{ cm} \Rightarrow r = (0.6) \text{ mm}$$



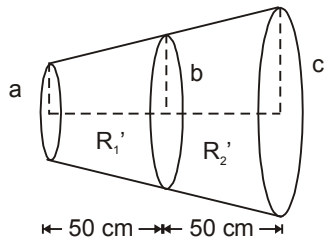
Also :

$$R = \frac{\rho l}{\pi ab}$$

According to situation

$$R_1' = \frac{\rho(50 \times 10^{-2})}{\pi(0.2)(0.6)(10^{-6})}$$

$$R_2' = \frac{\rho(50 \times 10^{-2})}{\pi(0.6)(1)(10^{-6})}$$



At balance point :

$$\frac{R_1}{R_2} = \frac{R_1'}{R_2'}$$

$$\Rightarrow \frac{x}{1} = \frac{1}{0.2}$$

$$\therefore x = 5\Omega$$

[:Q.4] In a particular system of units, a physical quantity can be expressed in terms of the electric charge e , electron mass m_e , Planck's constant h , and Coulomb's constant $k = \frac{1}{4\pi\epsilon_0}$, where ϵ_0 is the permittivity of vacuum. In terms of these physical constants, the dimension of the magnetic field is $[B] = [e]^\alpha [m_e]^\beta [h]^\gamma [k]^\delta$. The value of $\alpha + \beta + \gamma + \delta$ is _____.

[:ANS] 4

[:SOLN] $[B] = [e]^\alpha [m_e]^\beta [h]^\gamma [k]^\delta$

$$[M^1T^{-2}I^{-1}] = [IT]^\alpha [M]^\beta [ML^2T^{-1}]^\gamma [ML^3T^{-4}]^\delta$$

$$\text{So, } \beta + \gamma + \delta = 1 \quad \dots\text{(i)}$$

$$2\gamma + 3\delta = 0 \quad \dots\text{(ii)}$$

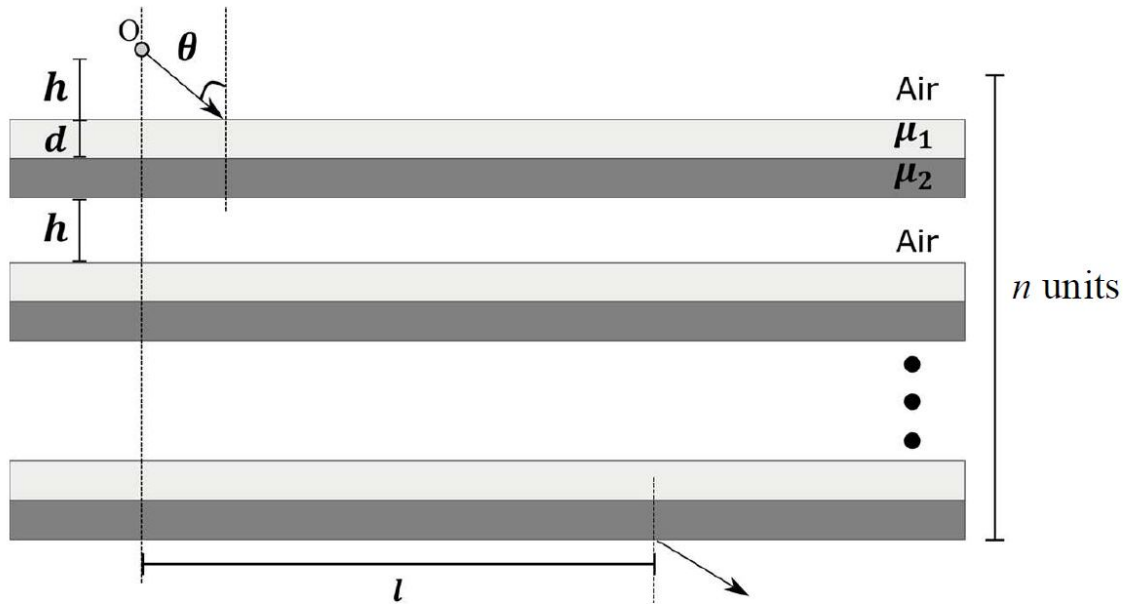
$$\alpha - \gamma - 4\delta = -2 \quad \dots\text{(iii)}$$

$$\alpha - 2\delta = -1 \quad \dots\text{(iv)}$$

On solving we get

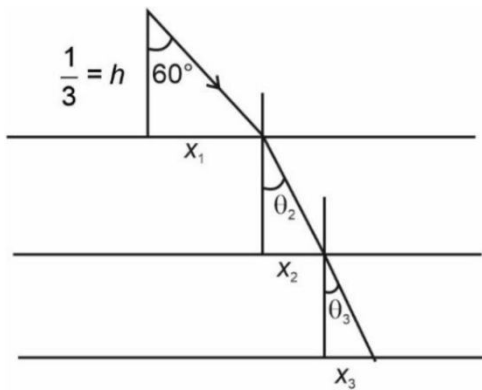
$$\alpha + \beta + \gamma + \delta = 4$$

[:Q.5] Consider a configuration of n identical units, each consisting of three layers. The first layer is a column of air of height $h = \frac{1}{3}$ cm, and the second and third layers are of equal thickness $d = \frac{\sqrt{3}-1}{2}$ cm, and refractive indices $\mu_1 = \sqrt{\frac{3}{2}}$ and $\mu_2 = \sqrt{3}$, respectively. A light source O is placed on the top of the first unit, as shown in the figure. A ray of light from O is incident on the second layer of the first unit at an angle of $\theta = 60^\circ$ to the normal. For a specific value of n , the ray of light emerges from the bottom of the configuration at a distance $l = \frac{8}{\sqrt{3}}$ cm, as shown in the figure. The value of n is _____.



[:ANS] 4

[:SOLN]



$$x_1 = \frac{1}{3} \times \tan 60^\circ = \frac{1}{\sqrt{3}} \text{ cm}$$

$$\text{and, } 1 \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2} \times \sin \theta_2$$

$$\Rightarrow \theta_2 = 45^\circ$$

$$\Rightarrow x_2 = d$$

$$\text{and, } 1 \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2} \times \sin \theta_3$$

$$\Rightarrow \theta_3 = 30^\circ$$

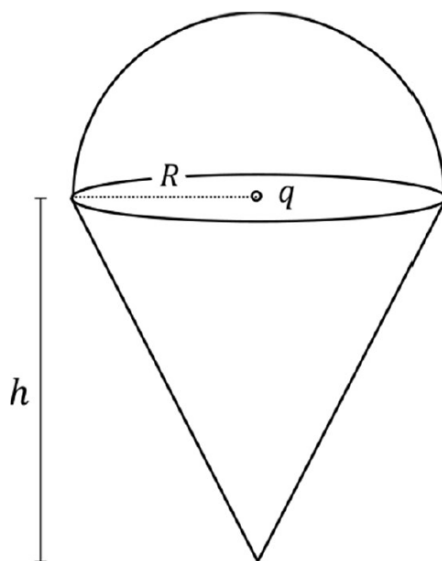
$$x_3 = \frac{d}{\sqrt{3}}$$

$$\therefore x_1 + x_2 + x_3 = \frac{1}{\sqrt{3}} + \frac{(\sqrt{3}-1)}{2} \left(1 + \frac{1}{\sqrt{3}}\right)$$

$$= \frac{2}{\sqrt{3}} \text{ cm}$$

$$\therefore n = \frac{1}{x_1 + x_2 + x_3} = \frac{8/\sqrt{3}}{2/\sqrt{3}} = 4$$

[:Q.6] A charge q is surrounded by a closed surface consisting of an inverted cone of height h and base radius R , and a hemisphere of radius R as shown in the figure. The electric flux through the conical surface is $\frac{nq}{6\epsilon_0}$ (in SI units). The value of n is _____.



[:ANS] 3

[:SOLN] From Gauss law,

$$\phi_{\text{hemisphere}} + \phi_{\text{Cone}} = \frac{q}{\epsilon_0} \quad \dots\dots (i)$$

$$\phi_{\text{hemisphere}} = \frac{q}{2\epsilon_0}; \text{ As charge is kept at the centre}$$

From equation (i)

$$\frac{q}{2\epsilon_0} + \phi_{\text{cone}} = \frac{q}{\epsilon_0}$$

$$\phi_{\text{cone}} = \frac{q}{2\epsilon_0}$$

$$\frac{4q}{6\epsilon_0} = \frac{q}{2\epsilon_0}$$

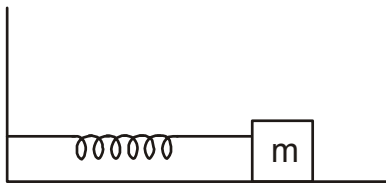
$$n = 3$$

Alternatively, $\phi \propto$ no of electric field lines passing through surface q is point charge which has uniformly distributed electric field lines thus half of electric field lines will pass through hemisphere & other half will pass through conical surface.

[:Q.7] On a frictionless horizontal plane, a bob of mass $m = 0.1$ kg is attached to a spring with natural length $l_0 = 0.1$ m. The spring constant is $k_1 = 0.009$ Nm^{-1} when the length of the spring $l > l_0$ and is $k_2 = 0.016$ Nm^{-1} when $l < l_0$. Initially the bob is released from $l = 0.15$ m. Assume that Hooke's law remains valid throughout the motion. If the time period of the full oscillation is $T = (n \pi)$ s, then the integer closed to n is _____.

[:ANS] 6

[:SOLN]



$$m = 0.1 \text{ kg}$$

$$l_0 = 0.1 \text{ m}$$

$$k_1 = 0.009 \text{ Nm}^{-1} (l > l_0)$$

$$k_2 = 0.016 \text{ Nm}^{-1} (l < l_0)$$

$$l (\text{initial}) = 0.15 \text{ m}$$

$$T_1 = \pi \sqrt{\frac{m}{k_1}} = \pi \sqrt{\frac{0.1}{0.009}}$$

$$= \pi \sqrt{\frac{100}{9}} = \pi \times \frac{10}{3}$$

$$T_2 = \pi \sqrt{\frac{m}{k_2}} = \pi \sqrt{\frac{0.1}{0.016}} = \pi \times \frac{10}{4}$$

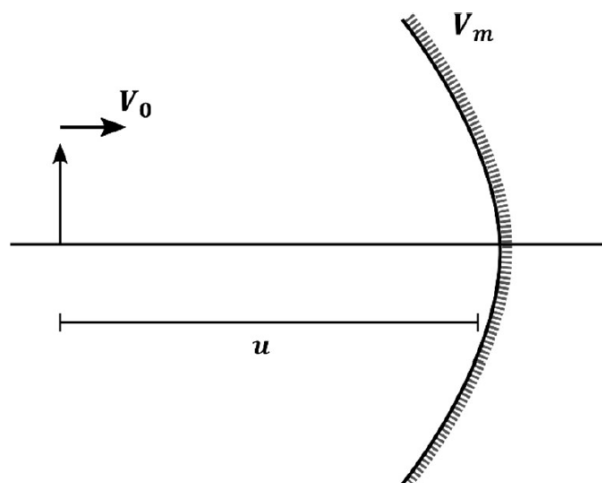
$$T = T_1 + T_2$$

$$= \pi \times 10 \times \left(\frac{1}{3} + \frac{1}{4} \right)$$

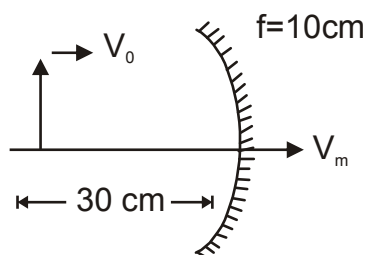
$$\pi \times 10 \times \frac{7}{12}$$

$$\approx \pi \times 6$$

[:Q.8] An object and a concave mirror of focal length $f = 10$ cm both move along the principal axis of the mirror with constant speeds. The object moves with speed $V_0 = 15$ cm s⁻¹ towards the mirror with respect to a laboratory frame. The distance between the object and the mirror at a given moment is denoted by u . When $u = 30$ cm, the speed of the mirror V_m is such that the image is instantaneously at rest with respect to the laboratory frame, and the object forms a real image. The magnitude of V_m is _____ cm s⁻¹.



[:ANS] 3



[:SOLN]

$$V_0 = 15 \text{ cm/s}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{v} + \frac{1}{-30} = \frac{1}{-10}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{30} - \frac{1}{10} = \frac{1-3}{30} = \frac{-2}{30} = -\frac{1}{15}$$

$$v = -15 \text{ cm}$$

$$m = -\frac{-15}{-30} = -\frac{1}{2}$$

$$\overline{V_{IM}} = -m^2 \times \overline{V_{OM}}$$

$$= -\frac{1}{4}(V_0 - V_m)$$

$$\overline{V_I} = \overline{V_{IM}} + \overline{V_M}$$

$$= -\frac{1}{4}(V_0 - V_m) + V_m$$

$$= -\frac{V_0}{4} + \frac{V_m}{4} + V_m$$

$$0 = -\frac{V_0}{4} + \frac{5V_m}{4}$$

$$V_0 = 5 V_m$$

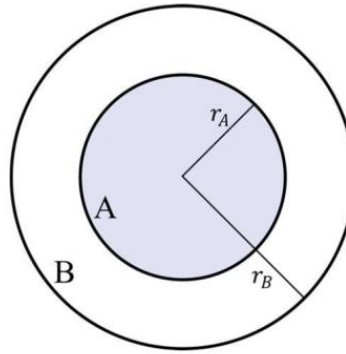
$$V_m = \frac{V_0}{5} = \frac{15}{5} = 3 \text{ cm / s}$$

SECTION 2 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +4	If ONLY (all) the correct option(s) is (are) chosen.
Partial Marks	: +3	If all the four options are correct but ONLY three options are chosen.
Partial Marks	: +2	If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks	: +1	If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered).
Negative Marks	: -2	In all other cases.

[:Q.9] In the figure, the inner (shaded) region A represents a sphere of radius $r_A = 1$, within which the electrostatic charge density varies with the radial distance r from the center as $\rho_A = kr$, where k is positive. In the spherical shell B of outer radius r_B , the electrostatic charge density varies as $\rho_B = \frac{2k}{r}$. Assume that dimensions are taken care of. All physical quantities are their SI units.



Which of the following statement(s) is(are) correct?

[A] If $r_B = \sqrt{\frac{3}{2}}$, then the electric field is zero everywhere outside B.

[B] If $r_B = \frac{3}{2}$, then the electric potential just outside B is $\frac{k}{\epsilon_0}$.

[C] If $r_B = 2$, then the total charge of the configuration is $15\pi k$.

[D] If $r_B = \frac{5}{2}$, then the magnitude of the electric field just outside B is $\frac{13\pi k}{\epsilon_0}$.

[ANS] B

[SOLN] $r_A = 1$

$$\rho_A = kr$$

$$\rho_B = \frac{2k}{r}$$

$$q_A = \int_0^1 kr \cdot 4\pi r^2 dr$$

$$= \frac{4\pi k}{4} = \pi k$$

$$q_B = \int_1^{r_B} \frac{2k}{r} \times 4\pi r^2 dr$$

$$= 8\pi k \int_1^{r_B} r dr$$

$$= 4\pi k (r_B^2 - 1)$$

(i) If $r_B = \sqrt{3/2}$, $q_A + q_B = \pi k + 4\pi k (3/2 - 1) = 3\pi k$

(Option is incorrect)

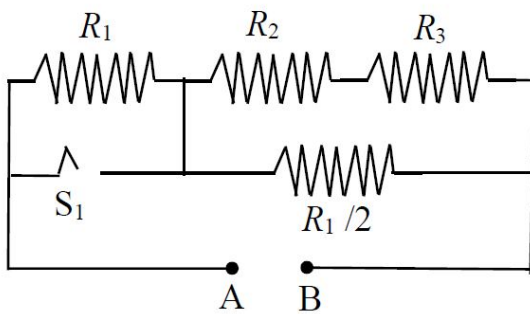
$$\begin{aligned}
 \text{(ii) If } r_B &= \frac{3}{2}, q_A + q_B = \pi k + 4\pi k \left(\frac{9}{4} - 1 \right) \\
 &= \pi k + \frac{4\pi k \times 5}{4} = 6\pi k \\
 v &= \frac{6\pi k}{4\pi\epsilon_0 \times 3/2} = \frac{K}{\epsilon_0} \quad (\text{Option is correct})
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii) } q_A + q_B &= \pi k + 4\pi k (r_B^2 - 1) \\
 &= \pi k + 4\pi k (4 - 1) \\
 &= \pi k + 12\pi k = 13\pi k \quad (\text{Option is incorrect})
 \end{aligned}$$

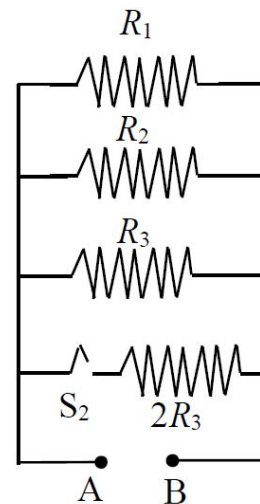
$$\begin{aligned}
 \text{(iv) } q_A + q_B &= \pi k + 4\pi k \left(\frac{25}{4} - 1 \right) \\
 &= \pi k + 21\pi k = 22\pi k \\
 E &= \frac{22\pi k \times 4}{4\pi\epsilon_0 \times 25} = \frac{22k}{25\epsilon_0} \quad (\text{Option is incorrect})
 \end{aligned}$$

[:Q.10] In Circuit – 1 and Circuit - 2 shown in the figures, $R_1 = 1 \Omega$, $R_2 = 2 \Omega$ and $R_3 = 3 \Omega$. P_1 and P_2 are the power dissipations in Circuit-1 and Circuit-2 when the switches S_1 and S_2 are in open conditions, respectively.

Q_1 and Q_2 are the power dissipations in Circuit-1 and Circuit-2 when the switches S_1 and S_2 are in closed conditions, respectively.



Circuit-1



Circuit-2

Which of the following statement(s) is(are) correct?

[A] When a voltage source of 6V is connected across A and b in both circuits, $P_1 < P_2$.

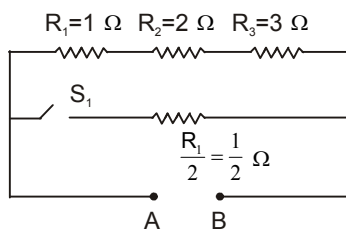
[B] When a constant current source of 2 Amp is connected across A and B in both circuits, $P_1 > P_2$

[C] When a voltage source of 6V is connected across A and B in Circuit-1, $Q_1 > P_1$.

[D] When a constant current source of 2 Amp is connected across A and B in both circuits, $Q_2 < Q_1$.

[ANS] A, B, C

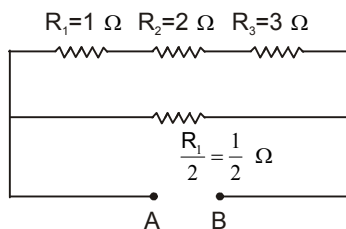
[SOLN] Circuit 1 when switch S_1 is open



$$R_{1eq} = \frac{5 \times \frac{1}{2}}{5 + \frac{1}{2}} + 1 = \frac{5}{11} + 1$$

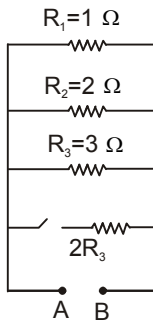
$$= \frac{16}{11} \Omega$$

Circuit 1 when switch S_1 is close



$$R'_{1eq} = \frac{5 \times \frac{1}{2}}{5 + \frac{1}{2}} = \frac{5}{11} \Omega$$

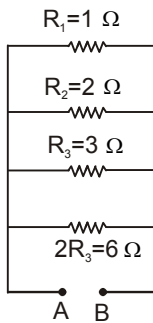
Circuit 2 when switch S_2 is open



$$\frac{1}{R_{2\text{eq}}} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{6+3+2}{6}$$

$$R_{2\text{eq}} = \frac{6}{11} \Omega$$

Circuit 2 when switch S_2 is close



$$\frac{1}{R'_{2\text{eq}}} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{6+3+2+1}{6} = \frac{12}{6}$$

$$R'_{2\text{eq}} = \frac{1}{2} \Omega$$

when 6 v connected

$$\frac{p_1}{p_2} = \frac{\frac{v^2}{R_{1\text{eq}}}}{\frac{v^2}{R_{2\text{eq}}}} = \frac{R_{2\text{eq}}}{R_{1\text{eq}}} = \frac{\frac{6}{11}}{\frac{6}{16}} = \frac{6}{16} < 1$$

$p_1 < p_2$ so option (B) is correct

For 6 V source

$$\frac{p_1}{Q_1} = \frac{\frac{v^2}{R_{1\text{eq}}}}{\frac{v^2}{R'_{1\text{eq}}}} = \frac{5}{16} < 1$$

$p_1 < Q_1 \Rightarrow Q_1 > p_1$ so option (C) is correct

For current source

$$\frac{Q_1}{Q_2} = \frac{i^2 R'_{1eq}}{i^2 R'_{2eq}} = \frac{10}{11} < 1$$

$Q_1 < Q_2 \Rightarrow Q_2 > Q_1$, So option (D) is incorrect

[Q.11] A bubble has surface tension S . The ideal gas inside the bubble has ratio of specific heats $\gamma = \frac{5}{3}$. The bubble is exposed to the atmosphere and it always retains its spherical shape. When the atmospheric pressure is P_{a1} , the radius of the bubble is found to be r_1 and the temperature of the enclosed gas is T_1 . When the atmospheric pressure is P_{a2} , the radius of the bubble and the temperature of the enclosed gas are r_2 and T_2 respectively. Which of the following statement(s) is(are) correct?

[A] If the surface of the bubble is a perfect heat insulator, then
$$\left(\frac{r_1}{r_2}\right)^5 = \frac{P_{a2} + \frac{2S}{r_2}}{P_{a1} + \frac{2S}{r_1}}$$

[B] If the surface of the bubble is a perfect heat insulator, then the total internal energy of the bubble including its surface energy does not change with the external atmospheric pressure.

[C] If the surface of the bubble is a perfect heat conductor and the change in atmospheric

temperature is negligible, then
$$\left(\frac{r_1}{r_2}\right)^3 = \frac{P_{a2} + \frac{4S}{r_2}}{P_{a1} + \frac{4S}{r_1}}$$

[D] If the surface of the bubble is a perfect heat insulator, then
$$\left(\frac{T_2}{T_1}\right)^{\frac{5}{2}} = \frac{P_{a2} + \frac{4S}{r_2}}{P_{a1} + \frac{4S}{r_1}}$$

[ANS] C, D

[SOLN] for bubble

$$P_{in} - P_{out} = \frac{4S}{r}$$

$$P_{in} = P_{out} + \frac{4S}{r}$$

For isothermal process

$$P_i V_i = P_f V_f$$

$$\left(P_{a_1} + \frac{4S}{r_1} \right) \left(\frac{4}{3} \pi r_1^3 \right) = \left(P_{a_2} + \frac{4S}{r_2} \right) \left(\frac{4}{3} \pi r_2^3 \right)$$

$$\left(\frac{r_1}{r_2} \right)^3 = \left(\frac{P_{a_2} + \frac{4S}{r_2}}{P_{a_1} + \frac{4S}{r_1}} \right)$$

so option (c) is correct

since for adiabatic process

$$T_i^\gamma P_i^{1-\gamma} = T_f^\gamma P_f^{1-\gamma}$$

$$\left(\frac{T_f}{T_i} \right)^\gamma = \left(\frac{P_i}{P_f} \right)^{1-\gamma}$$

$$\left[\frac{T_2}{T_1} \right]^{5/3} = \left(\frac{P_{a_1} + \frac{4S}{r_1}}{P_{a_2} + \frac{4S}{r_2}} \right)^{-2/3}$$

$$\left[\frac{T_2}{T_1} \right]^{5/2} = \left(\frac{P_{a_2} + \frac{4S}{r_2}}{P_{a_1} + \frac{4S}{r_1}} \right) \text{ So option D is correct}$$

[:Q.12] A disk of radius R with uniform positive charge density σ is placed on the xy plane with its center at the origin. The Coulomb potential along the z -axis is

$$V(z) = \frac{\sigma}{2\epsilon_0} \left(\sqrt{R^2 + z^2} - z \right).$$

A particle of positive charge q is placed initially at rest at a point on the z axis with $z = z_0$ and $z_0 > 0$. In addition to the Coulomb force, the particle experiences a vertical force $\vec{F} = -c\hat{k}$ with $c > 0$. Let $\beta = \frac{2c\epsilon_0}{q\sigma}$. Which of the following statement(s) is(are) correct?

[A] For $\beta = \frac{1}{4}$ and $z_0 = \frac{25}{7}R$, the particle reaches the origin.

[B] For $\beta = \frac{1}{4}$ and $z_0 = \frac{3}{7}R$, the particle reaches the origin.

[C] For $\beta = \frac{1}{4}$ and $z_0 = \frac{R}{\sqrt{3}}$, the particle returns back to $z = z_0$.

[D] For $\beta > 1$ and $z_0 > 0$, the particle always reaches the origin.

[ANS] A, C, D

$$[\text{SOLN}] \quad \beta = \frac{2C\epsilon_0}{q\sigma}$$

$$C = \frac{q\sigma\beta}{2\epsilon_0}$$

$$F = -C\hat{k} = \frac{-q\sigma\beta}{2\epsilon_0}\hat{k}$$

by work energy theorem

$$W_{\text{electric}} + W_{\text{external}} = K_f - K_i$$

$$q(V_f - V_i) + CZ = K_f - 0$$

$$K_f = \frac{q\sigma}{2\epsilon_0} \left[\sqrt{R^2 + Z^2} - Z \right] - \frac{q\sigma R}{2\epsilon_0} + \frac{q\sigma\beta}{2\epsilon_0} Z$$

$$(A) \quad \beta = \frac{1}{4}; \quad Z = \frac{25}{7}R$$

$$= \frac{q\sigma R}{2\epsilon_0} \left[\sqrt{1 + \frac{625}{49}} - \frac{25}{7} - 1 + \frac{25}{28} \right] = +ve$$

So charge can reach at origin

Option A is correct

$$(B) \quad \beta = \frac{1}{4} \text{ and } Z = \frac{3}{7}R$$

$$K_f = -ve$$

So charge can not reach at origin

Option B is incorrect

$$(C) \quad K_f = -ve$$

i.e. can not reach at origin so return back, option C is correct.

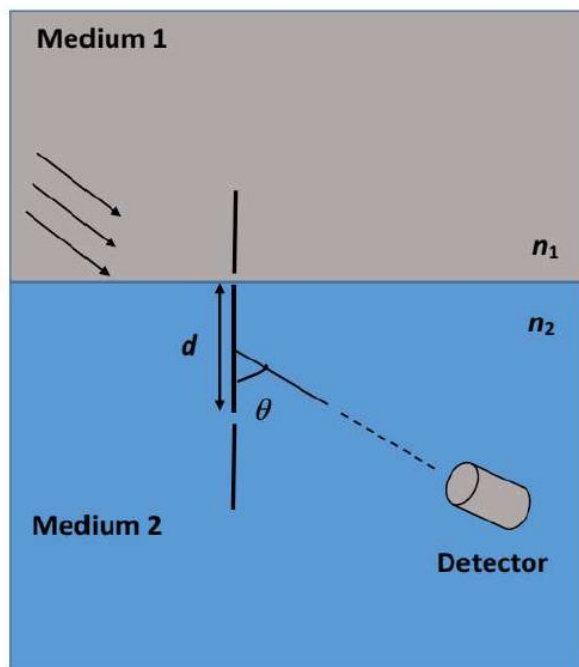
$$(D) \quad K_f = \frac{q\sigma R}{2\epsilon_0} \left[\sqrt{1 + \frac{Z_0^2}{R^2}} - \frac{Z_0}{R} - 1 + \frac{\beta}{R} \right]$$

$$K_f = +ve$$

So always reaches at origin

Option D is correct.

- [:Q.13]** A double slit setup is shown in the figure. One of the slits is in medium 2 of refractive index n_2 . The other slit is at the interface of this medium with another medium 1 of refractive index n_1 ($n_1 \neq n_2$). The line joining the slits is perpendicular to the interface and the distance between the slits is d . The slit widths are much smaller than d . A monochromatic parallel beam of light is incident on the slits from medium 1. A detector is placed in medium 2 at a large distance from the slits, and at an angle θ from the line joining them, so that θ equals the angle of refraction of the beam. Consider two approximately parallel rays from the slits received by the detector.

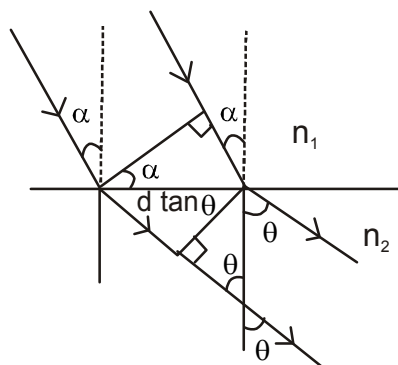


Which of the following statement(s) is(are) correct?

- [:A] The phase difference between the two rays is independent of d .
 [:B] The two rays interfere constructively at the detector.
 [:C] The phase difference between the two rays depends on n_1 but is independent of n_2 .
 [:D] The phase difference between the two rays vanishes only for certain values of d and the angle of incidence of the beam, with θ being the corresponding angle of refraction.

[:ANS] A, B

[:SOLN]



Optical path difference

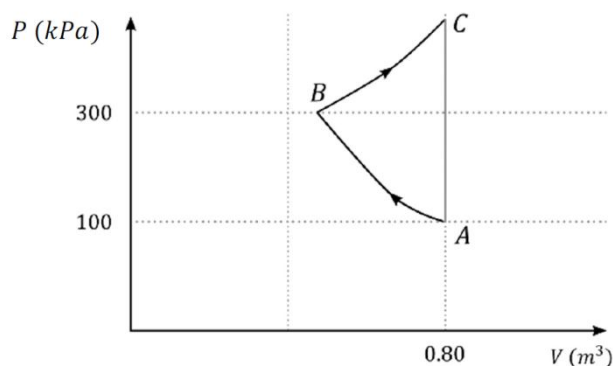
$$\Delta x = n_1 (d \tan \theta) \sin \alpha - n_2 (d \tan \theta) \sin \theta$$

$$= (n_1 \sin \alpha - n_2 \sin \theta) d \tan \theta$$

$$= 0$$

$$\Rightarrow \Delta \phi = 0$$

[Q.14] In the given P-V diagram, a monoatomic gas ($\gamma = \frac{5}{3}$) is first compressed adiabatically from state A to state B. Then it expands isothermally from state B to state C. [Given: $(\frac{1}{3})^{0.6} \approx 0.5$, $\ln 2 \approx 0.7$].



Which of the following statement(s) is(are) correct?

[A] The magnitude of the total work done in the process $A \rightarrow B \rightarrow C$ is 144 kJ.

[B] The magnitude of the work done in the process $B \rightarrow C$ is 84 kJ.

[C] The magnitude of the work done in the process $A \rightarrow B$ is 60 kJ.

[D] The magnitude of the work done in the process $C \rightarrow A$ is zero.

[ANS] B, C, D

[SOLN] For adiabatic process

(A \rightarrow B)

$$P_A V_A^\gamma = P_B V_B^\gamma$$

$$10^5 \times (0.8)^{5/3} = 3 \times 10^5 (V_B)^{5/3}$$

$$\Rightarrow V_B = 0.4$$

$$W_{AB} = \frac{P_A V_A - P_B V_B}{\gamma - 1}$$

$$= \frac{10^5 \times 0.8 - 3 \times 10^5 \times 0.4}{\frac{5}{3} - 1} = -60 \text{ kJ}$$

$$|W_{AB}| = 60 \text{ kJ}$$

$$\Rightarrow W_{BC} = nRT \ln \frac{V_C}{V_B}$$

$$= P_B V_B \ln \frac{V_C}{V_B}$$

$$= 3 \times 10^5 \times 0.4 \ln \frac{0.8}{0.4}$$

$$= 84 \text{ kJ}$$

$$W_{CA} = P\Delta V = 0$$

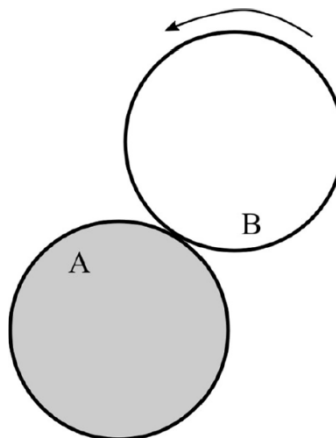
$$W_{ABC} = W_{AB} + W_{BC} + W_{CA}$$

$$= -60 + 84 + 0 = 24 \text{ kJ}$$

SECTION3 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
 Full Marks : +3 If **ONLY** the correct option is chosen;
 Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
 Negative Marks : -1 In all other cases.

[:Q.15] A flat surface of a thin uniform disk A of radius R is glued to a horizontal table. Another thin uniform disk B of mass M and with the same radius R rolls without slipping on the circumference of A, as shown in the figure. A flat surface of B also lies on the plane of the table. The center of mass of B has fixed angular speed ω about the vertical axis passing through the center of A. The angular momentum of B is $nM\omega R^2$ with respect to the center of A. Which of the following is the value of n ?



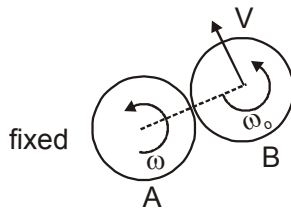
[:A] 2

[:B] 5

[:C] $\frac{7}{2}$ [:D] $\frac{9}{2}$

[:ANS] B

[:SOLN]



$$V = \omega (2R)$$

$$V = \omega_0 R \text{ (for no slipping)}$$

$$\omega_0 = 2\omega$$

$$\vec{L} = m\vec{r} \times \vec{v}_c + I_c \vec{\omega}_0$$

$$= 2MRV + \frac{1}{2}MR^2\omega_0$$

$$= 4MR^2\omega + \frac{1}{2}MR^2(2\omega)$$

$$= 5MR^2\omega$$

$$\therefore n = 5$$

[:Q.16] When light of a given wavelength is incident on a metallic surface, the minimum potential needed to stop the emitted photoelectrons is 6.0 V . This potential drops to 0.6 V if another source with wavelength four times that of the first one and intensity half of the first one is used. What are the wavelength of the first source and the work function of the metal, respectively? [Take $\frac{hc}{e} = 1.24 \times 10^{-6} \text{ J m C}^{-1}$]

[:A] $1.72 \times 10^{-7} \text{ m}$, 1.20 eV [:B] $1.72 \times 10^{-7} \text{ m}$, 5.60 eV [:C] $3.78 \times 10^{-7} \text{ m}$, 5.60 eV [:D] $3.78 \times 10^{-7} \text{ m}$, 1.20 eV

[:ANS] A

$$[:\text{SOLN}] \quad \therefore \frac{hc}{\lambda} - \phi = 6\text{eV} \dots(\text{i})$$

$$\text{and } \frac{hc}{4\lambda} - \phi = 0.6\text{eV} \dots(\text{ii})$$

$$(\text{i}) - (\text{ii}) \Rightarrow \frac{hc}{\lambda} \left(1 - \frac{1}{4}\right) = 5.4 \text{ eV}$$

$$\Rightarrow \frac{hc}{e\lambda} \times \frac{3}{4} = 5.4$$

$$\Rightarrow \frac{1.24 \times 10^{-6}}{\lambda} \times \frac{3}{4} = 5.4$$

$$\lambda = \frac{1.24 \times 3}{4 \times 5.4} \times 10^{-6} \text{ m}$$

$$\therefore \lambda = 1.72 \times 10^{-7} \text{ m}$$

from equation (i)

$$\frac{1.24 \times 10^{-6} \text{ eV}}{1.72 \times 10^{-7}} - \phi = 6\text{eV}$$

$$\Rightarrow 7.20\text{eV} - \phi = 6\text{eV}$$

$$\therefore \phi = 1.20\text{eV}$$

[:Q.17] Area of the cross-section of a wire is measured using a screw gauge. The pitch of the main scale is 0.5 mm. The circular scale has 100 divisions and for one full rotation of the circular scale, the main scale shifts by two divisions. The measured readings are listed below.

Measurement condition	Main scale reading	Circular scale reading
Two arms of gauge touching each other without wire	0 division	4 divisions
Attempt-1 : With wire	4 divisions	20 divisions
Attempt-2 : With wire	4 divisions	16 divisions

What are the diameter and cross-sectional area of the wire measured using the screw gauge?

$$[:\text{A}] \quad 2.22 \pm 0.02 \text{ mm}, \pi(1.23 \pm 0.02)\text{mm}^2$$

$$[:\text{B}] \quad 2.22 \pm 0.01 \text{ mm}, \pi(1.23 \pm 0.01)\text{mm}^2$$

$$[:\text{C}] \quad 2.14 \pm 0.02 \text{ mm}, \pi(1.14 \pm 0.02)\text{mm}^2$$

$$[:\text{D}] \quad 2.14 \pm 0.01 \text{ mm}, \pi(1.14 \pm 0.01)\text{mm}^2$$

[:ANS] C

$$[:SOLN] \quad \therefore \text{Least Count (L.C)} = \frac{2 \times 0.5}{100} = \frac{1}{100} \text{ mm}$$

$$\text{Zero error} = 4 \times \frac{1}{100} \text{ mm} = 0.4 \text{ mm}$$

Attempt -1

$$\text{Main scale reading (MSR)} = 4 \times 0.5 \text{ mm} = 2 \text{ mm}$$

$$\text{Circular scale reading (CSR)} = 20 \times \frac{1}{100} \text{ mm} = 0.20 \text{ mm}$$

$$\therefore (\text{Reading})_1 = \text{MSR} + \text{CSR} - \text{Zero error}$$

$$= 2 \text{ mm} + 0.20 \text{ mm} - 0.04 \text{ mm} = 2.16 \text{ mm}$$

Attempt -2

$$\text{MSR} = 4 \times 0.5 \text{ mm} = 2 \text{ mm}$$

$$\text{CSR} = 16 \times \frac{1}{100} \text{ mm} = 0.16 \text{ mm}$$

$$\therefore (\text{Reading})_1 = \text{MSR} + \text{CSR} - \text{Zero error}$$

$$= 2 \text{ mm} + 0.16 \text{ mm} - 0.04 \text{ mm} = 2.12 \text{ mm}$$

$$\begin{aligned} \therefore \text{Reading} &= \frac{(\text{Reading})_1 + (\text{Reading})_2}{2} \\ &= \frac{2.16 \text{ mm} + 2.12 \text{ mm}}{2} = 2.14 \text{ mm} \end{aligned}$$

$$\therefore \text{Mean error} = \frac{0.02 \text{ mm} + 0.02 \text{ mm}}{2} = 0.02 \text{ mm}$$

$$\therefore \text{diameter} = (2.14 \pm 0.02) \text{ mm}$$

$$\therefore \text{Area} = \frac{\pi d^2}{4} = \frac{\pi (2.14 \text{ mm})^2}{4} = \pi (1.14 \text{ mm}^2)$$

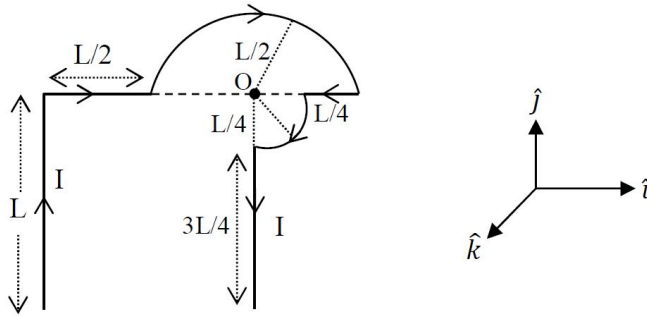
$$\therefore \Delta A = \frac{\pi}{4} (2d)(\Delta d) = \frac{\pi d}{2} (\Delta d)$$

$$= \frac{\pi (2.14 \text{ mm})}{2} (0.02 \text{ mm})$$

$$= \pi (0.0214) \text{ mm}^2 \approx \pi (0.02) \text{ mm}^2$$

$$\therefore \text{Area with error} = \pi (1.14 \pm 0.02) \text{ mm}^2$$

[:Q.18] Which one of the following options represents the magnetic field \vec{B} at O due to the current flowing in the given wire segments lying on the xy plane?



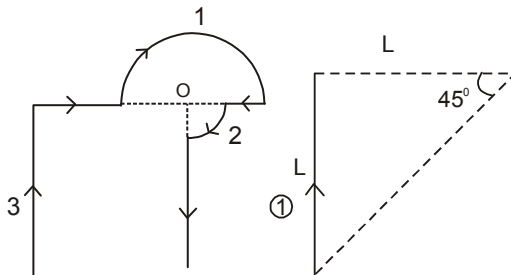
[:A] $\vec{B} = \frac{-\mu_0 I}{L} \left(\frac{3}{2} + \frac{1}{4\sqrt{2}\pi} \right) \hat{k}$

[:B] $\vec{B} = \frac{-\mu_0 I}{L} \left(\frac{3}{2} + \frac{1}{2\sqrt{2}\pi} \right) \hat{k}$

[:C] $\vec{B} = \frac{-\mu_0 I}{L} \left(1 + \frac{1}{4\sqrt{2}\pi} \right) \hat{k}$

[:D] $\vec{B} = \frac{-\mu_0 I}{L} \left(1 + \frac{1}{4\pi} \right) \hat{k}$

[:ANS] **C**



[:SOLN]

\therefore Net magnetic field at O is

$$\vec{B}_{\text{net}} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$$

(\therefore Magnetic field at O due to current wire passing through O is zero)

$$\frac{\mu_0 I}{4} \left(\frac{L}{2} \right) (-\hat{k}) + \frac{\mu_0 I}{8} \left(\frac{L}{4} \right) (-\hat{k}) + \frac{\mu_0 I}{4\pi(L)} (\sin 45^\circ + \sin 0^\circ) (-\hat{k})$$

$$= - \left(\frac{\mu_0 I}{2L} + \frac{\mu_0 I}{2L} + \frac{\mu_0 I}{4\sqrt{2}\pi L} \right) \hat{k}$$

$$= \frac{-\mu_0 I}{L} \left(1 + \frac{1}{4\sqrt{2}\pi} \right) \hat{k}$$